Majority Rule on Heterogeneous Networks

R. Lambiotte

In collaboration with M. Ausloos,
and J. Holyst

UNIVERSITÉ de Liège

CREEN

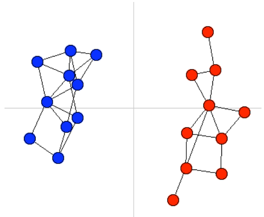
Contents

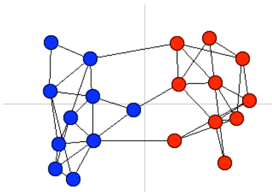
Majority Rule on heterogeneous networks

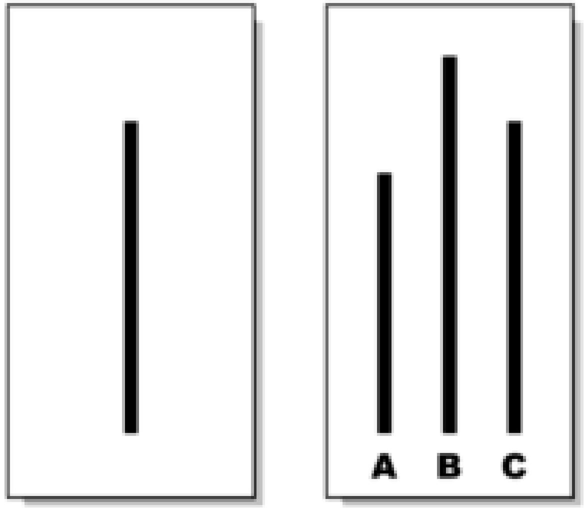
Opinion formation models, the "social atom": Voter, Galam, Sznajd, etc.

Simplest interaction possible: imitation. People copy the behaviour of their friend, acquaintances, neighbours, etc.

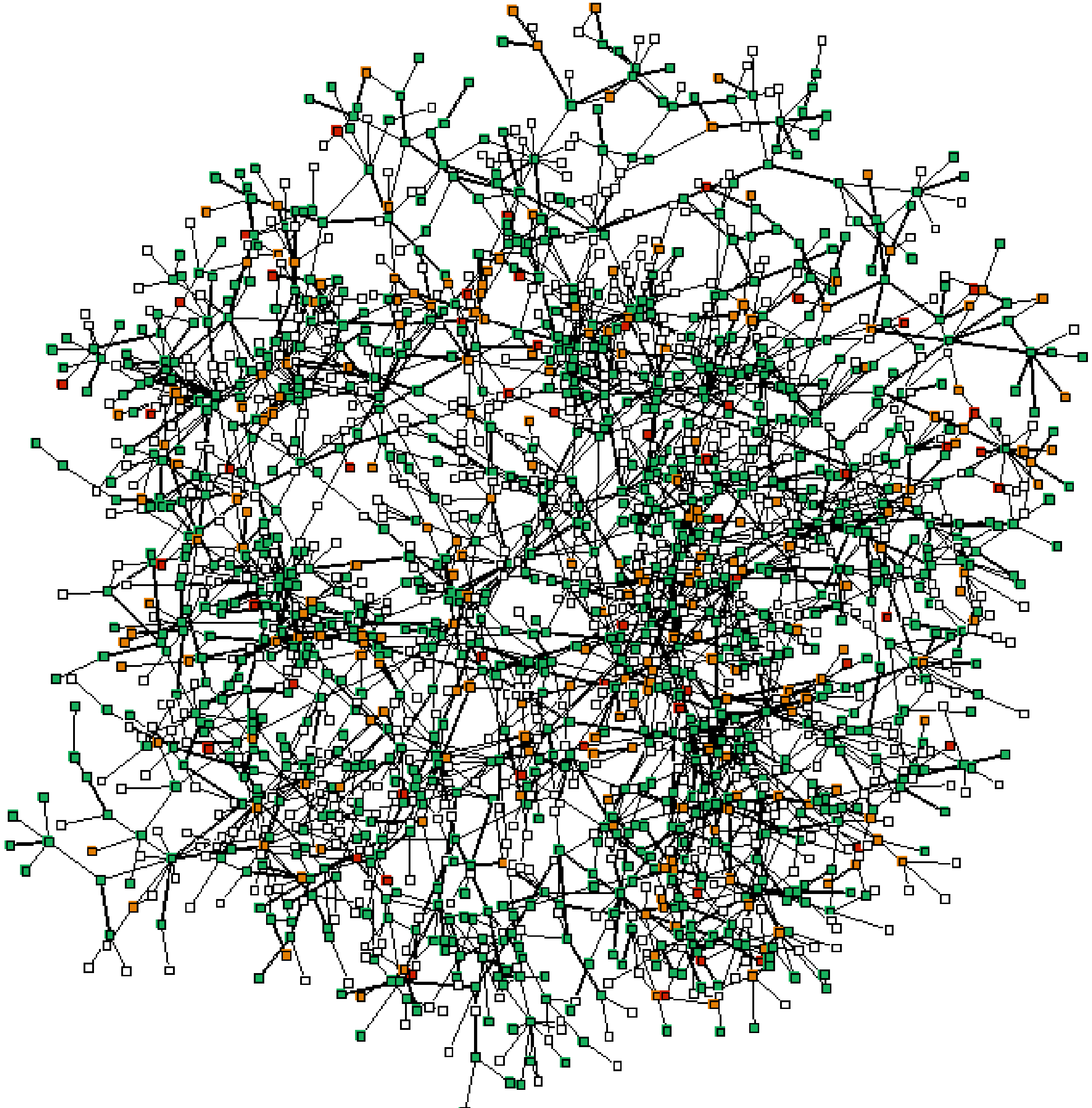
E.g. Solomon Asch experiment:





