Hierarchical networks: (information) diffusion and mapping

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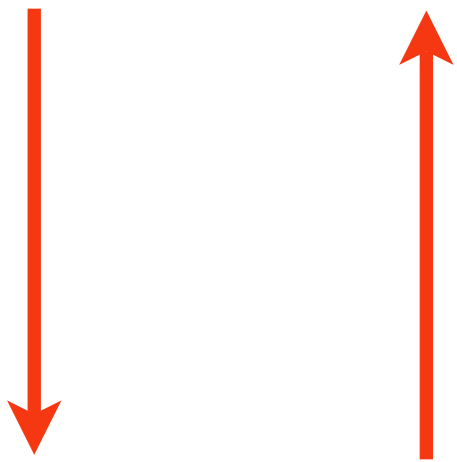
100 years of living science
100

Topological properties

(local, global)

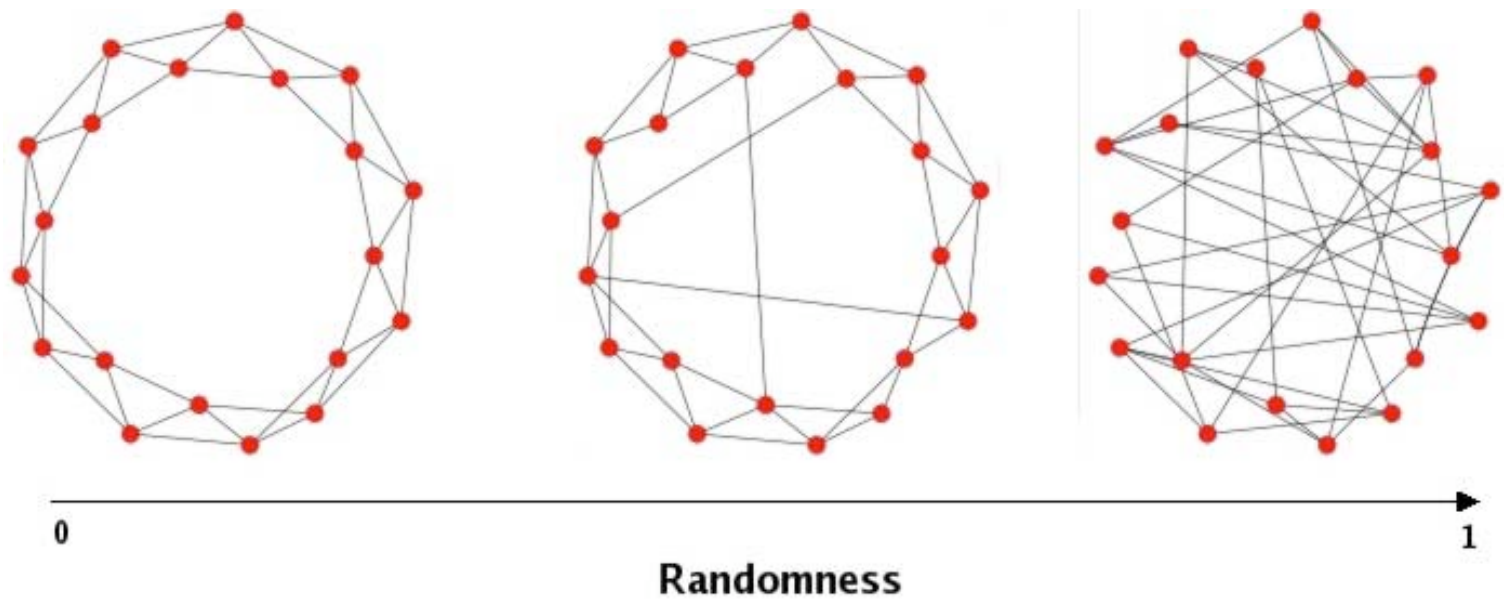


How does structure affect dynamics?

Can dynamics reveal the structure? (data mining)

Dynamical processes

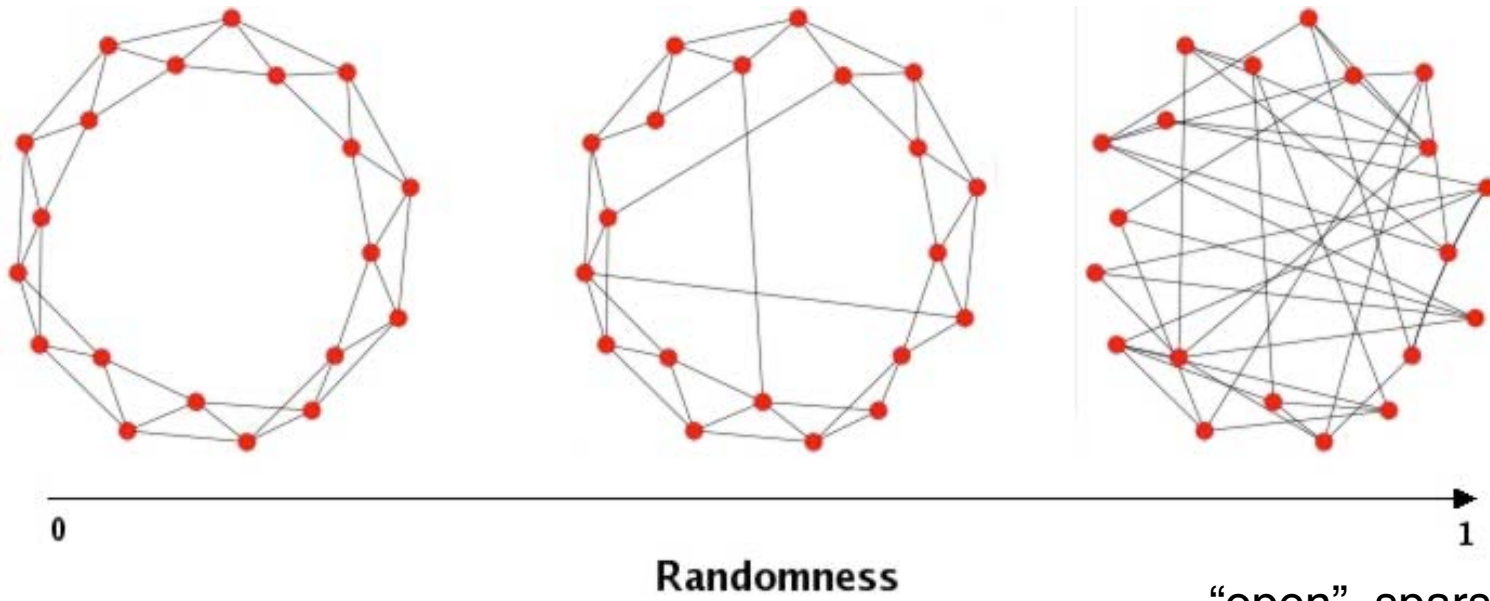
Universal properties (common mechanisms?)



Small-world: High clustering and short diameter

Information diffusion

What is the most efficient topology?



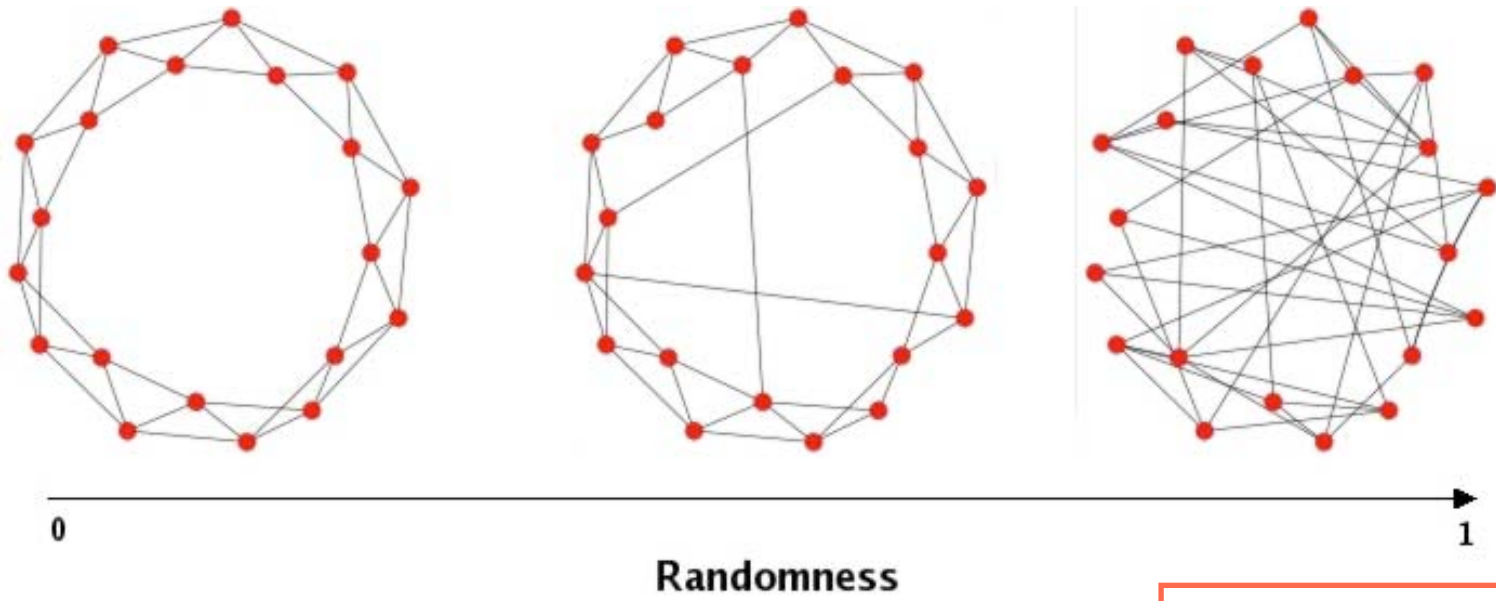
"closed", dense, or cohesive networks => likely to foster trust, facilitate the enforcement of social norms, and enable the creation of a common culture(Coleman, 1988)

opposite benefits

"open", sparse, or brokered networks: likely to connect people with different ideas, interests and perspectives (Burt, Granovetter)

Information diffusion

What is the most efficient topology?



