# Community and multiplexity in complex networks

R. Lambiotte
Department of Mathematics
University of Namur

Namur Center for Complex Systems

# Networks

Mathematical tool to describe systems composed of elements and of the relations between them (directed or not)

Natural language to describe complex systems

Possibility to analyse systems of a very different nature within a single framework

Identification of universal properties among diverse kinds of systems => generic organisation principles

Internet

Transport networks

Power grids

Protein interaction networks

Food webs

Metabolic networks

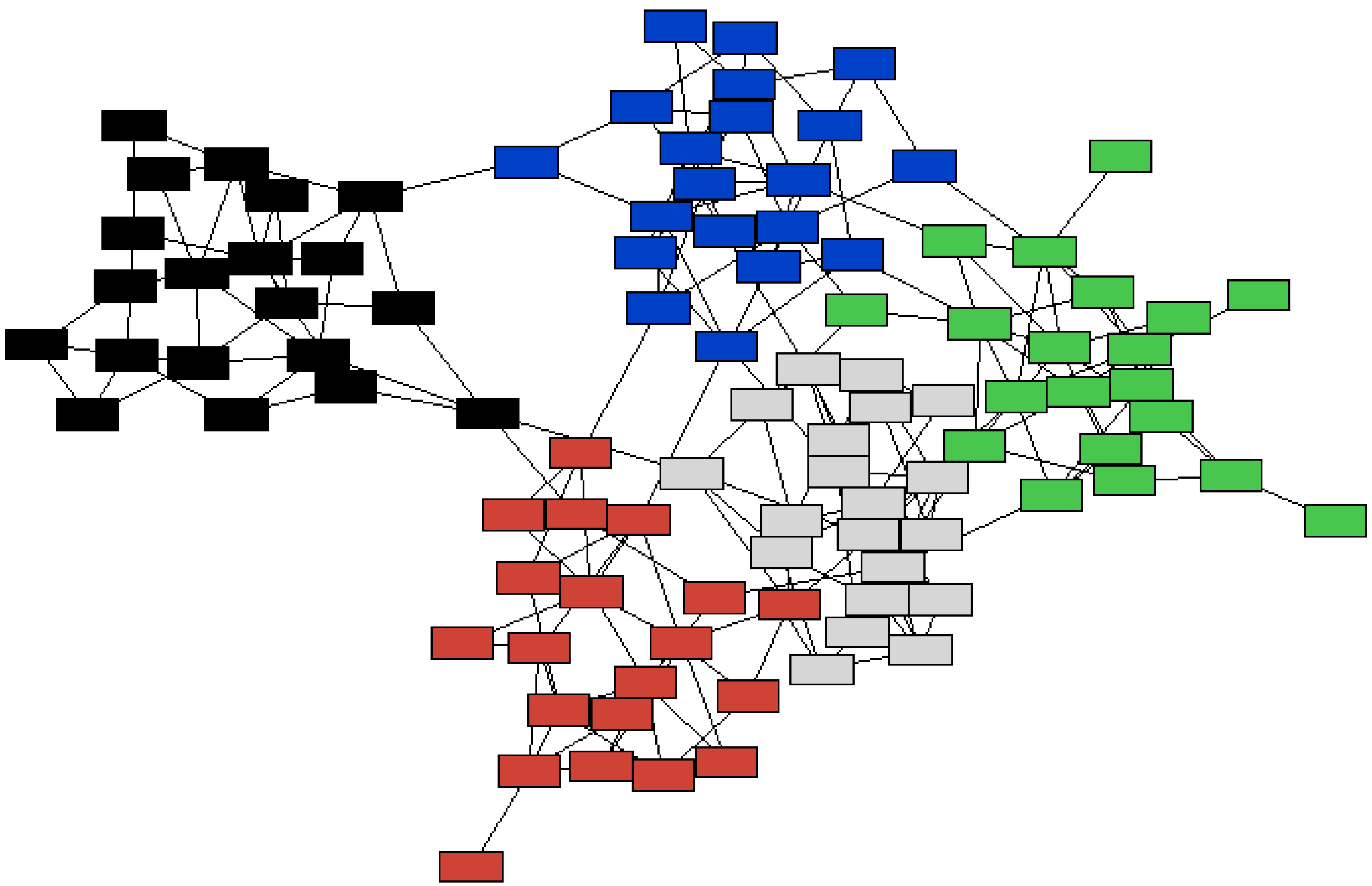
Social networks

The brain

Etc.