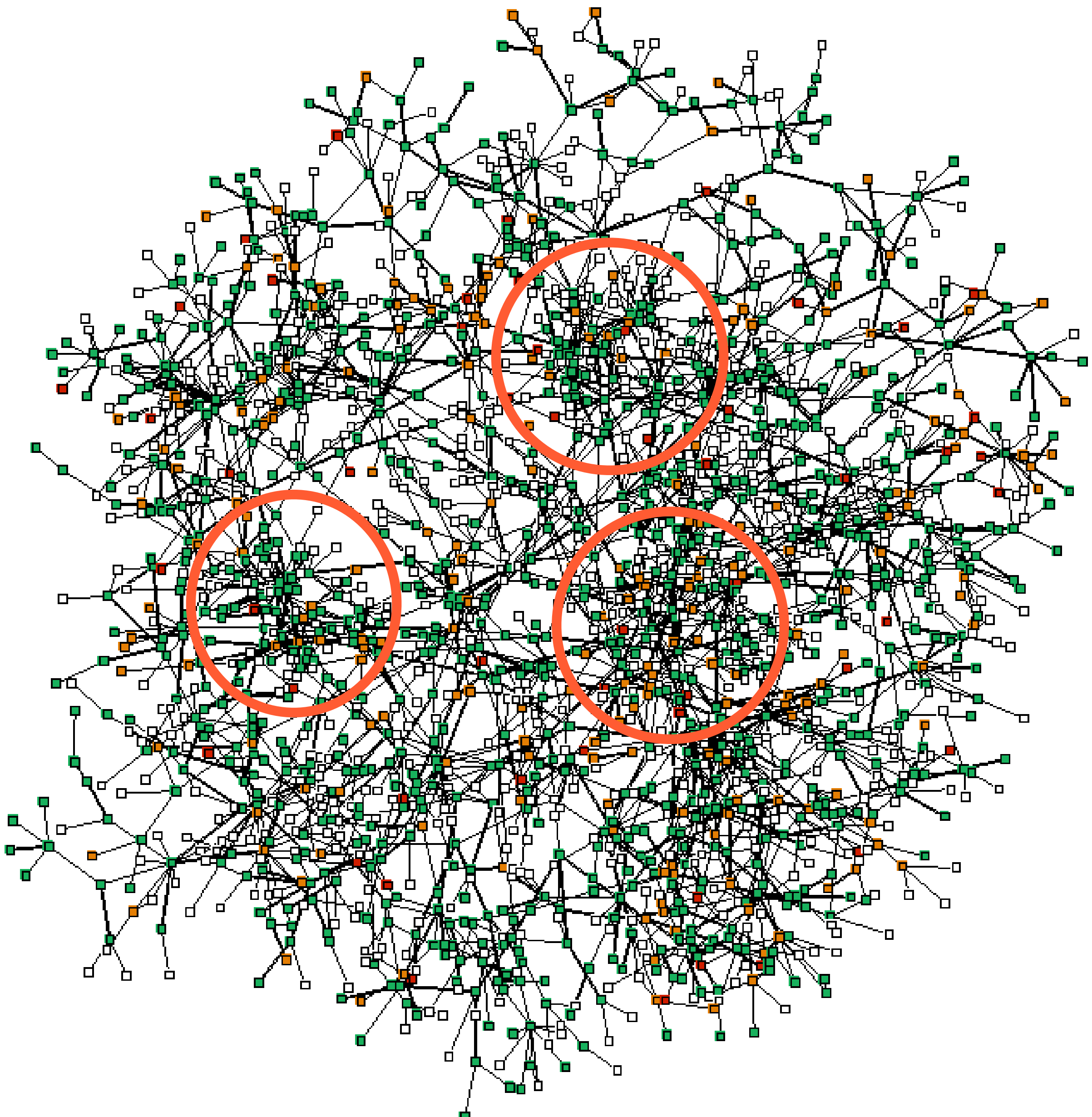


Role of the underlying topology



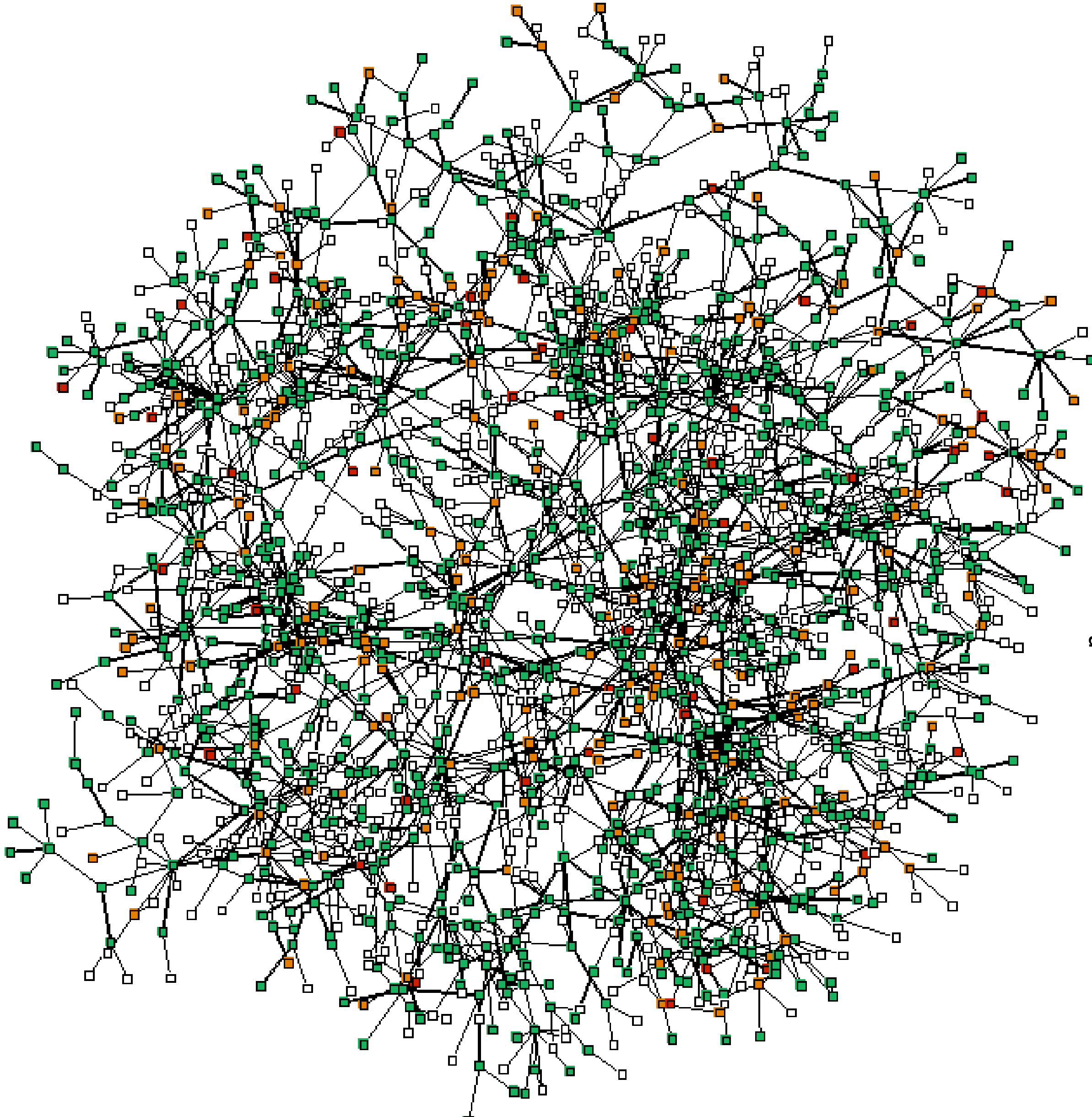
2000 users from a Belgian
Mobile Phone Network
1 day of communication
~ 1 million active users

Role of the underlying topology: modular structures

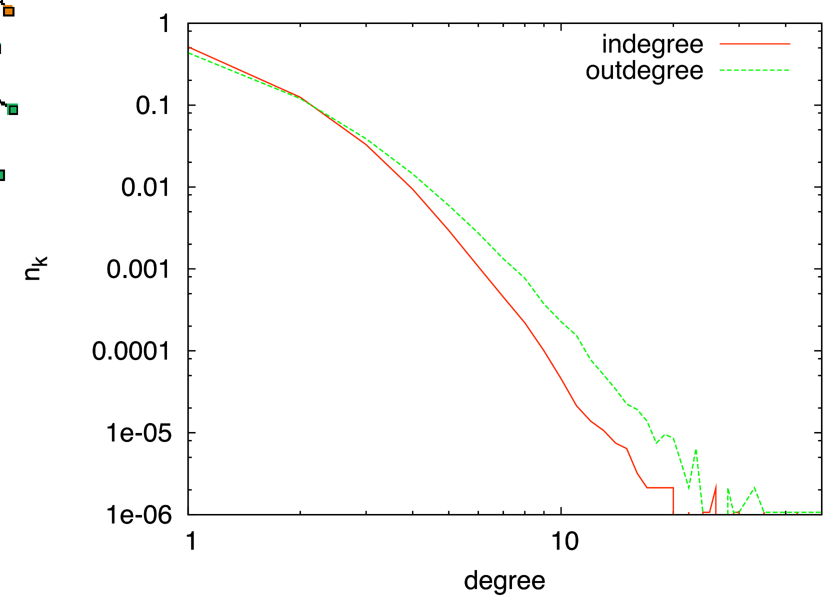


Highly connected communities, while nodes in different communities are sparsely connected