"closed", dense, or cohesive networks => likely to foster trust, facilitate the enforcement of social norms, and enable the creation of a common culture

"open", sparse, or brokered networks: likely to connect people with different ideas, interests and perspectives

optimal for diffusive models (RW)

Information diffusion

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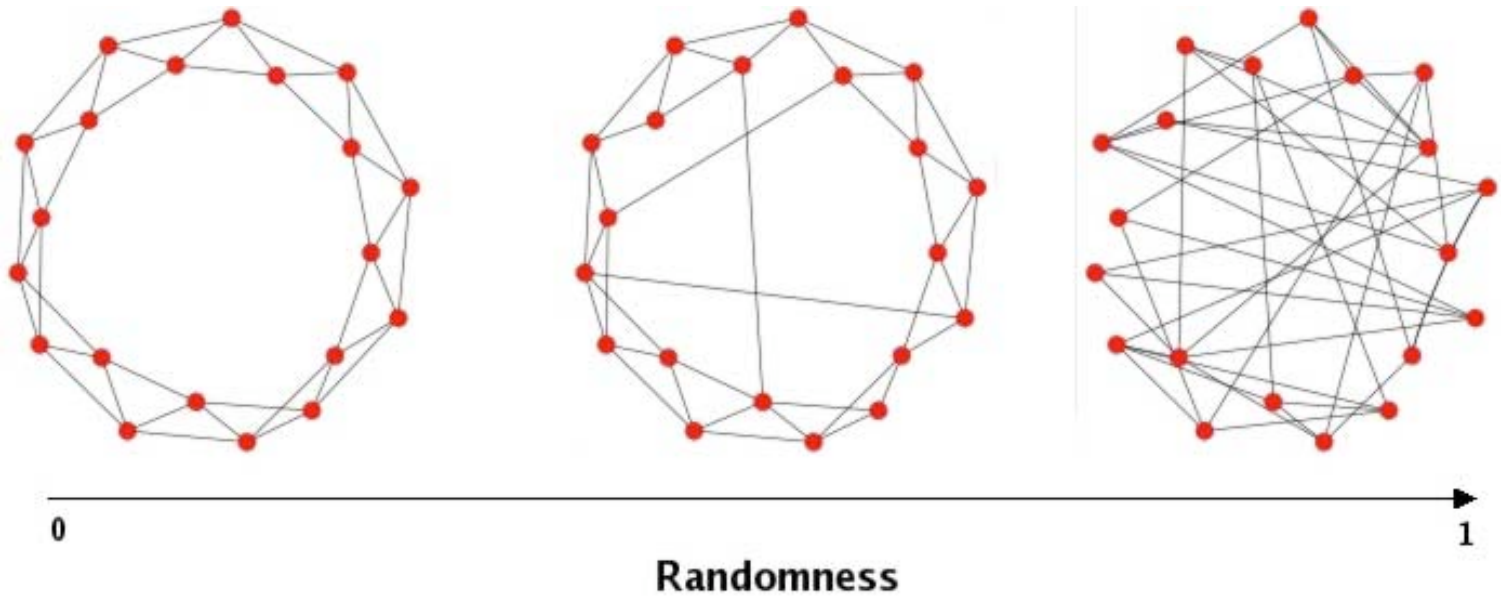


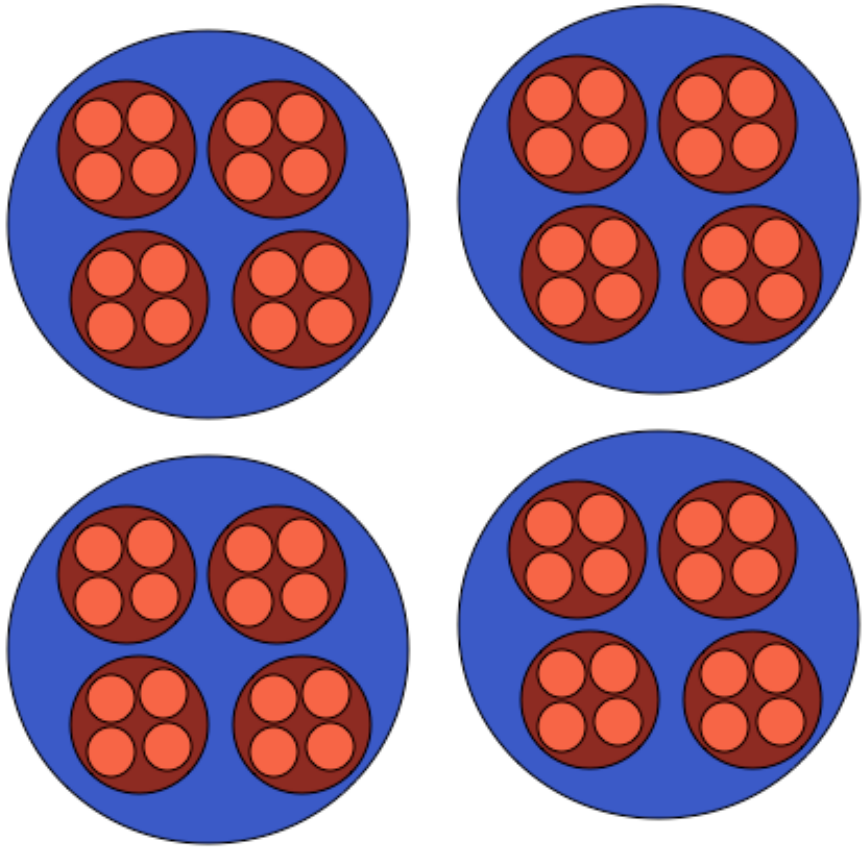
"closed", dense, or cohesive networks => likely to foster trust, facilitate the enforcement of social norms, and enable the creation of a common culture

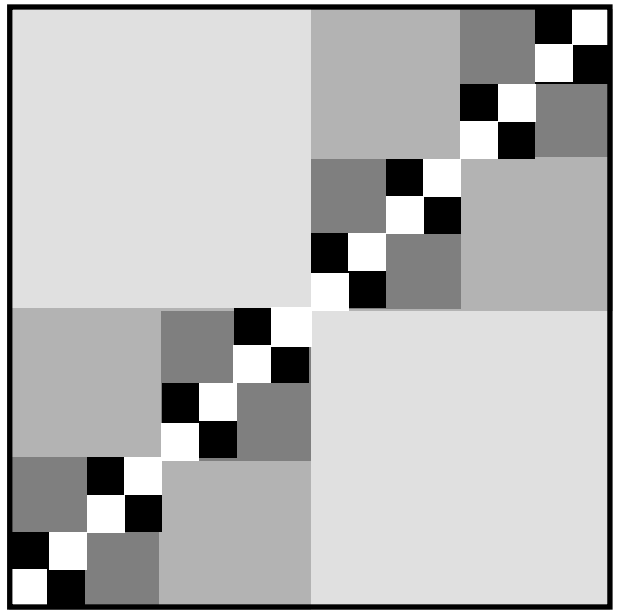
"open", sparse, or brokered networks: likely to connect people with different ideas, interests and perspectives

importance of redundancy/simultaneous exposure to accelerate diffusion

Cascade Dynamics of Complex Propagation. D. Centola, V. M. Eguiluz and M. W. Macy, Physica A 374,449-456 (2007).
The Role of Second Trials in Cascades of Information over Networks, C. de Kerchove et al., Phys. Rev. E, 79 (2009) 016114

Universal properties (common mechanisms?)



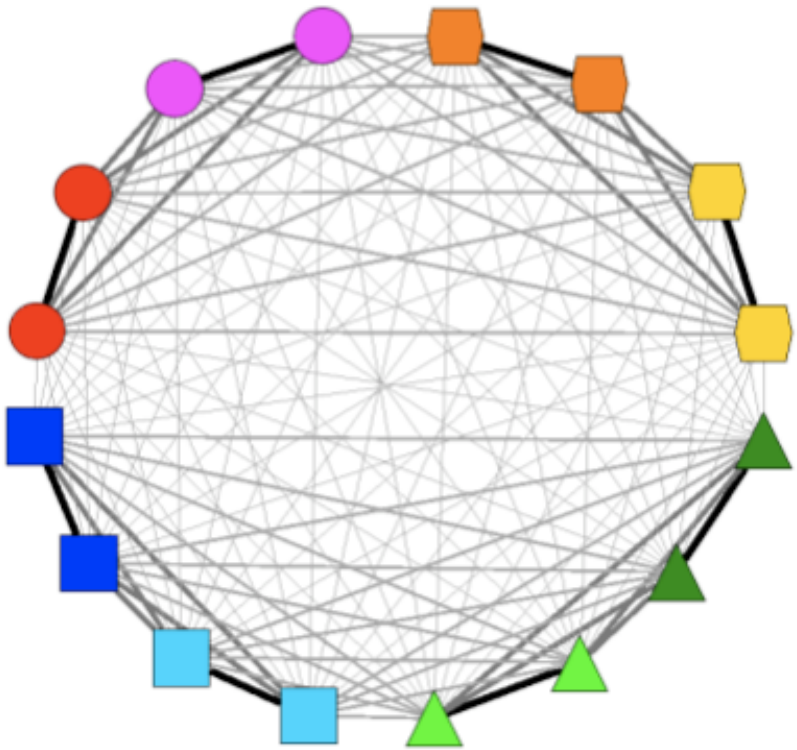


Hierarchical networks: modules within modules

Simon, H. (1962). The architecture of complexity. Proceedings of the American Philosophical Society, 106, 467–482.

Hierarchical networks

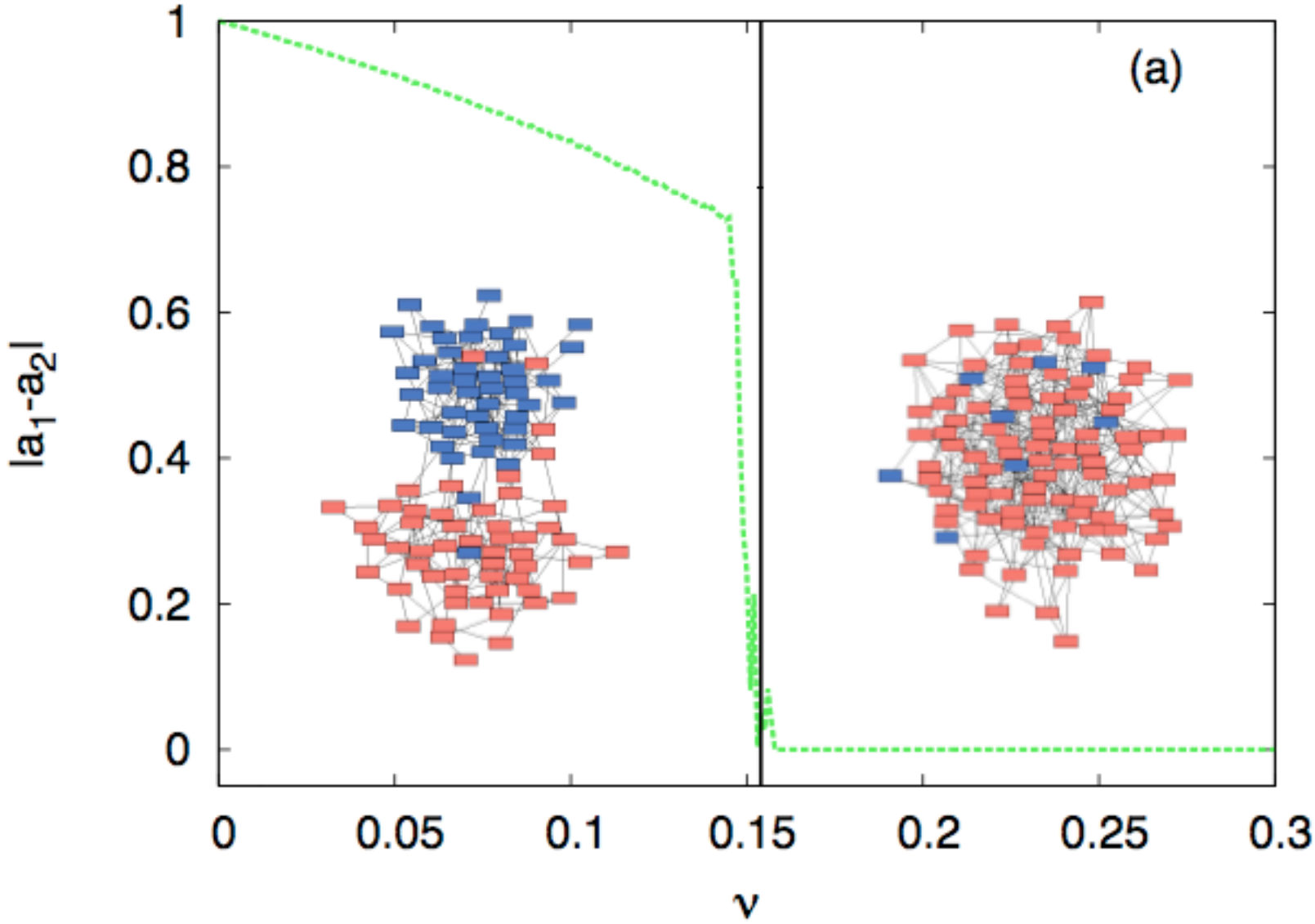
Small-world AND nearly-decomposable



The system is made of interacting building blocks at different scales
Robustness: The system is weakly affected if a block disappears
Global connectivity **and** Separation of (time) scales

Simon, H. (1962). The architecture of complexity. Proceedings of the American Philosophical Society, 106, 467–482.