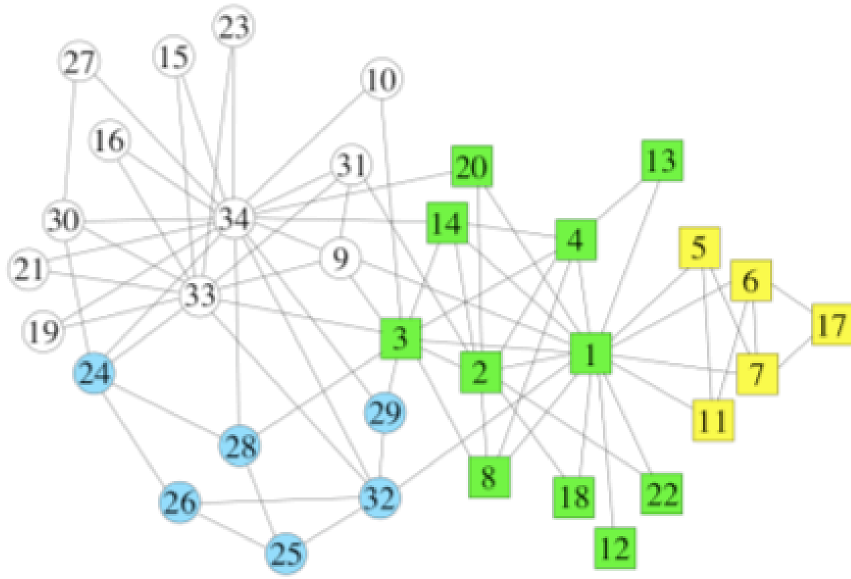
# Modular Networks

Many networks are inhomogeneous and are made of modules: many links within modules and a few links between different modules