Role of the underlying topology: degree heterogeneity

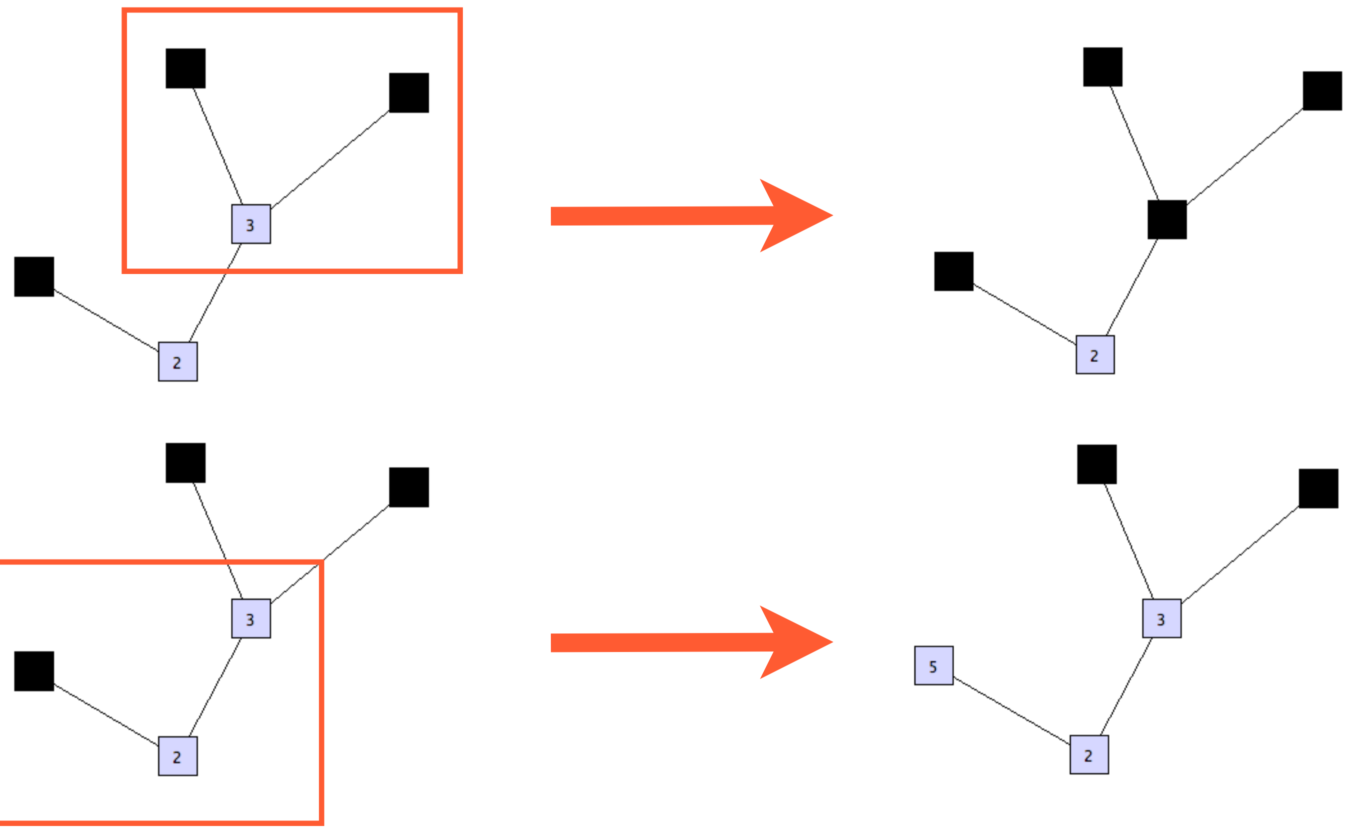


Broad degree distribution: presence of hubs



Majority Rule

At each time step, one node is selected. Two processes:
i) With probability q, random change
ii) Else, a discussion between 2 of its neighbours takes place and the local majority is reached



Mean-field: Fully-Connected Network

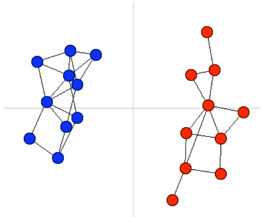
$$A_{t+1} = A_t + q(\frac{1}{2} - a_t) - 3(1-q)a_t(1 - 3a_t + 2a_t^2)$$

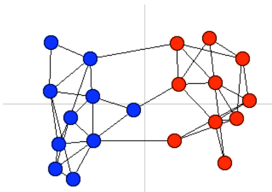
A_t average number of nodes with opinion α at time *t*

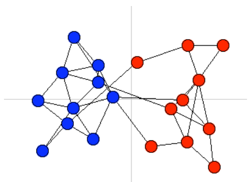
$a_t \equiv \frac{A_t}{N}$ average proportion of nodes with opinion α at time *t*

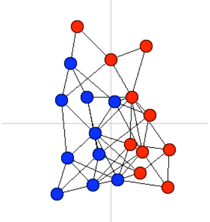
The disordered solution $a = \frac{1}{2}$ is always a stationary solution but it loses its stability below $q_c = \frac{3}{5}$. In that case, the system asymptotically reaches the ordered solution

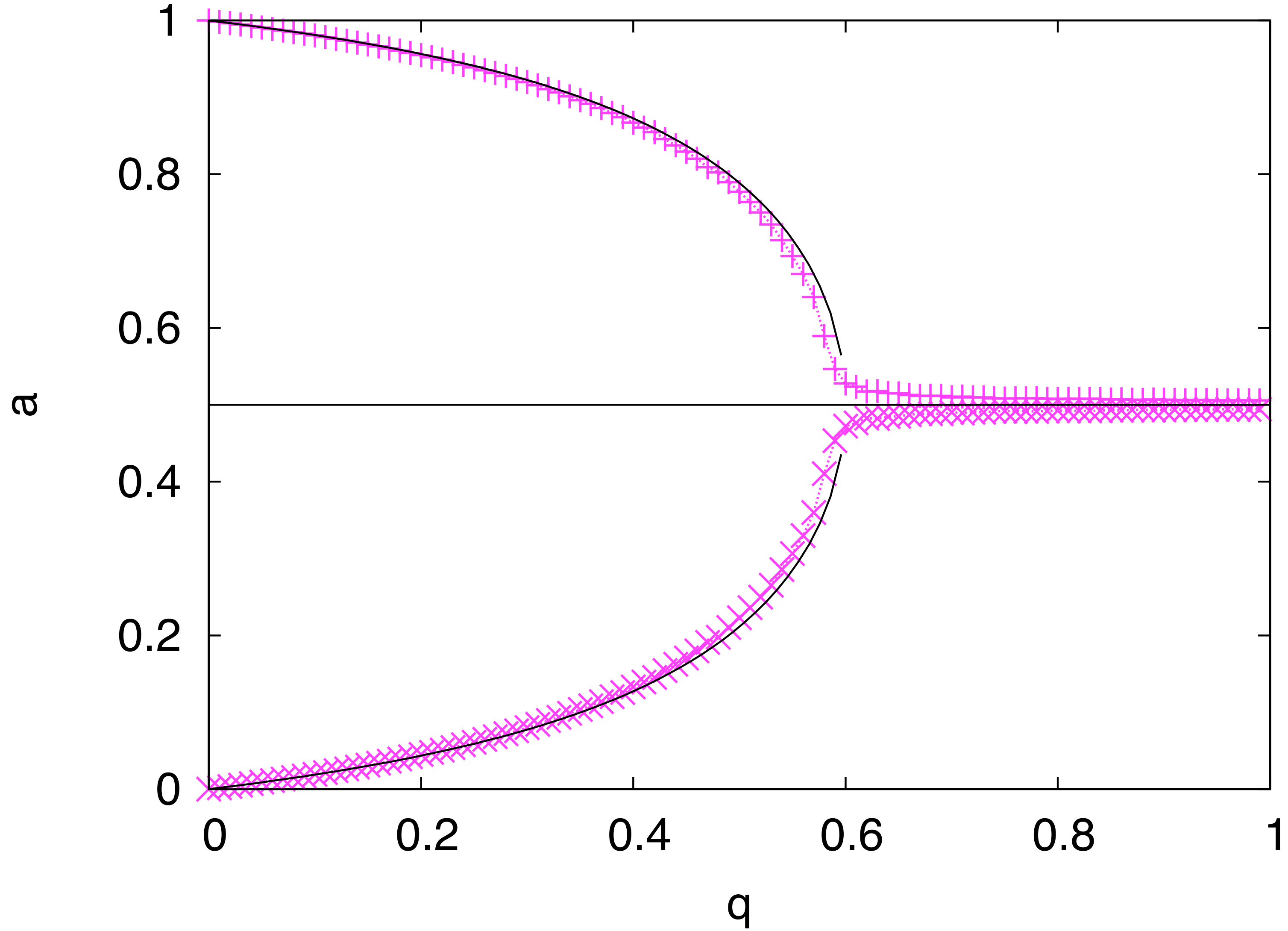
$$a_{\pm} = \frac{1}{2} \pm \sqrt{\frac{3-5q}{12(1-q)}}$$