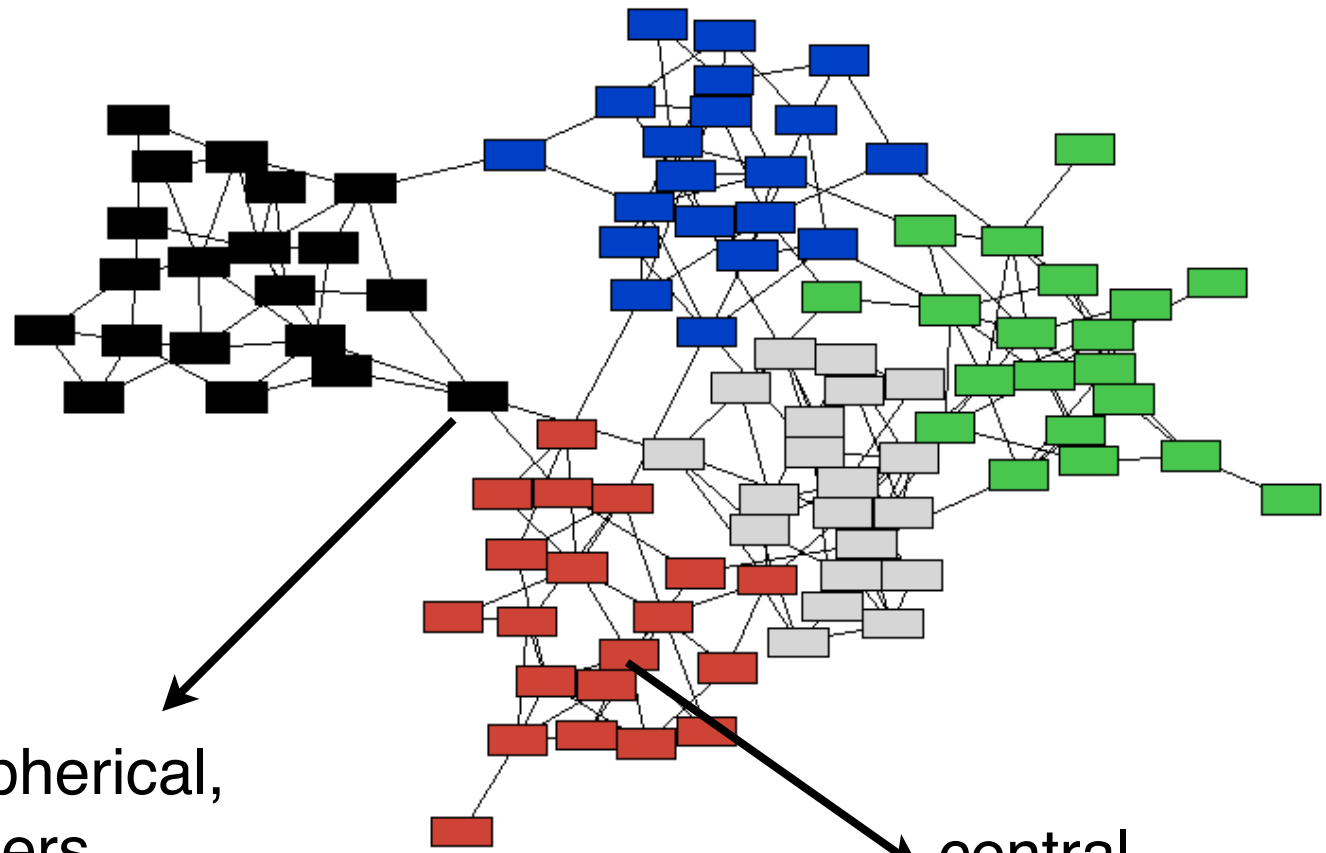
Separation of time scales: diversity



R. Lambiotte and M. Ausloos, J. Stat. Mech., (2007) P08026

Modular networks

Inhomogeneity: different location, different roles



peripherical,
brokers

central,
“core”,
cohesive

Communities, Knowledge Creation and Information Diffusion, R. Lambiotte and P. Panzarasa, Journal of Informetrics, 3 (2009) 180–190

Modular Networks

Uncovering communities/modules helps to understand the structure of the network, to uncover similar nodes and to draw a readable map of the network (when N is large).

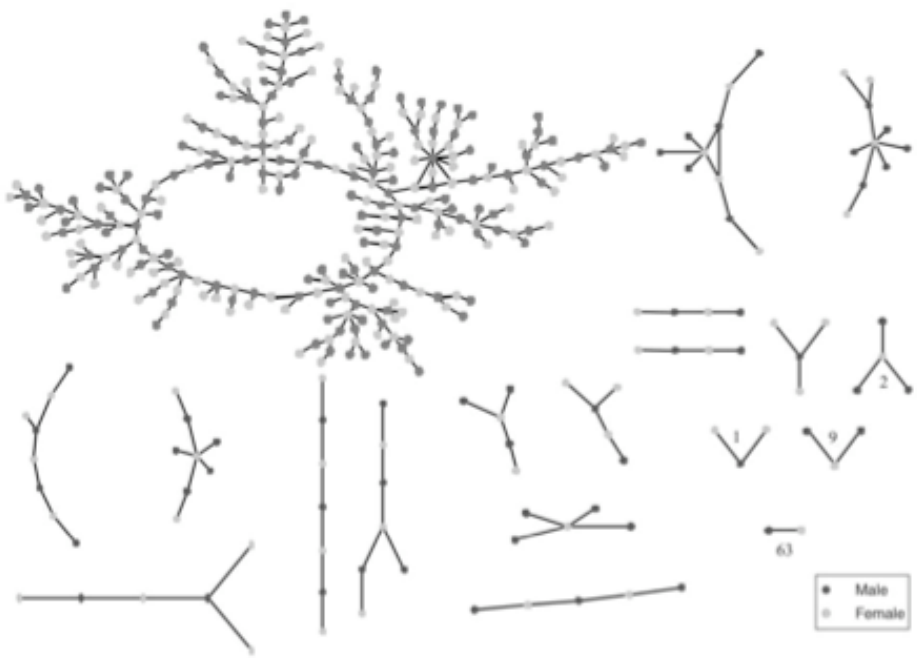
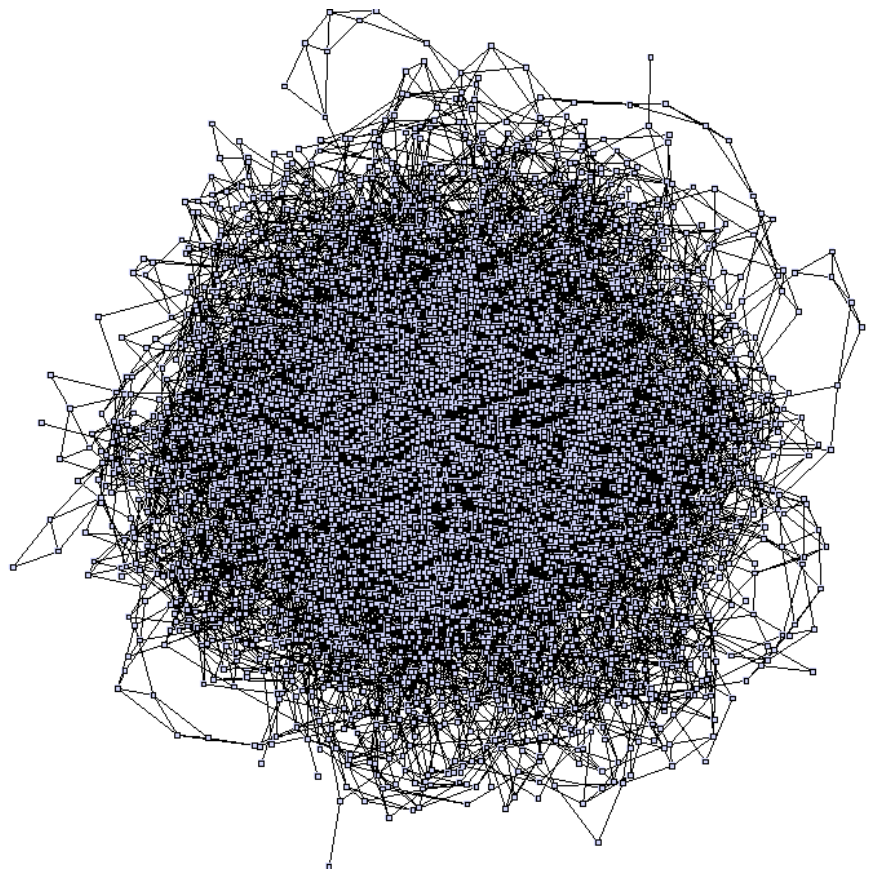