Observed in social, biological and information networks

a



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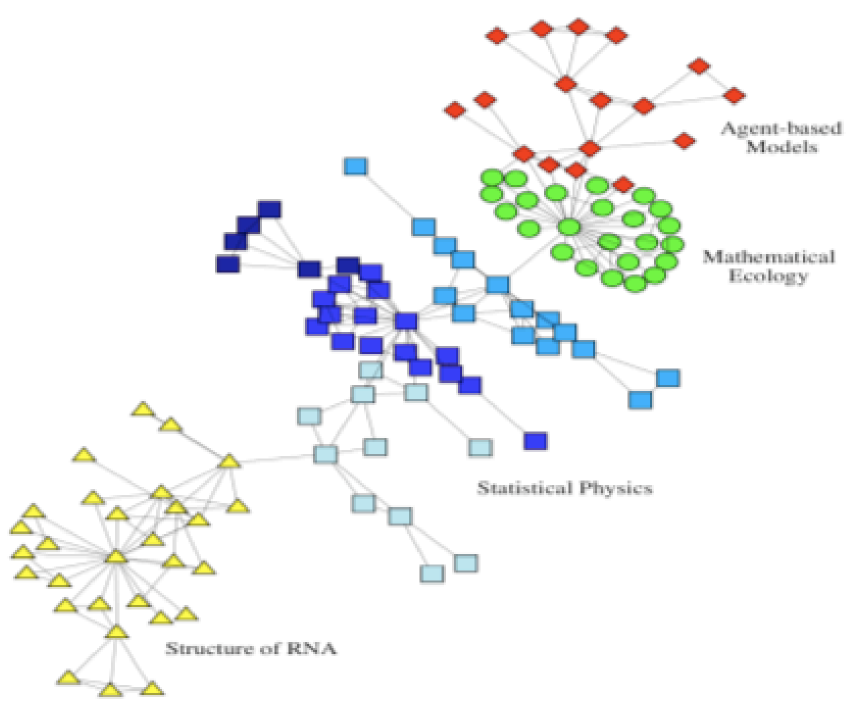
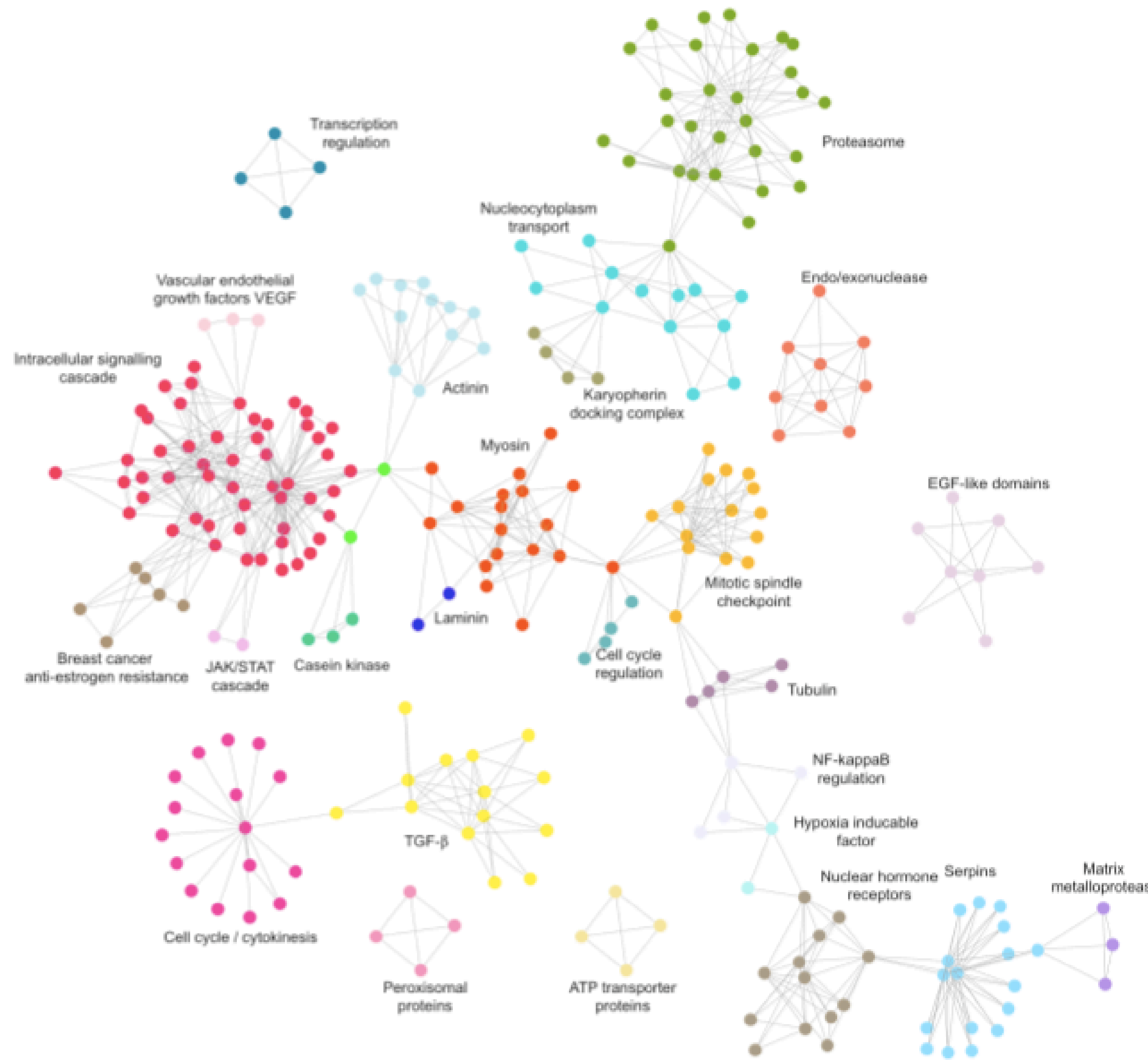