Under the critical value, a collective opinion has emerged due to the *imitation* between neighbouring nodes.

Coupled Random Networks

Distinct communities within networks are defined as subsets of nodes which are more densely linked when compared to the rest of the network.

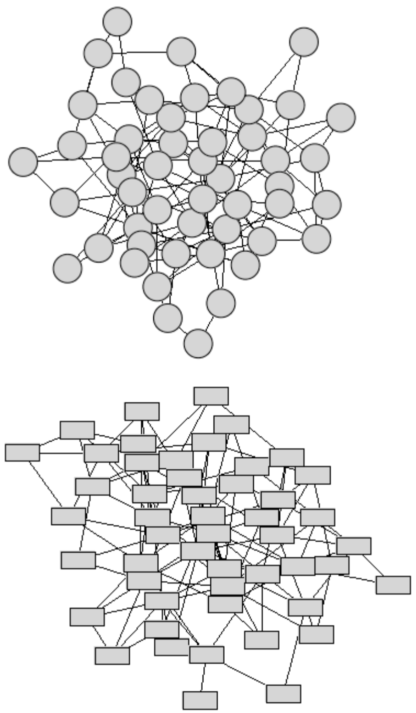
The network is composed of N nodes divided into two types of nodes, 1 and 2. Different types of nodes have a probability p_{cross} to be linked, while nodes of the same type have a probability p_{in} to be linked.

The inter-connectivity between the communities is tunable through the parameter

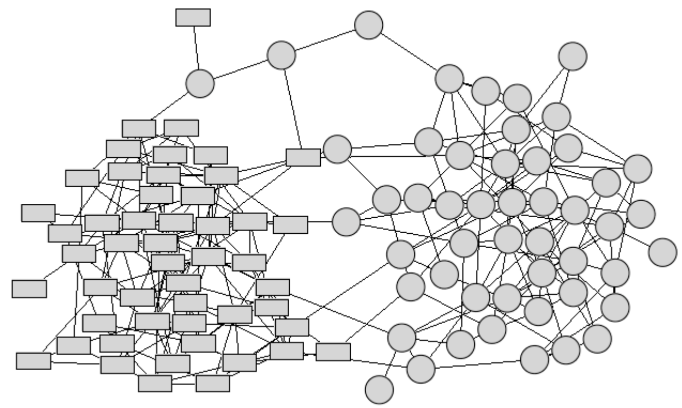
$$\nu = p_{cross}/p_{in}$$

Coexistence of opposite opinions in a network with communities, R. Lambiotte and M. Ausloos, submitted to *JSTAT* (2007) P08026
Majority Model on a network with communities, R. Lambiotte, M. Ausloos and J. Holyst, *Phys. Rev. E,* 75 (2007) 030101(R)