FIG. 2.—The direct relationship structure at Jefferson High



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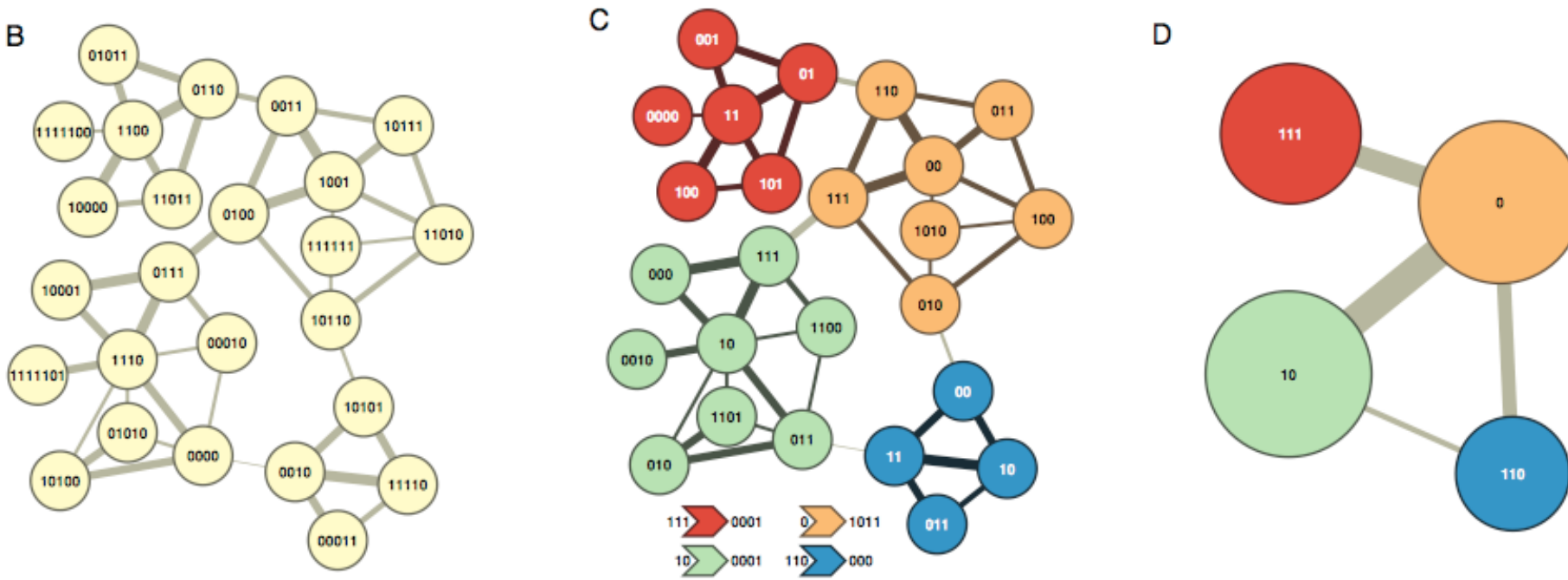
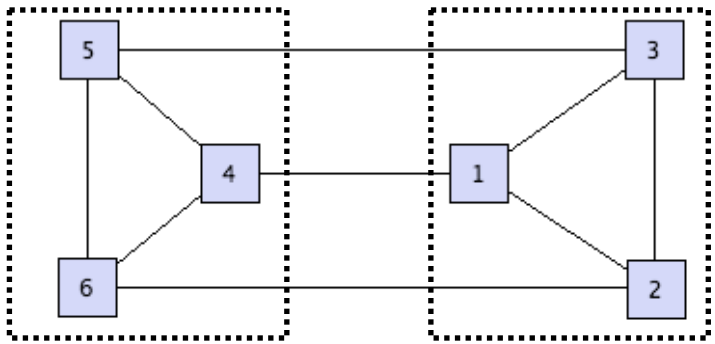


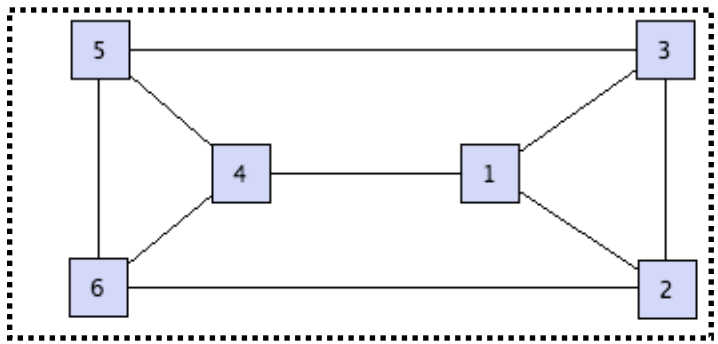
Martin Rosvall and Carl T. Bergstrom, PNAS 105, 1118 –1123 (2008)

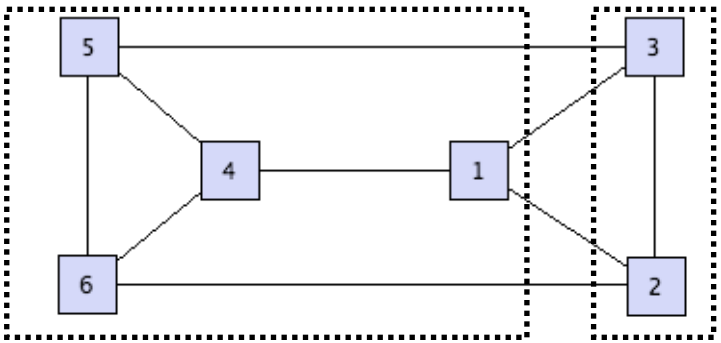
Quality of a partition

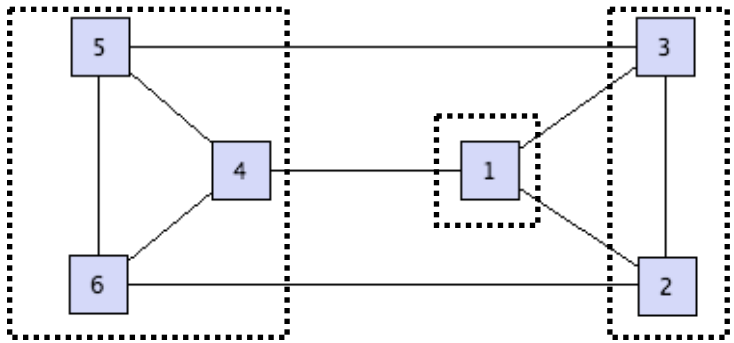
How good are those partitions of the same graph?

Among those, what is the best partition?









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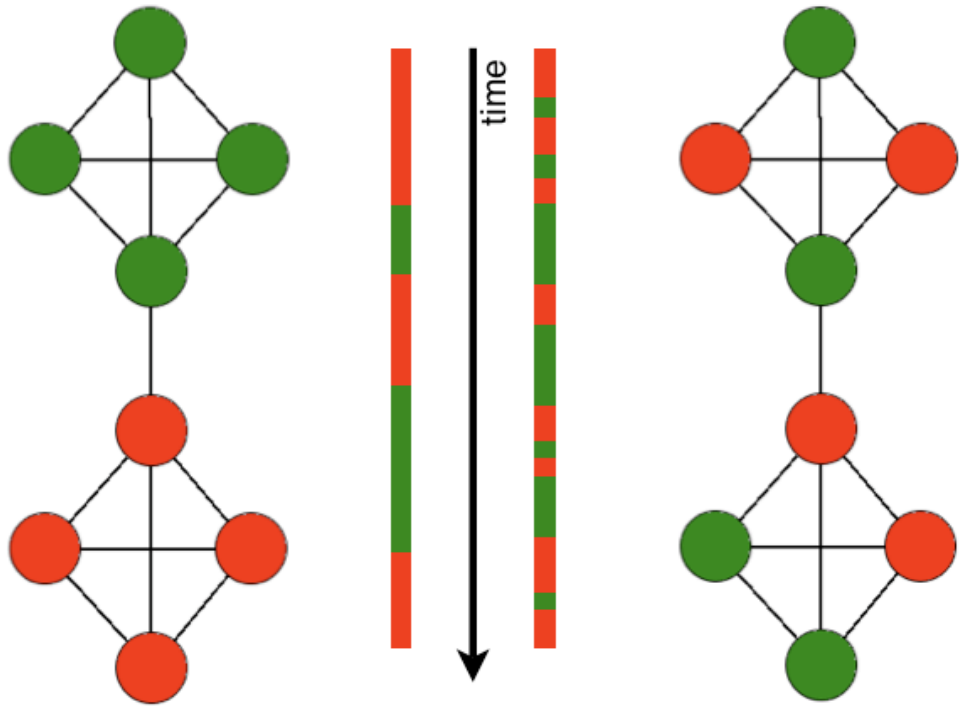
Stability

The quality of a partition is determined by the patterns of a flow within the network: a flow should be trapped for long time periods within a community before escaping it.

The stability of a partition is defined by the statistical properties of a random walker moving on the graph:



Stability: time as a resolution parameter

Let us consider a continuous-time random walk with Poisson waiting times

$$\dot{p}_i = \sum_j \frac{A_{ij}}{k_j} p_j - p_i \quad \xrightarrow{\text{equilibrium}} \quad p_i^* = k_i/2m$$

$$R(t) = \sum_{i,j} \left[\left(e^{t(B-I)}\right)_{ij} \frac{k_j}{2m} - \frac{k_i k_j}{(2m)^2} \right] \delta(c_i, c_j)$$

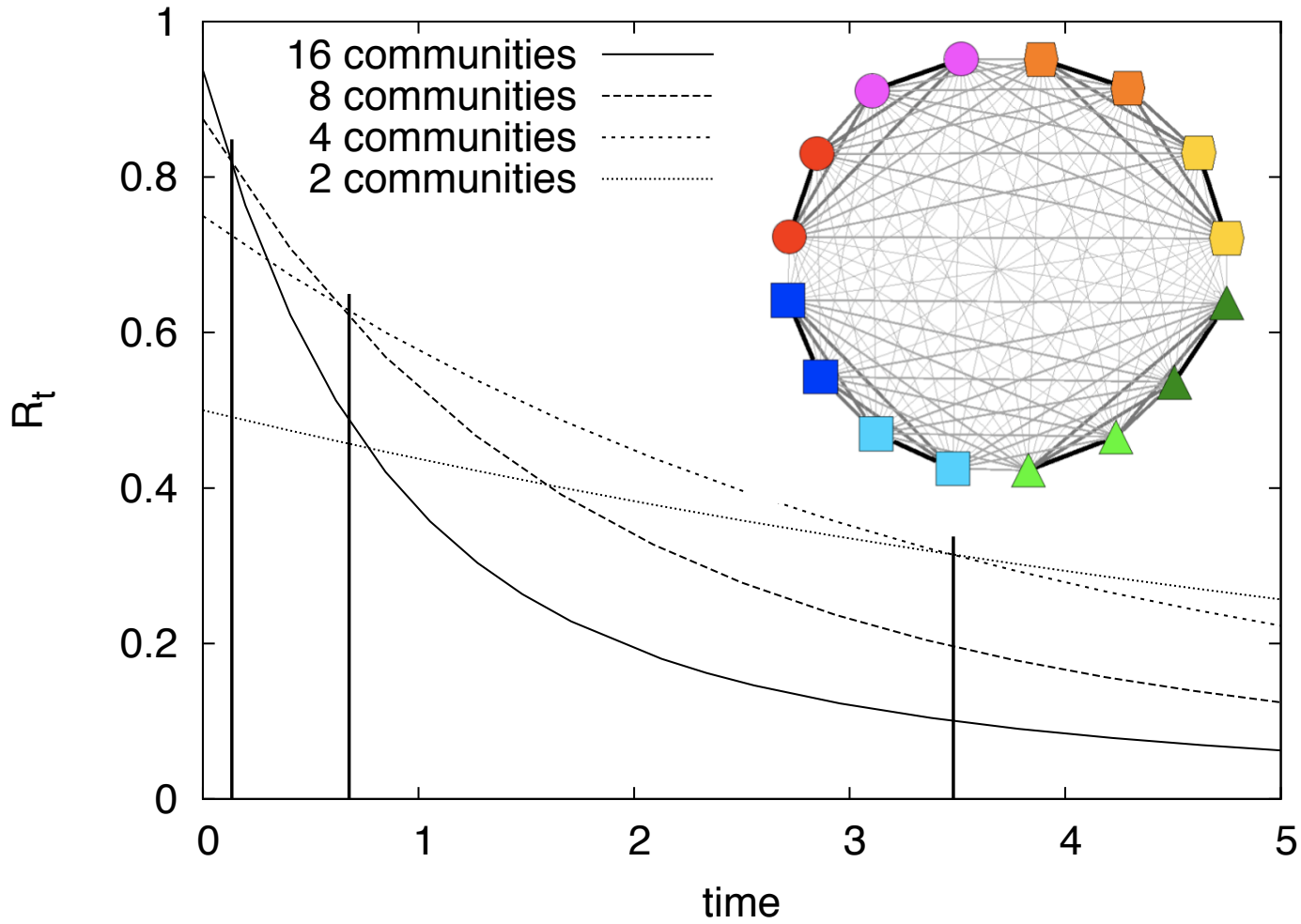
$$B_{ij} = A_{ij}/k_j$$

Probability that a walker is in the same community initially and at time t

Same probability for independent walkers

Stability: time as a resolution parameter

Time is a “resolution parameter”: larger and larger communities when time is increased