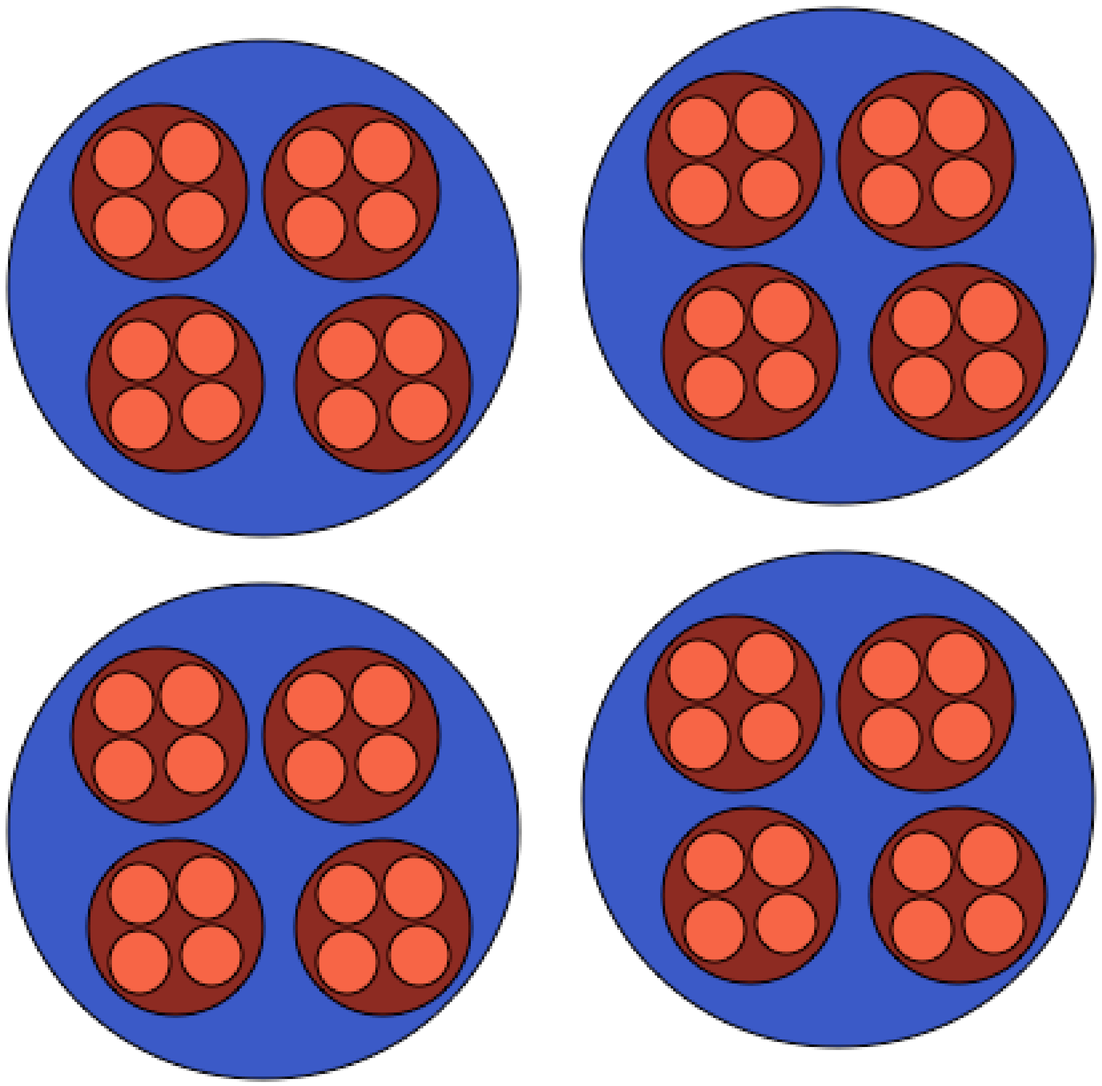



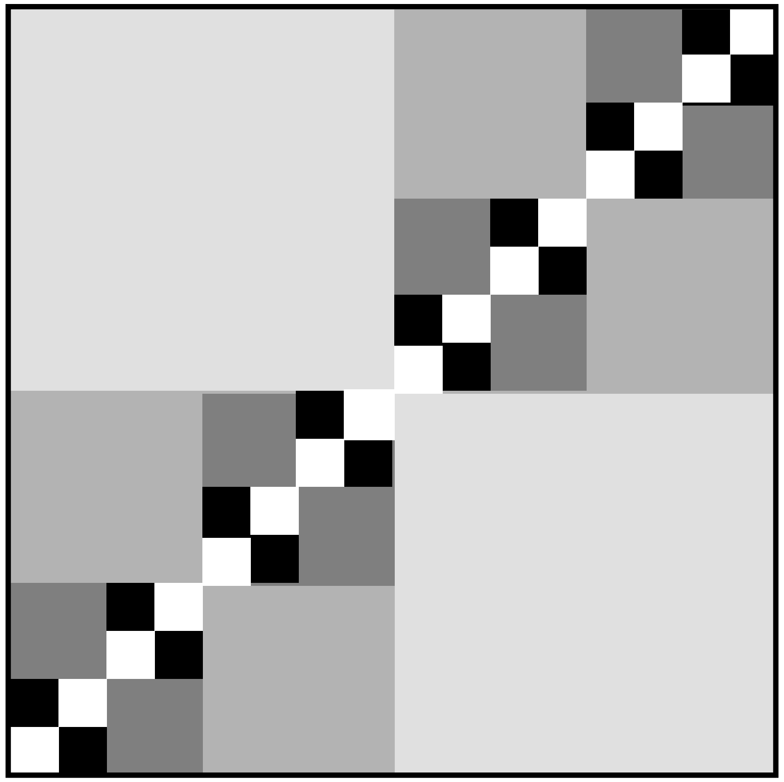

S. Fortunato, Community detection in graph, Physics Reports, vol. 486, pp. 75-174 (2010)