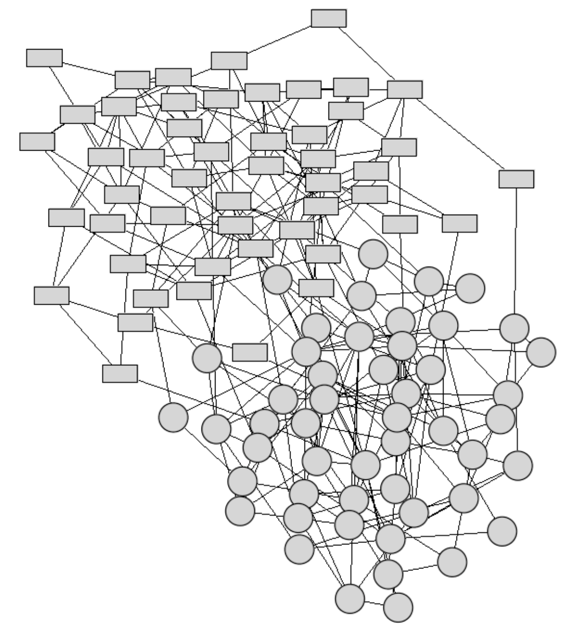
Coupled Random Networks



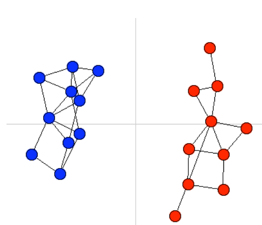
$\nu = 0$

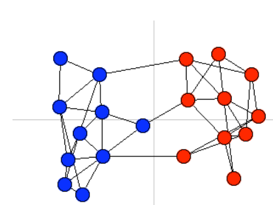


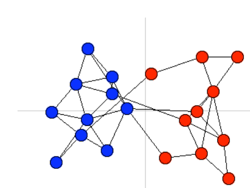
$\nu = 0.05$

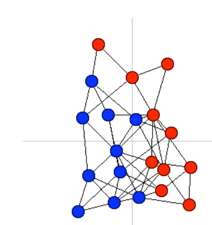


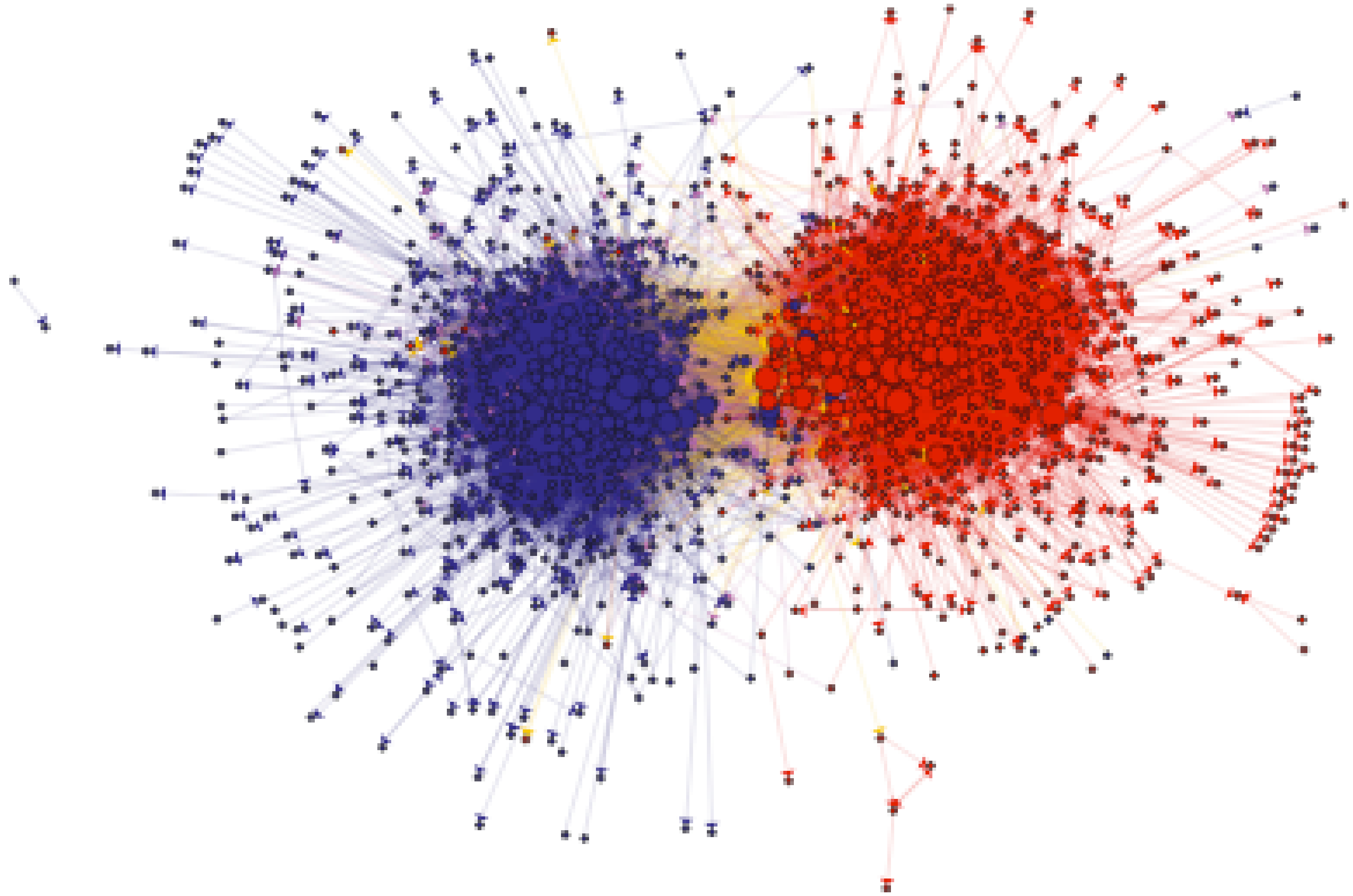
$\nu = 0.1$



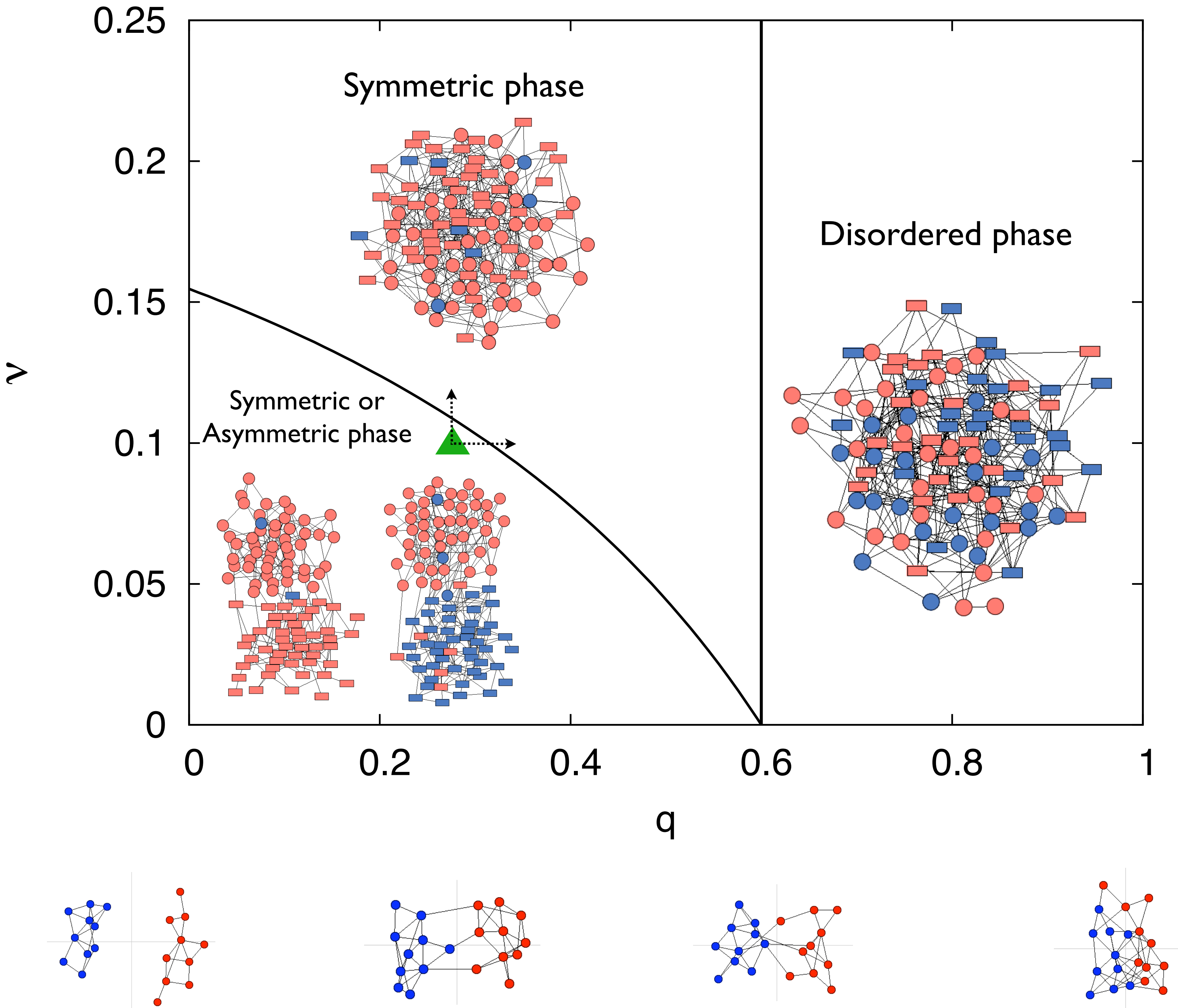