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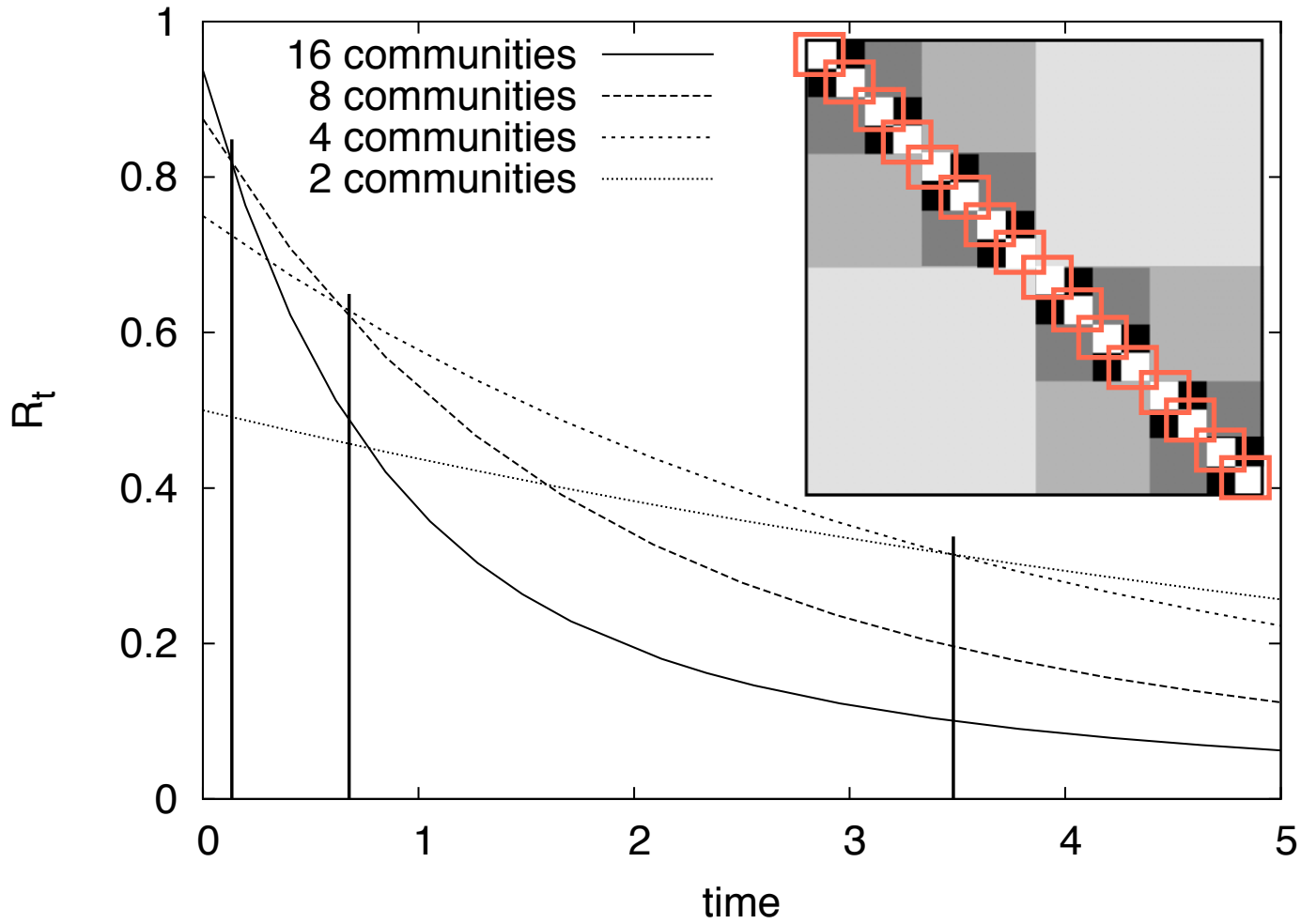

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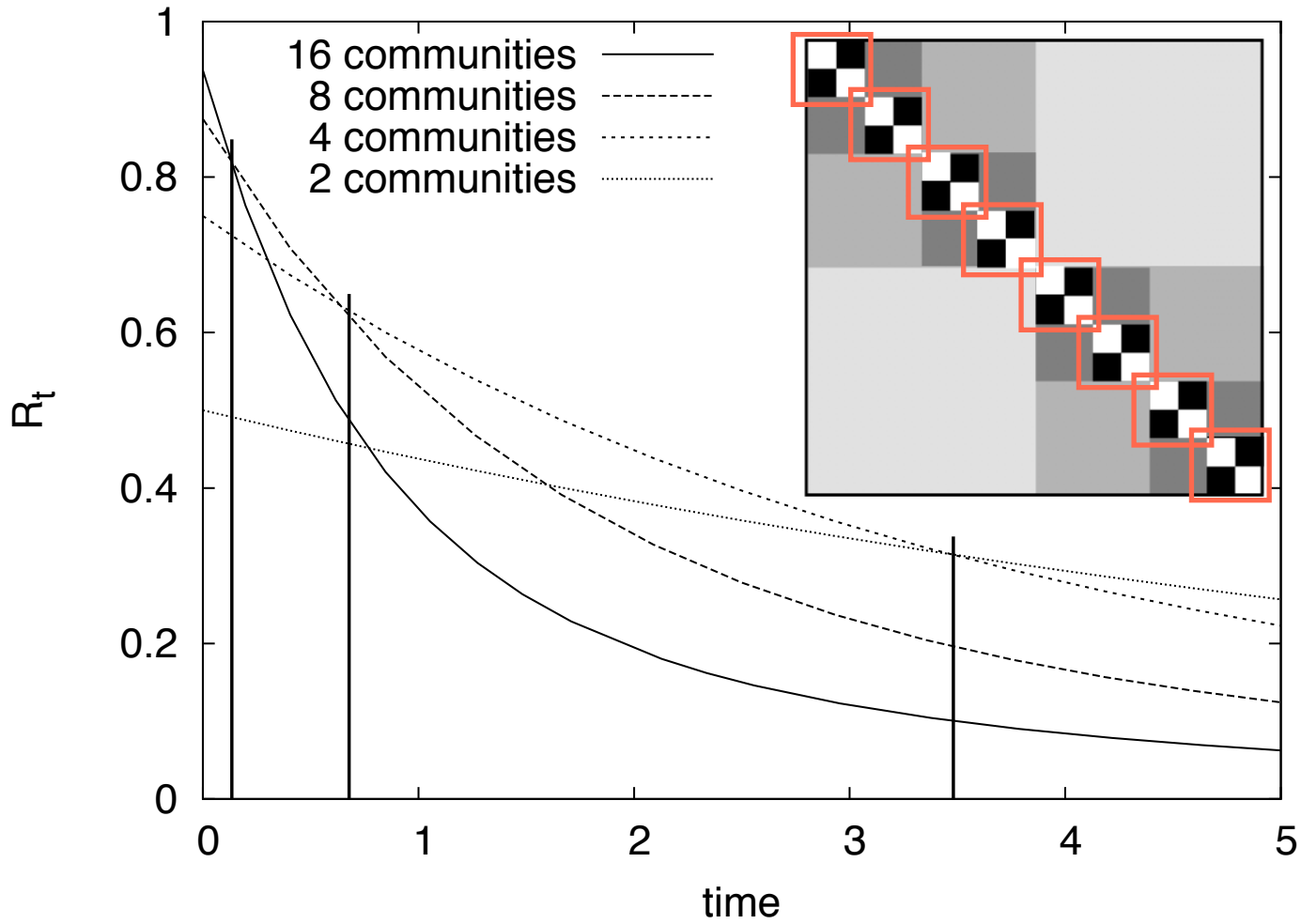

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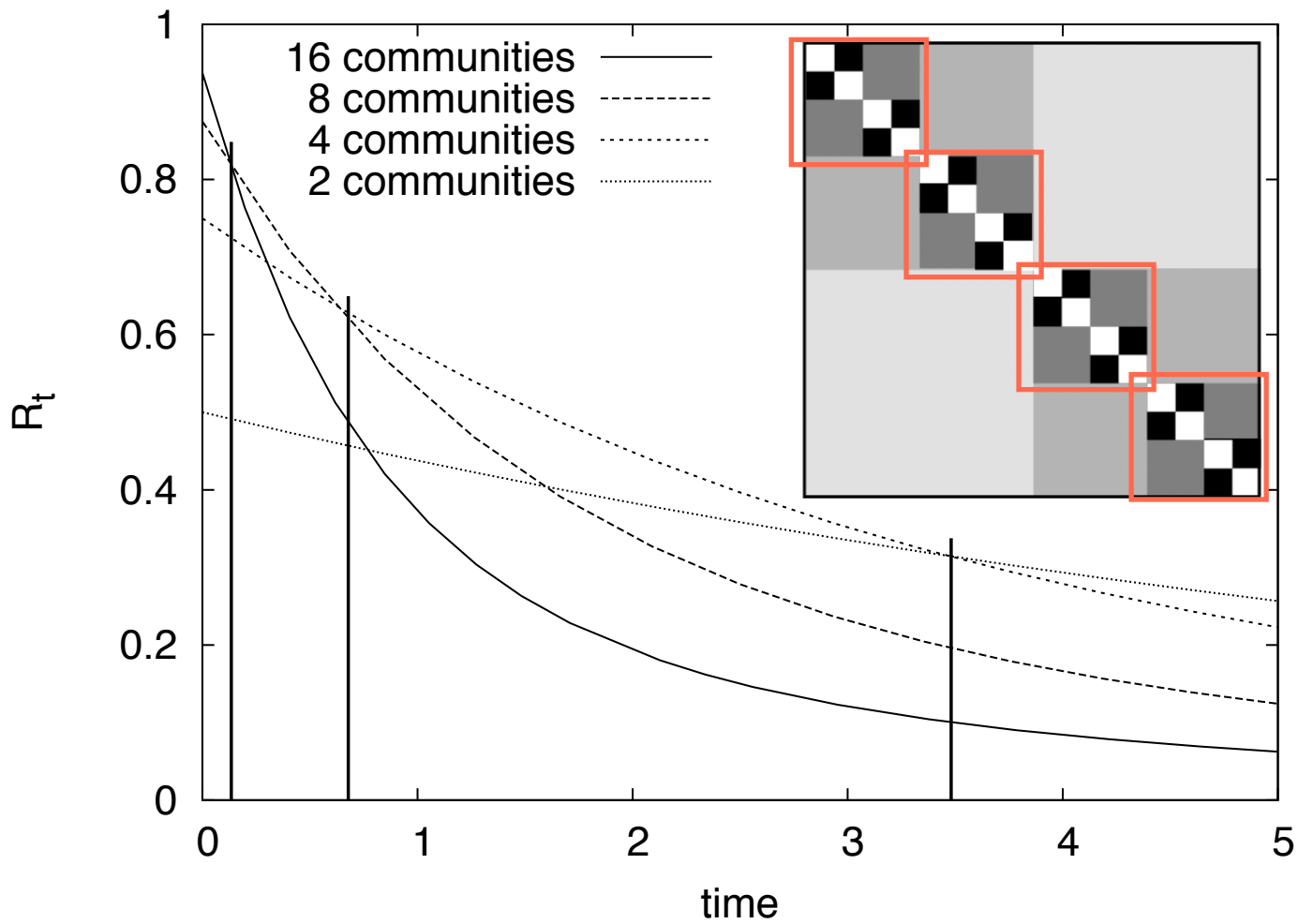