# Hierarchical Networks

Networks have a hierarchical/multi-scale structure: modules within modules

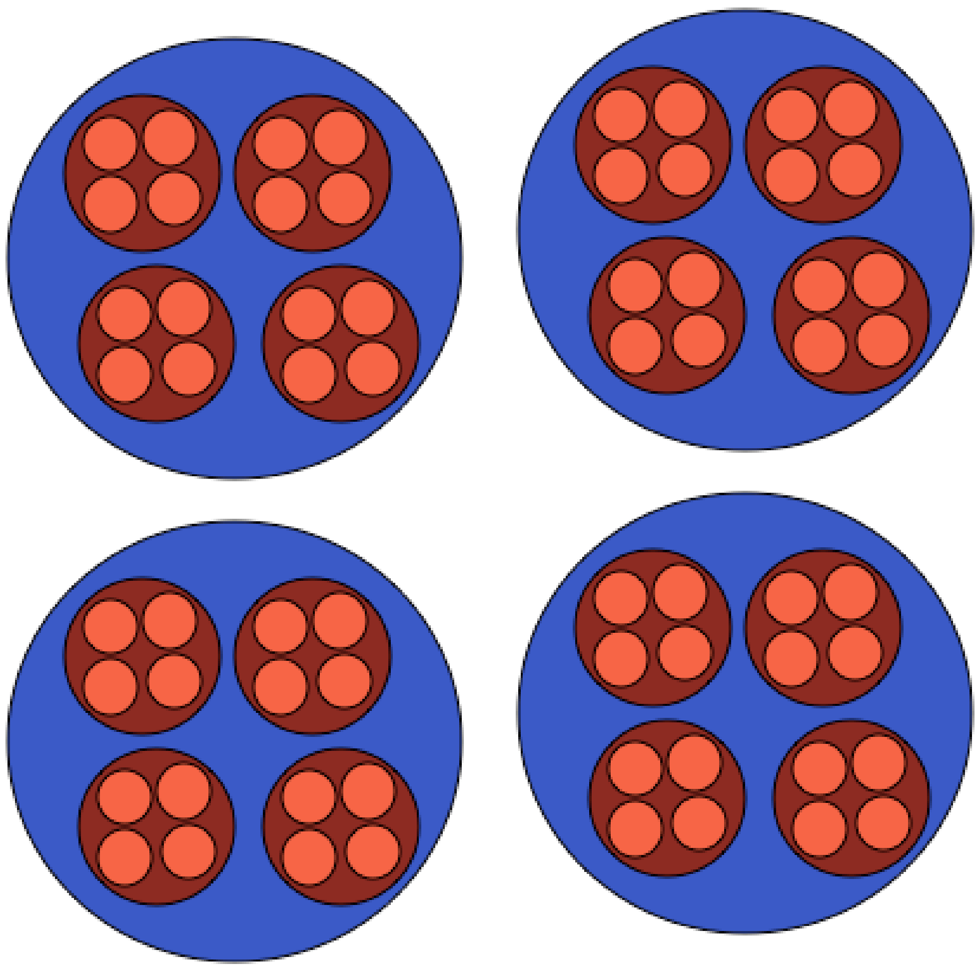
Nested organization