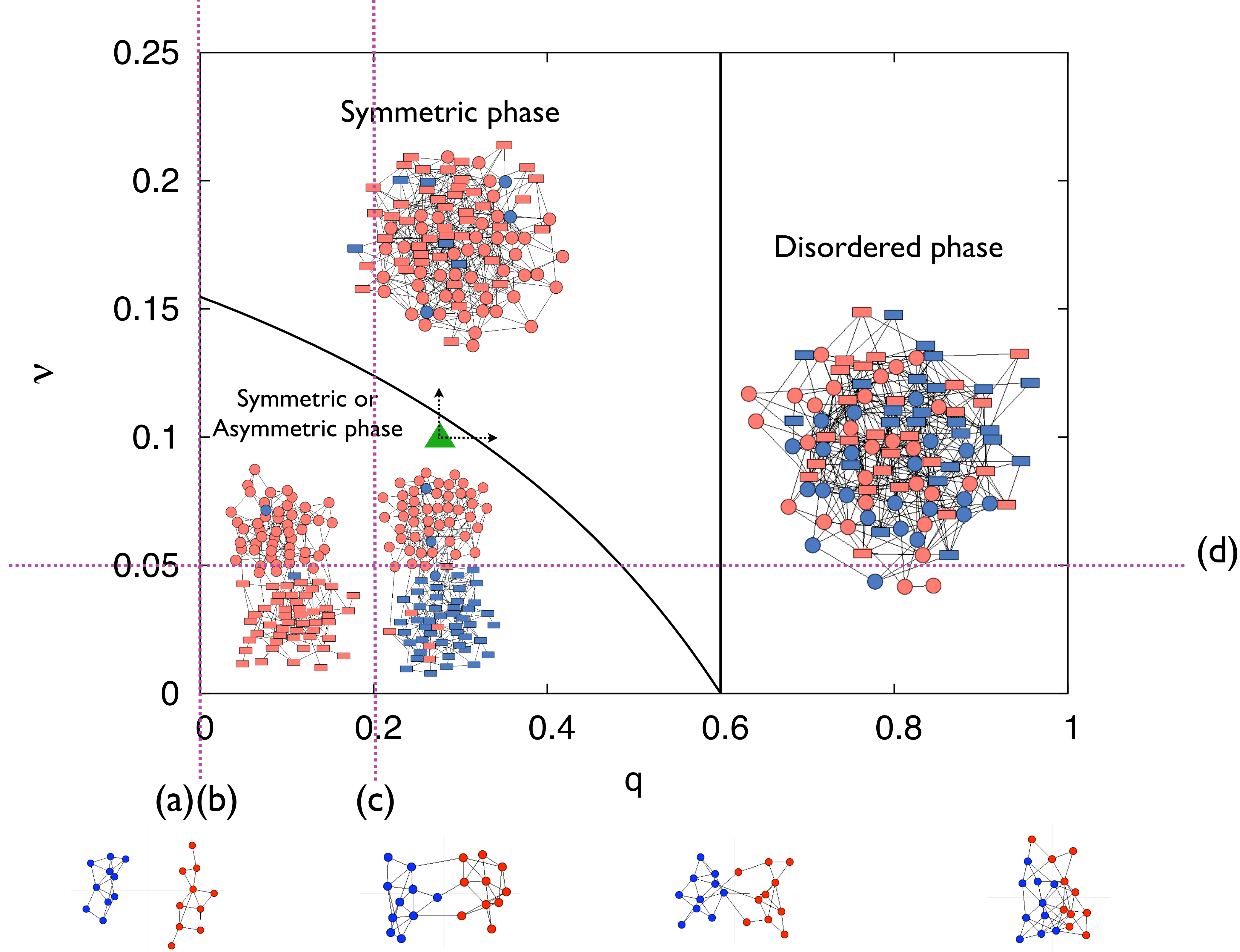




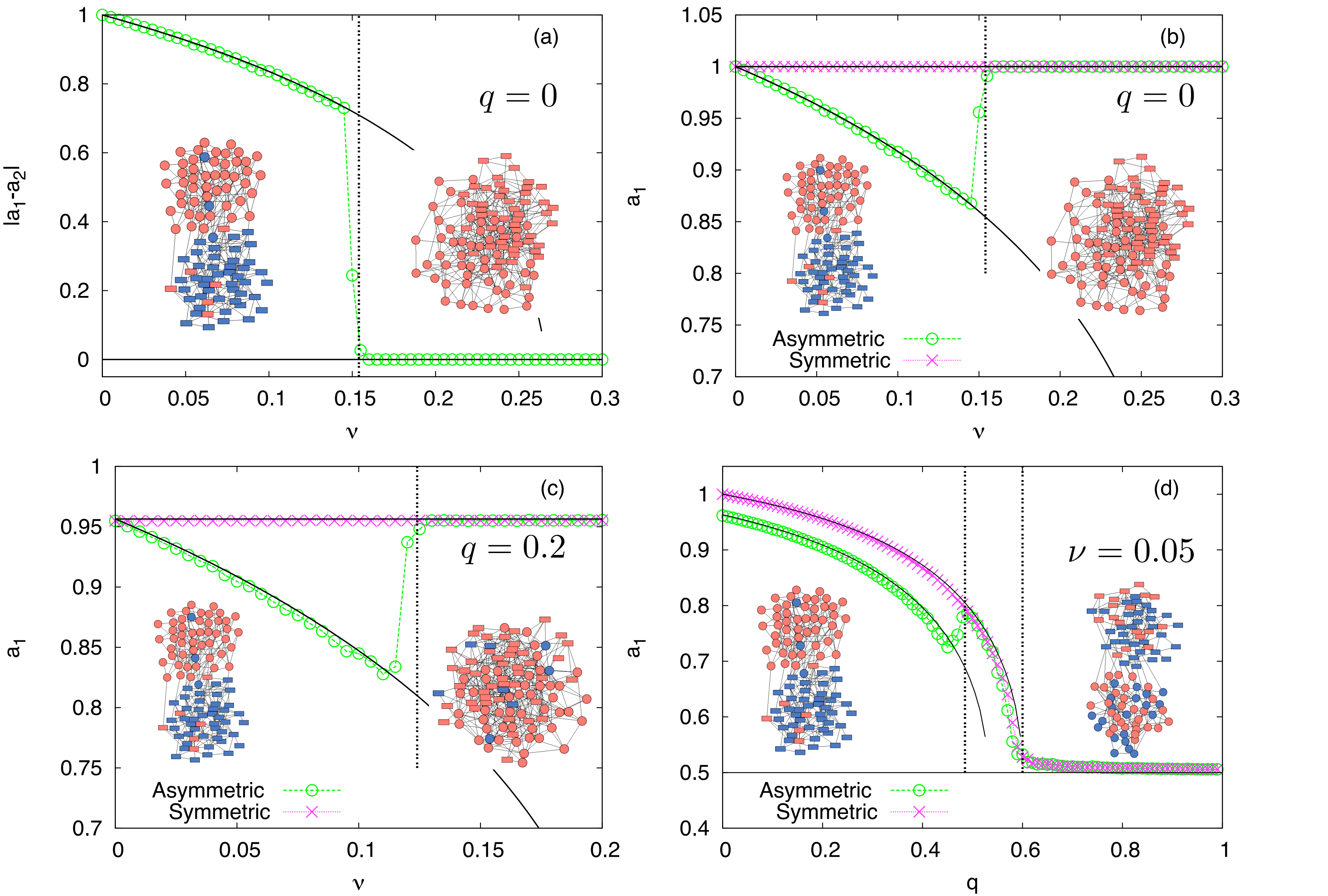
Lada A. Adamic and Natalie Glance, LinkKDD-2005, Chicago, IL, Aug 21, 2005



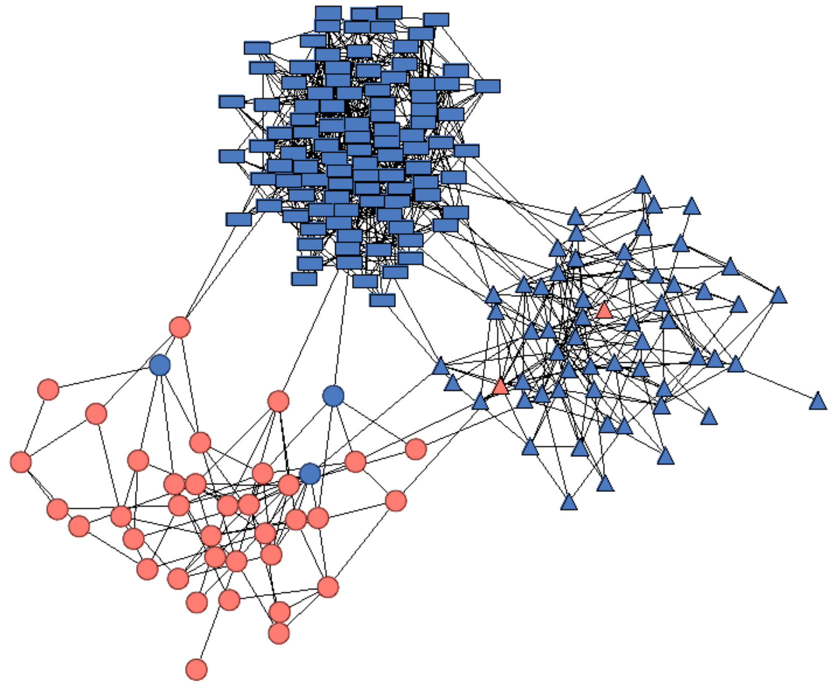
Symmetric phase
Disordered phase
Symmetric or
Asymmetric phase
0.25
0.2
0.15
0.1
0.05
0
ν
0
0.2
0.4
0.6
0.8
1
q