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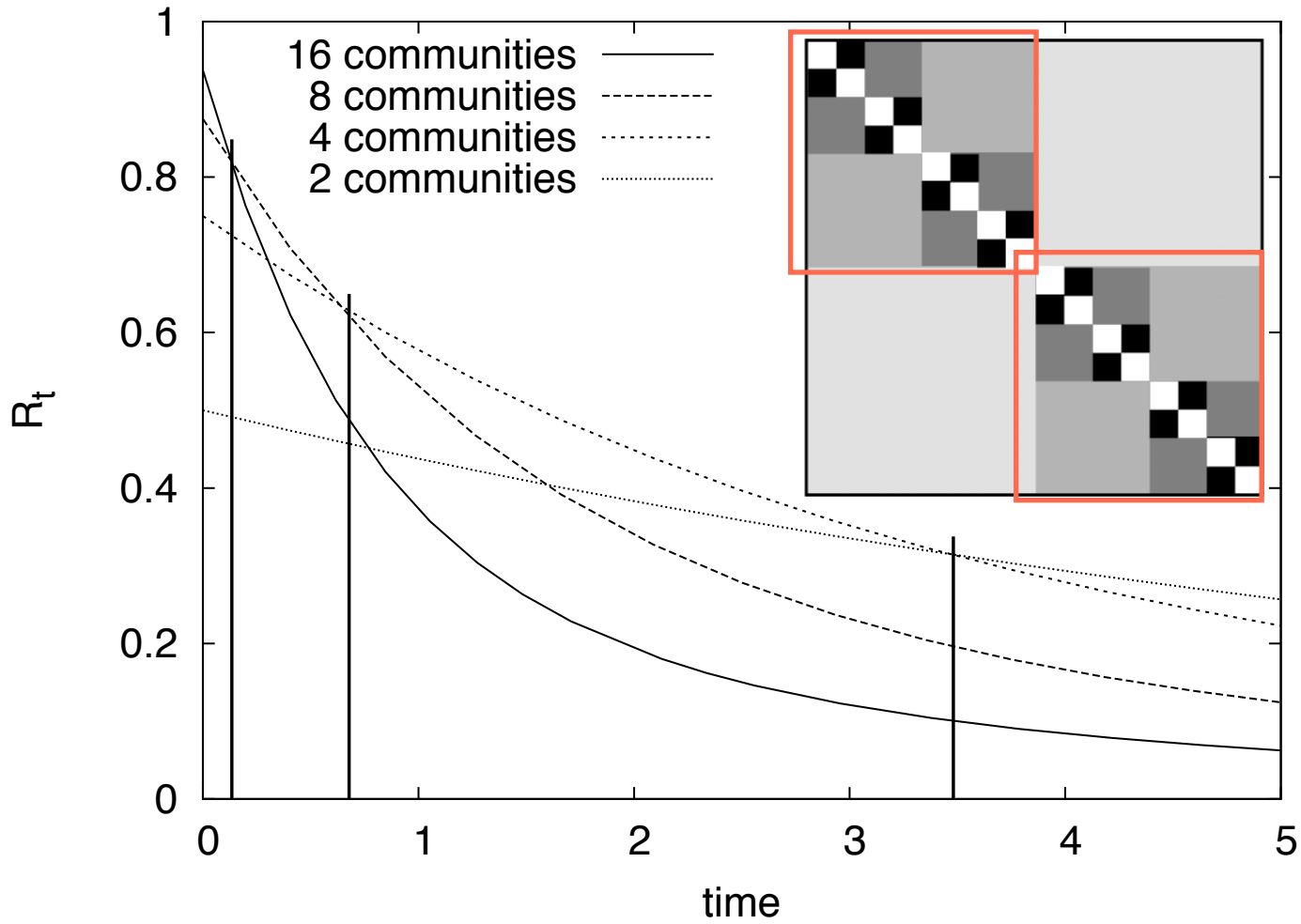


In practice: Selection of the most relevant scales

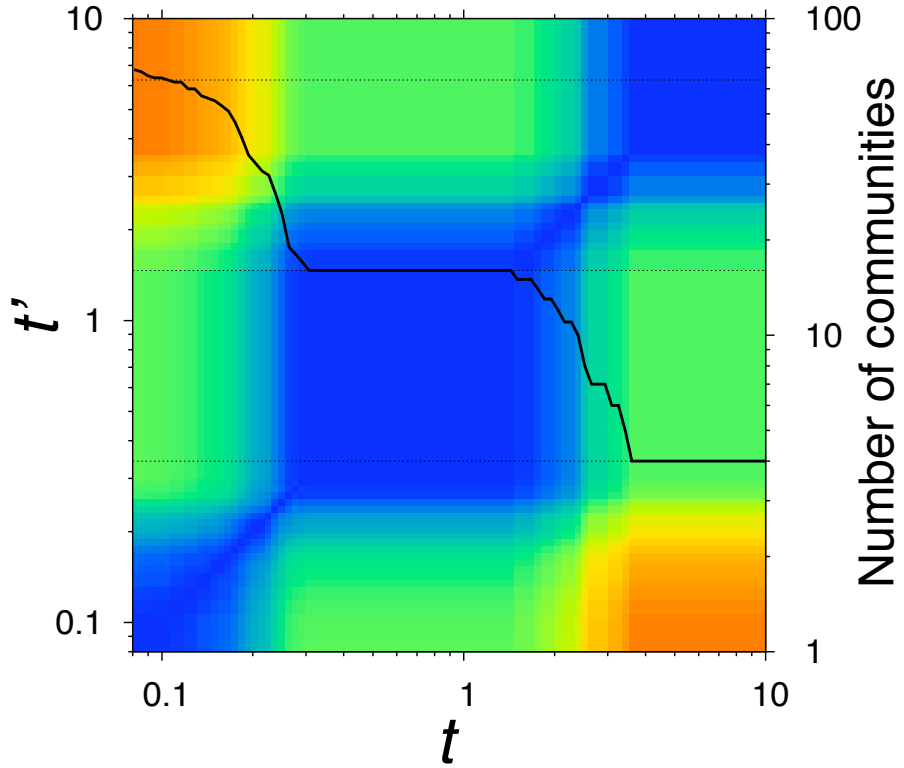
The optimization of *R(t)* over a period of time leads to a sequence of partitions that are optimal at different time scales.
How to select the most relevant scales of description?

The significance of a particular scale is usually associated to a certain notion of the robustness of the optimal partition. Here, robustness indicates that a small modification of the optimization algorithm, of the network, or of the quality function does not alter this partition.

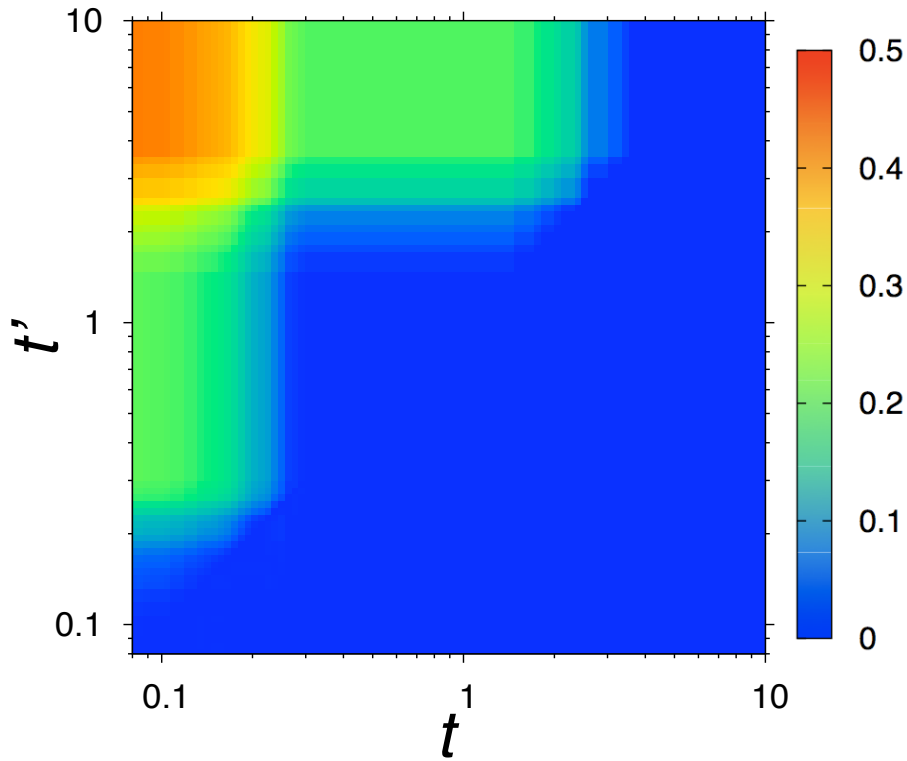
We look for regions of time where the optimal partitions are very similar. The similarity between two partition is measured by the *normalised variation of information*.

D.J. Fenn, M.A. Porter, M. McDonald, S. Williams, N.F. Johnson, N.S. Jones, *Chaos* 19, 033119 (2009).
M. Meilă, *J. Multivariate Anal.* 98, 873 (2007).

Normalised variation of information vanishes only if partitions P_t and $P_{t'}$ are identical.