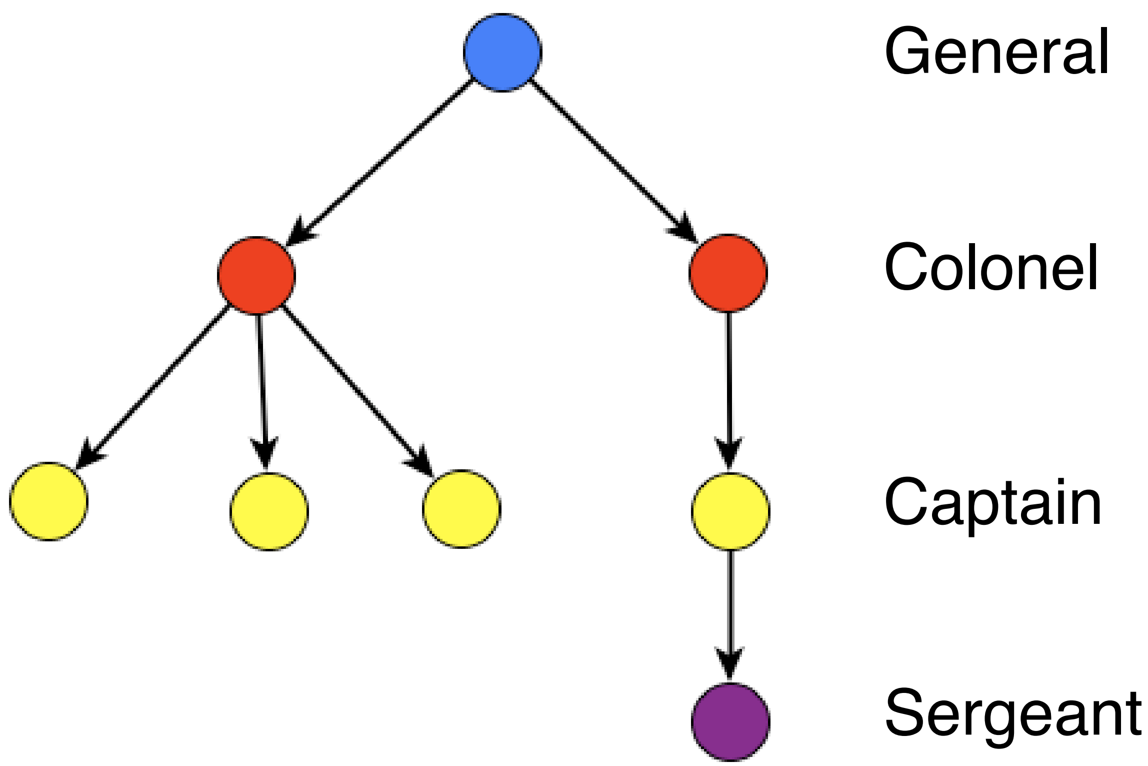


# Different definitions for hierarchy

Hierarchy = multi-scale structure:
modules within modules



Hierarchy = subordination