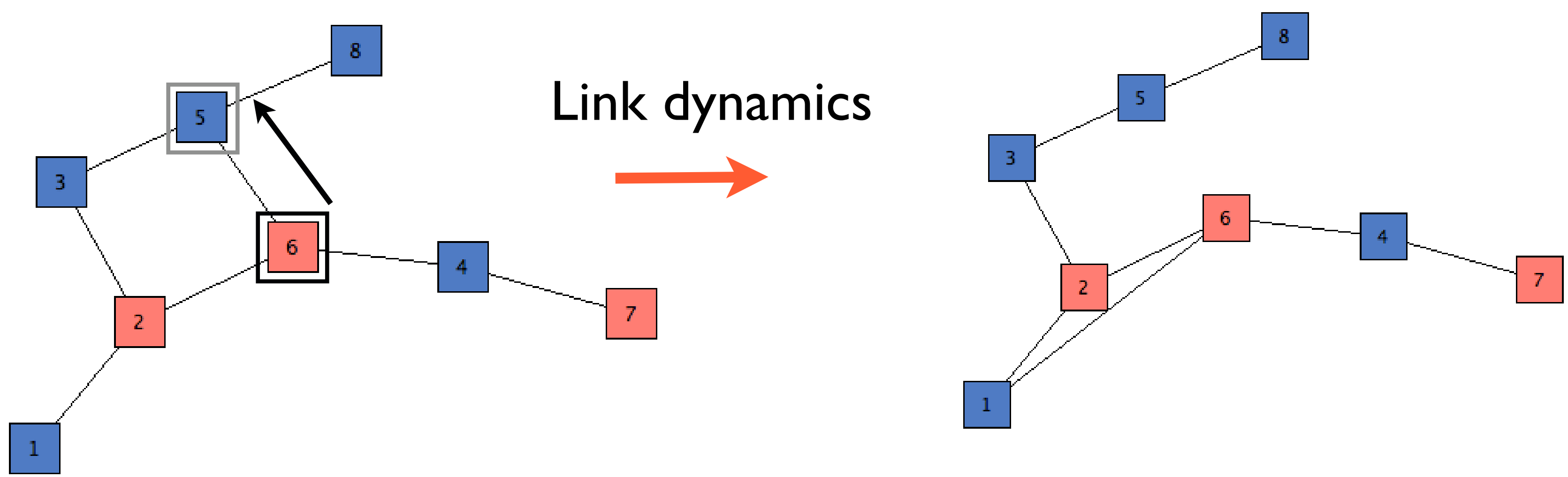
0.25
0.2
0.15
0.1
0.05
0
ν
0
0.2
0.4
0.6
0.8
1
q
Symmetric phase
Disordered phase
Symmetric or
Asymmetric phase
(a)(b)
(c)
(d)


(a)
$q = 0$
$|a_1 - a_2|$
ν
0
0.05
0.1
0.15
0.2
0.25
0.3
0.2
0.4
0.6
0.8
1
(b)
$q = 0$
a_1
0.7
0.75
0.8
0.85
0.9
0.95
1
1.05
Asymmetric
Symmetric
(c)
$q = 0.2$
a_1
ν
0.7
0.75
0.8
0.85
0.9
0.95
1
Asymmetric
Symmetric
(d)
$\nu = 0.05$
a_1
q
0.4
0.5
0.6
0.7
0.8
0.9
1
Asymmetric
Symmetric

Many communities?



Co-evolution?

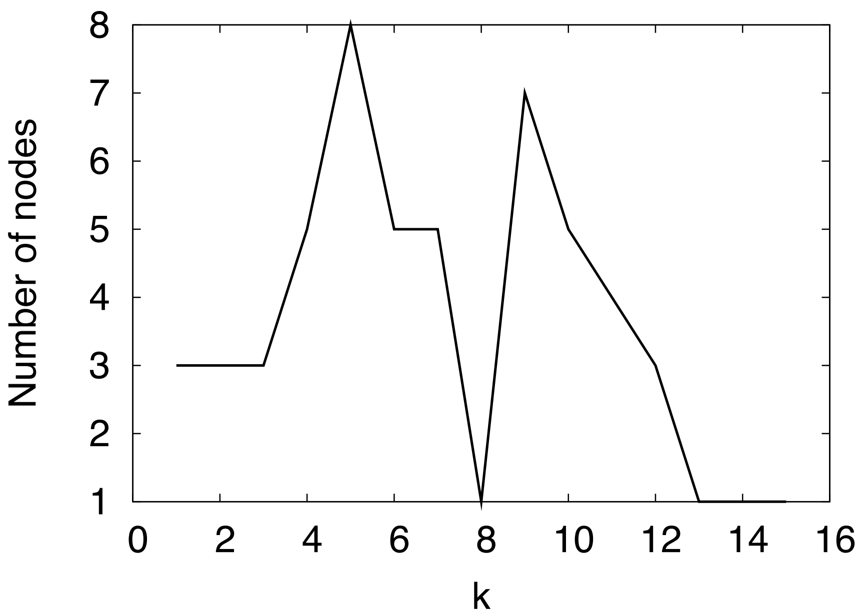


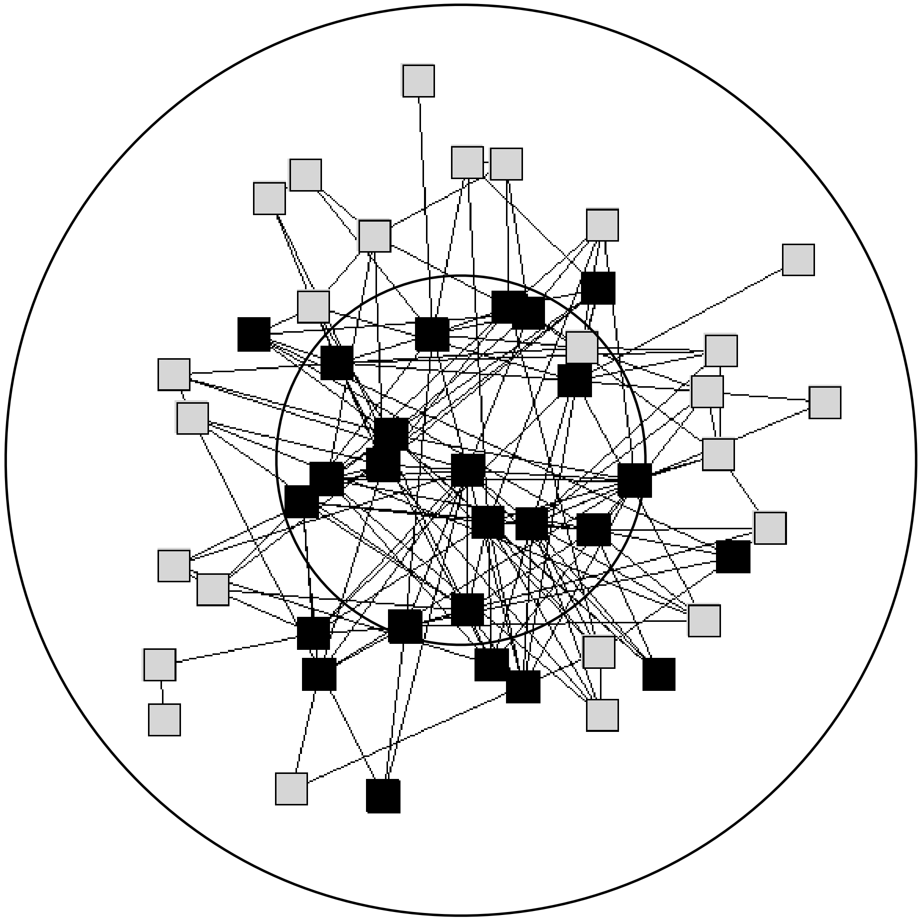
Role of the degree heterogeneity

Dichotomous networks: there are two kinds of nodes, each kind i being characterised by a degree k_i