Normalized conditional entropy vanishes only if each community of P_t is the union of communities of $P_{t'}$



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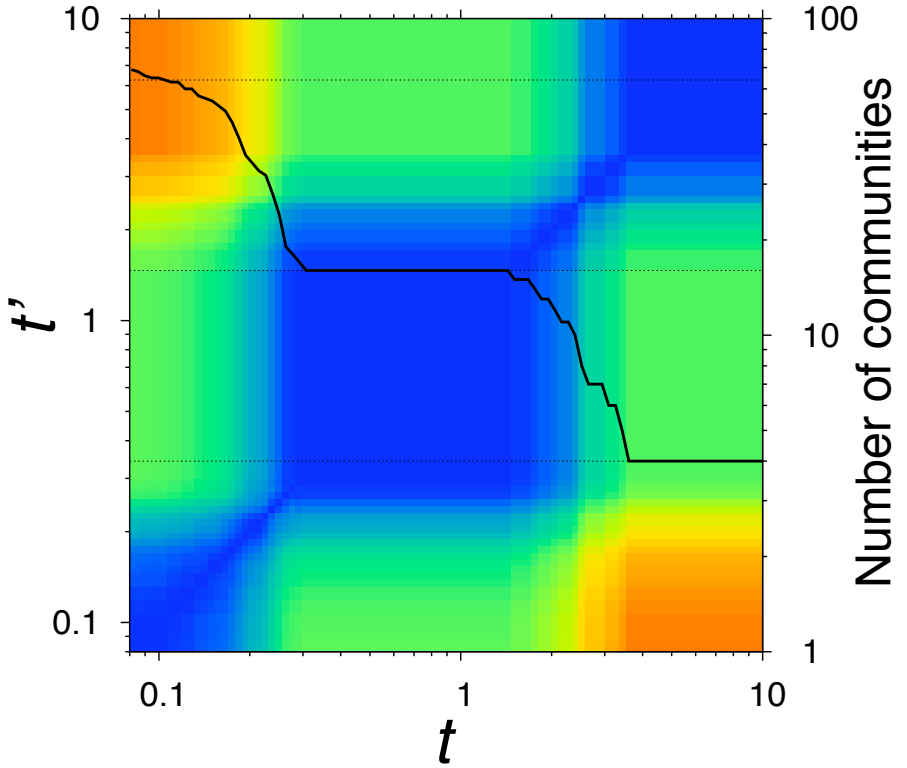
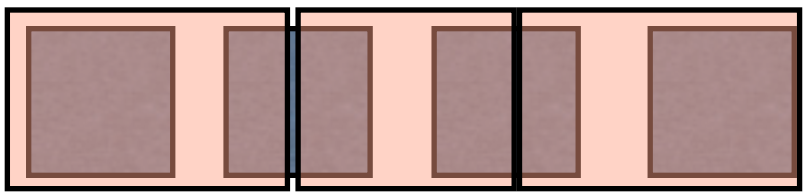
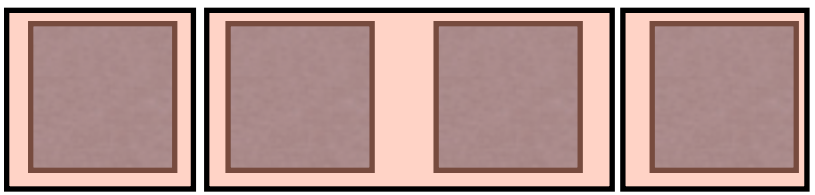
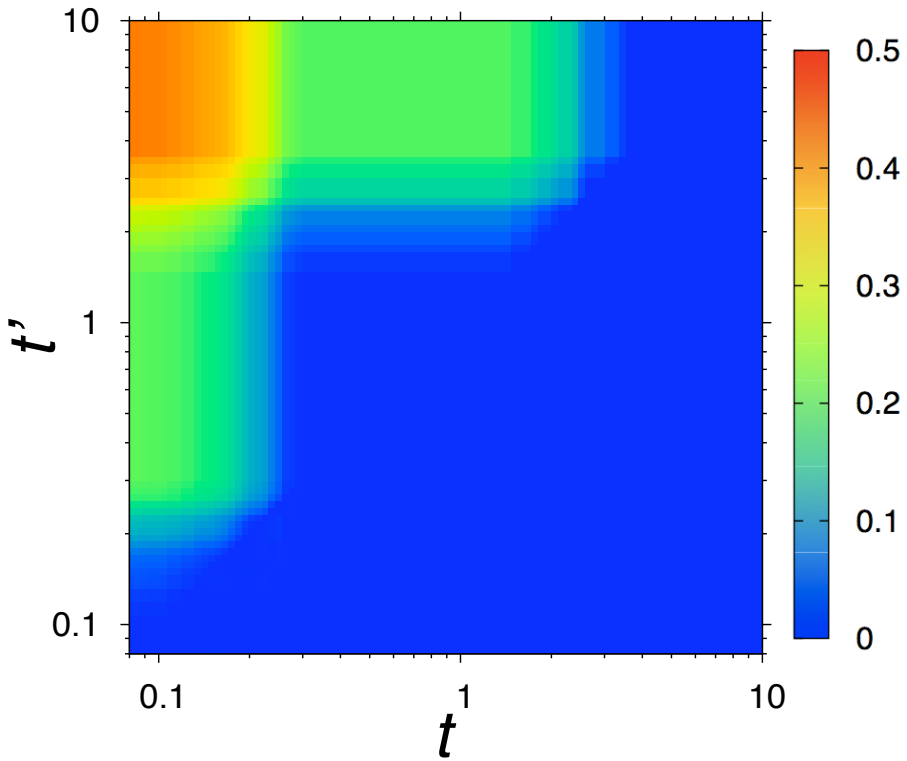


No hierarchy:



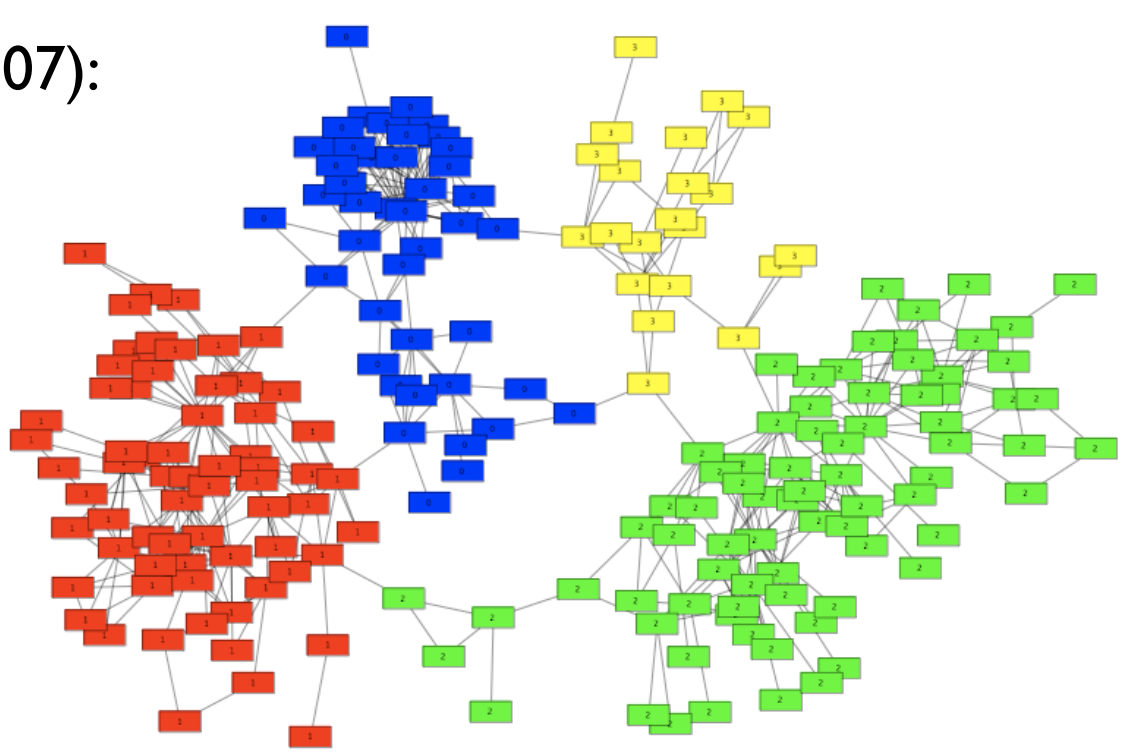
Hierarchy:

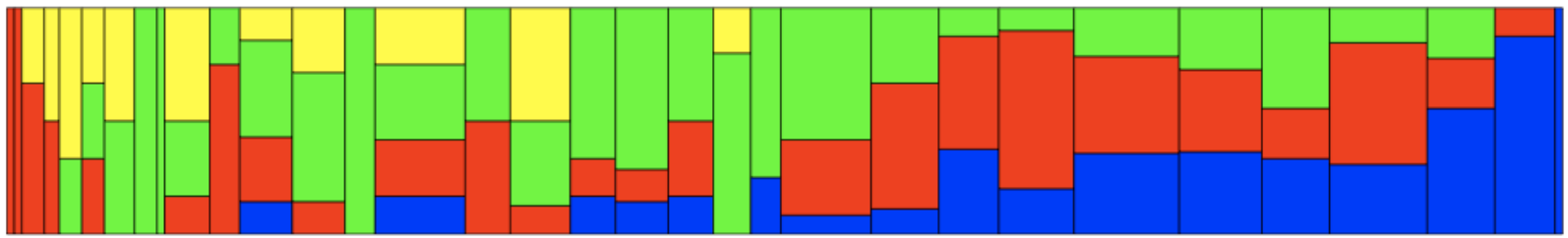




Uncovering researchers' mobility through self-citation networks

W. Ebeling (207):



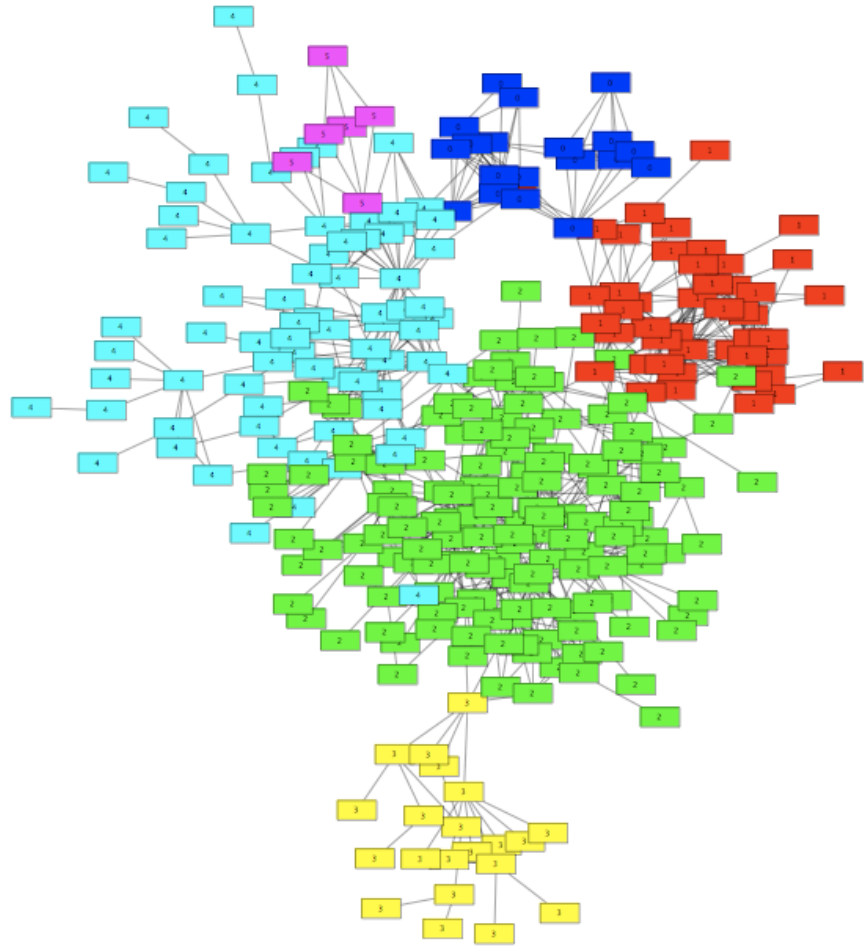


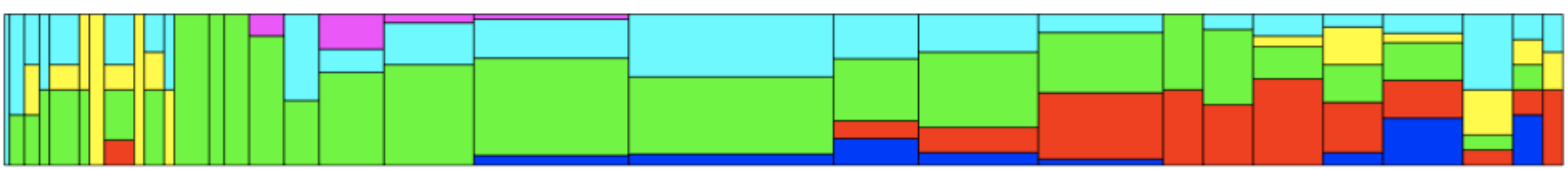
1974 *2006*

Self-Citations, Co-Authorships and Keywords: A New Method for Detecting Scientists' Field Mobility?, I. Hellsten, R. Lambiotte, A. Scharnhorst and M. Ausloos, Scientometrics, 72 (2007) 469

Uncovering researchers' mobility throughself-citation networks

M. Ausloos (312):





1976 *2007*

Thanks to:

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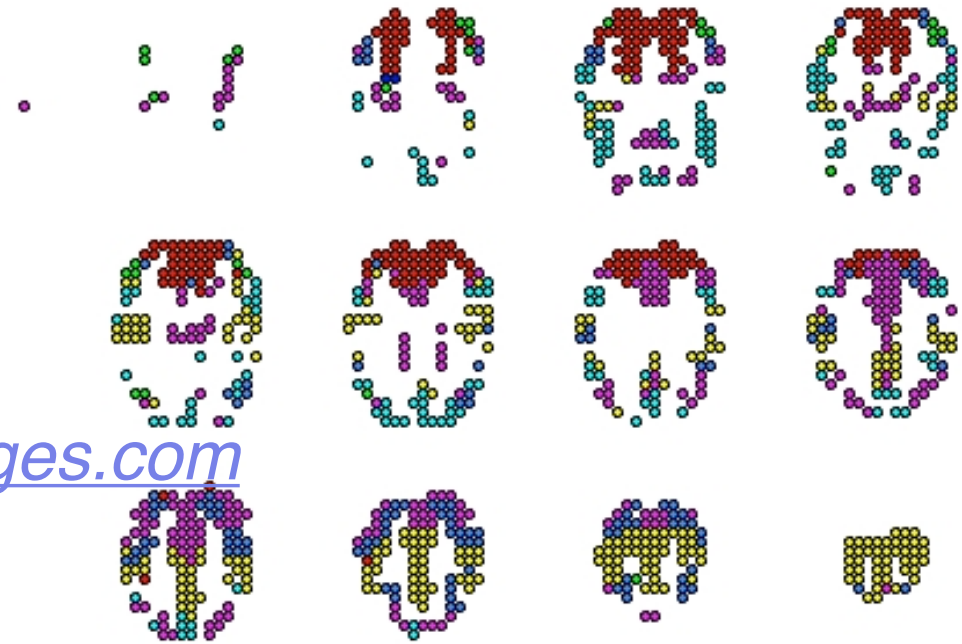
P. Panzarasa (Queen Mary)
D. Meunier (Cambridge)
V. Blondel (Louvain)
J.-L. Guillaume (Paris)
M. Ausloos (Liege)
A. Scharnhorst (Amsterdam)

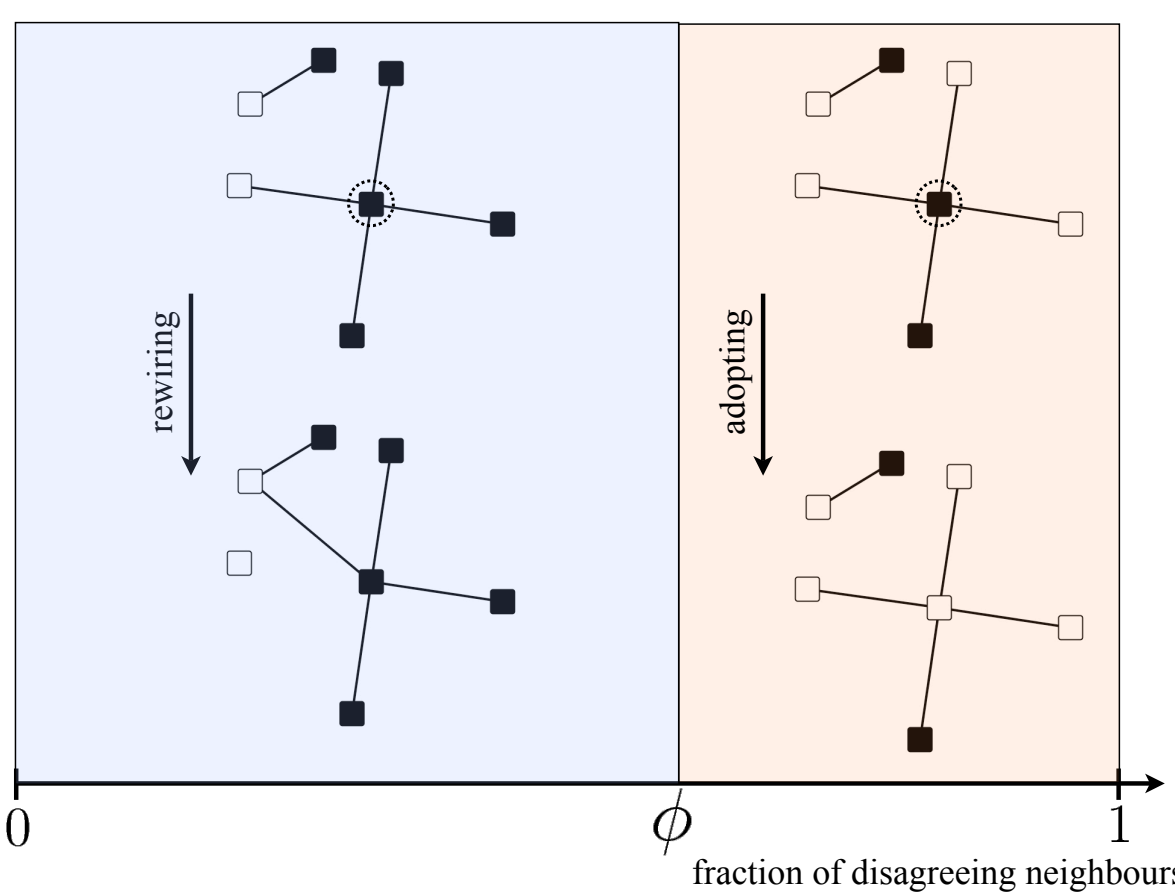


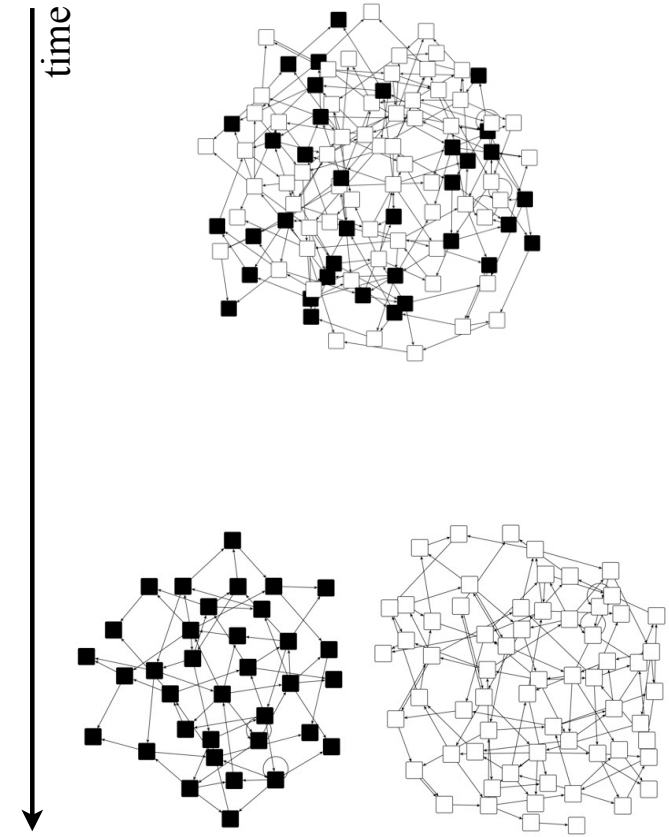
http://findcommunities.googlepages.com

http://www.lambiotte.be

- *Communities, Knowledge Creation and Information Diffusion, R. Lambiotte and P. Panzarasa, Journal of Informetrics, 3 (2009) 180–190*
- *Majority Model on a network with communities, R. Lambiotte, M. Ausloos and J. Holyst, Phys. Rev. E, 75 (2007) 030101(R)*
- *Self-Citations, Co-Authorships and Keywords: A New Method for Detecting Scientists' Field Mobility?, I. Hellsten, R. Lambiotte, A. Scharnhorst and M. Ausloos, Scientometrics, 72 (2007) 469*
- *Laplacian Dynamics and Multiscale Modular Structure in Networks, R. Lambiotte, J.-C. Delvenne and M. Barahona, arXiv:0812.1770*
- *Fast unfolding of community hierarchies in large networks, V.D. Blondel, J.-L. Guillaume, R. Lambiotte and E. Lefebvre, J. Stat. Mech., (2008) P10008*
- Hierarchical modularity in human brain functional networks, D. Meunier, R. Lambiotte, A. Fornito, K.D. Ersche, E.T. Bullmore, *Frontiers in NeuroInformatics in press*
- Line Graphs, Link Partitions and Overlapping Communities, T. Evans and R. Lambiotte, *Phys. Rev. E*, **80** (2009) 016105

Coevolution





Coevolutionary Threshold Dynamics, R. Lambiotte and J.C. Gonzalez-Avella, arXiv:0906.3389