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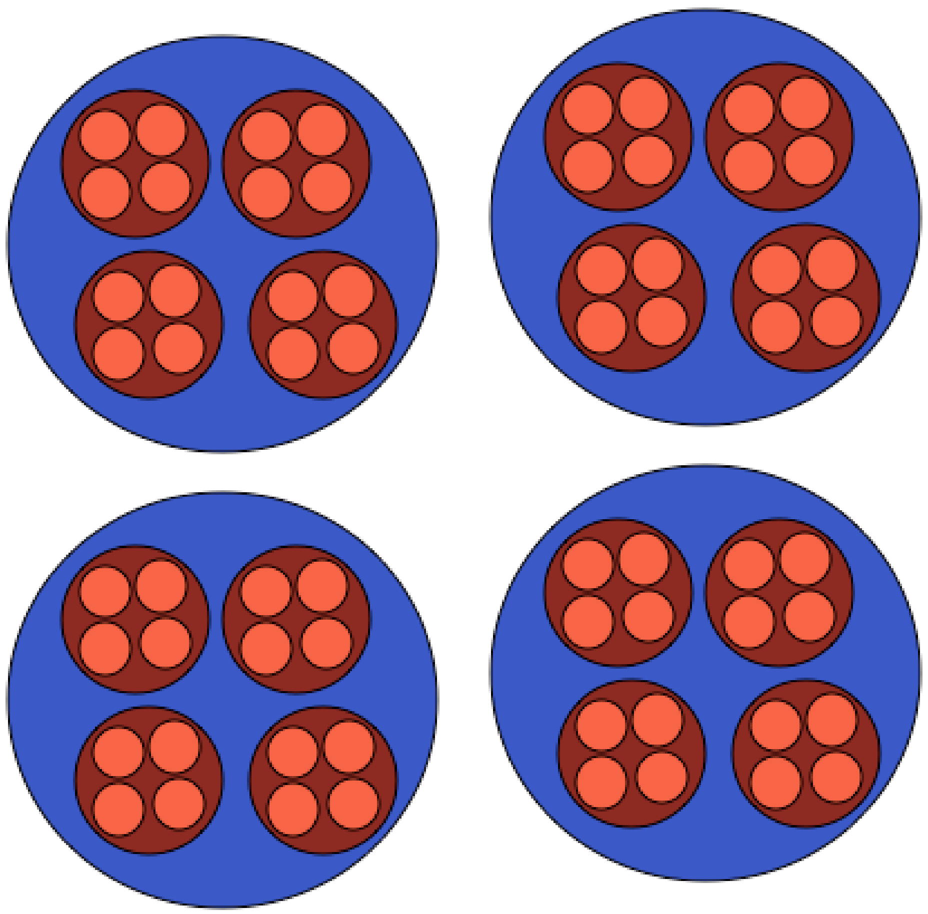
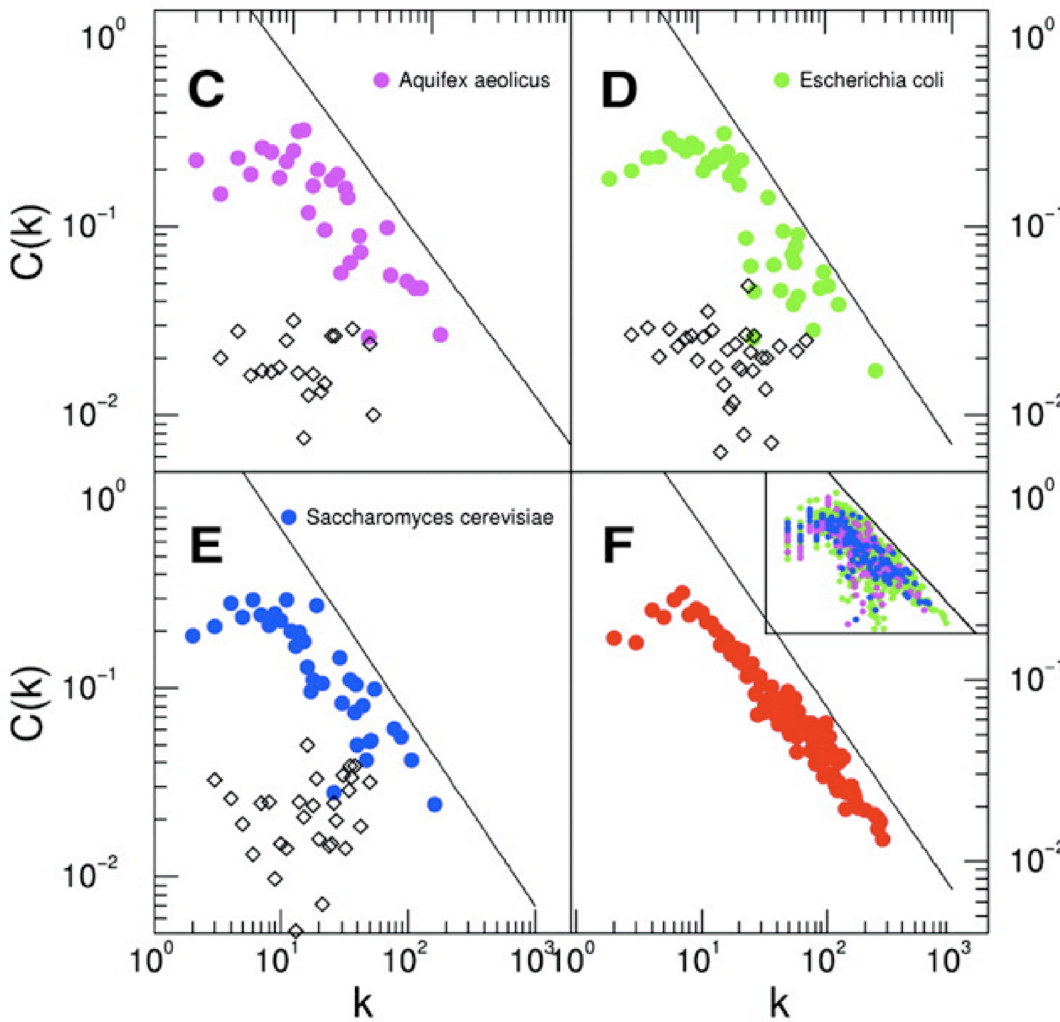
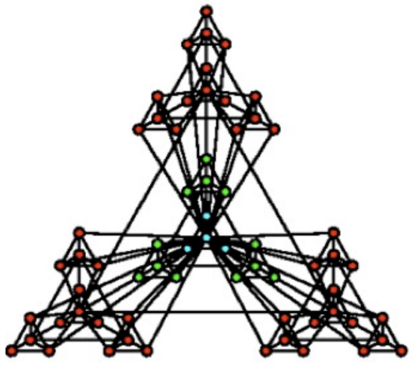