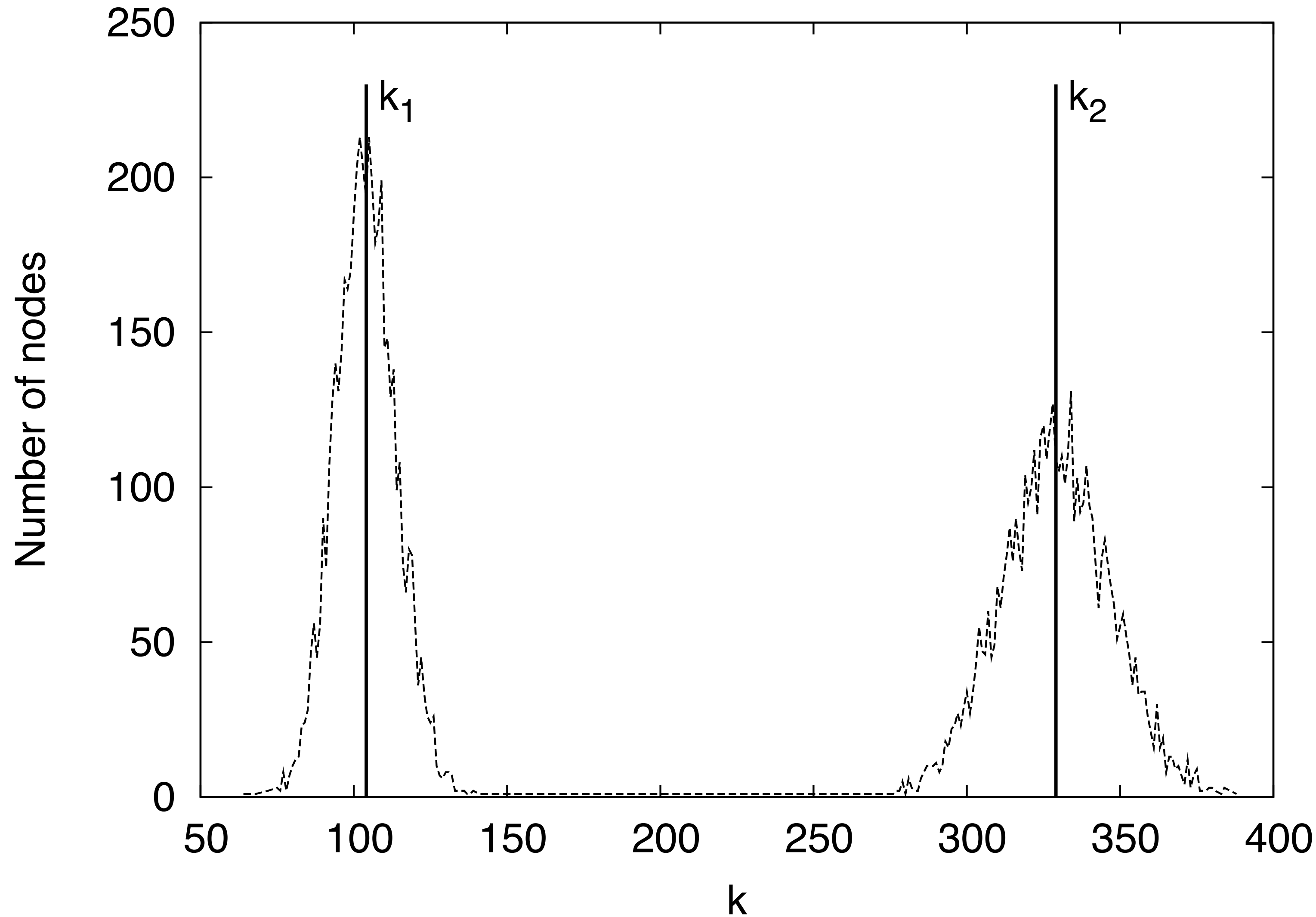
Hidden variable $p_1 = 0.05$

Hidden variable $p_2 = 0.25$



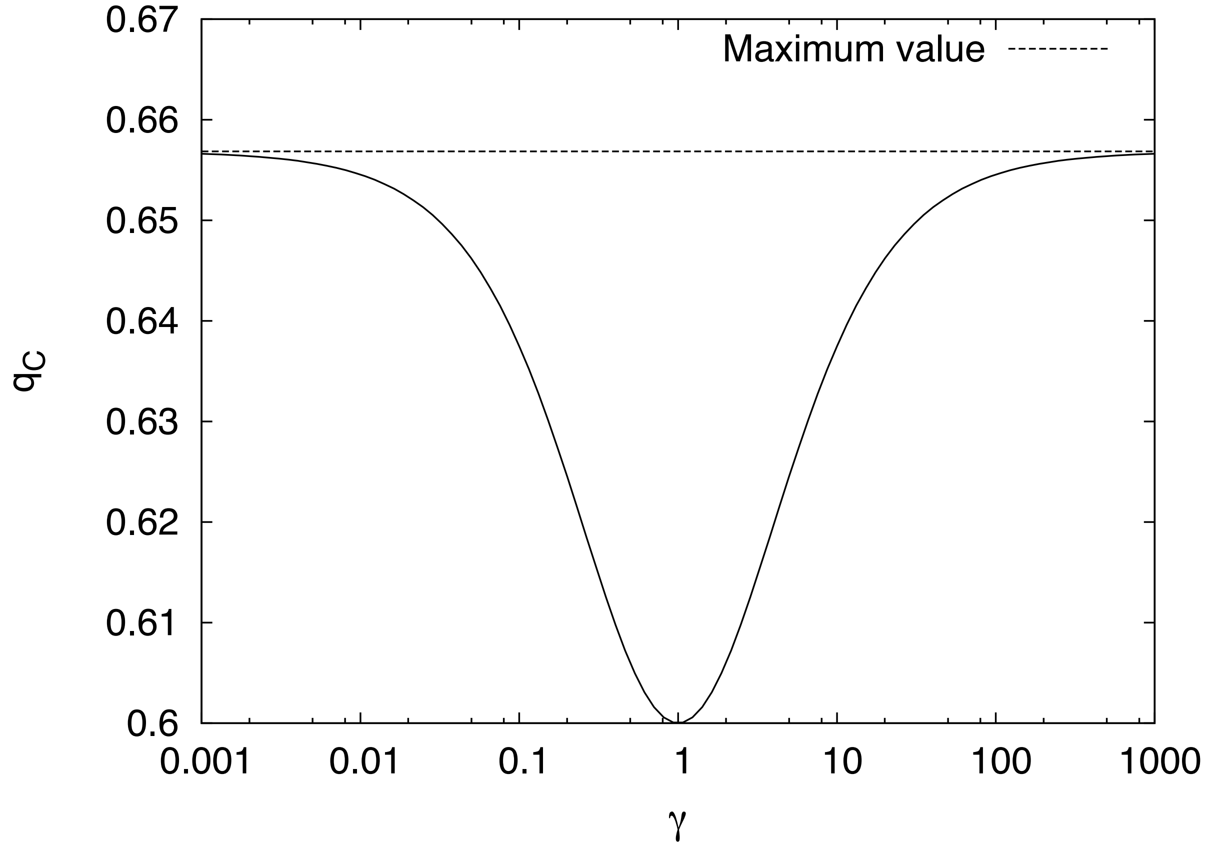


How does degree heterogeneity affect an order-disorder transition?, R. Lambiotte, *EPL*, **78** (2007) 68002



Degree heterogeneity: $\gamma = k_2/k_1$

Degree heterogeneity displaces the location of the transition



A change of the underlying topology (degree distribution) might trigger a phase transition