Hierarchy of nodes with different degrees of “modularity” (clustering)



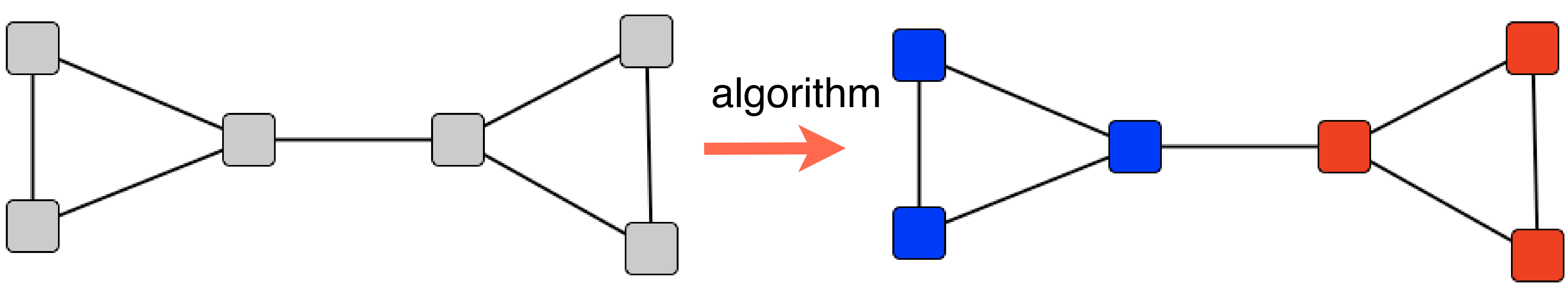



Hierarchical Organization of Modularity in Metabolic Networks
E. Ravasz, A. L. Somera, D. A. Mongru, Z. N. Oltvai, A.-L. Barabási, Science (2002)

# Community detection

Is it possible to uncover the (multi-scale) modular organisation of networks in an automated fashion?
Given a graph, we look for an algorithm able to uncover its modules without specifying their number or their size (related to but different from classical partitioning problems, see later)

- find modules and only the modules: the method automatically finds the true modular organisation
- multi-scale modularity?
- scalability (millions of nodes)



# Why looking for modules?

Graphs help us to comprehend in a visual way its global organisation. This works extremely well when the graph is small but, as soon as the system is made of hundreds or thousands of nodes, a brute force representation typically leads to a meaningless cloud of nodes.

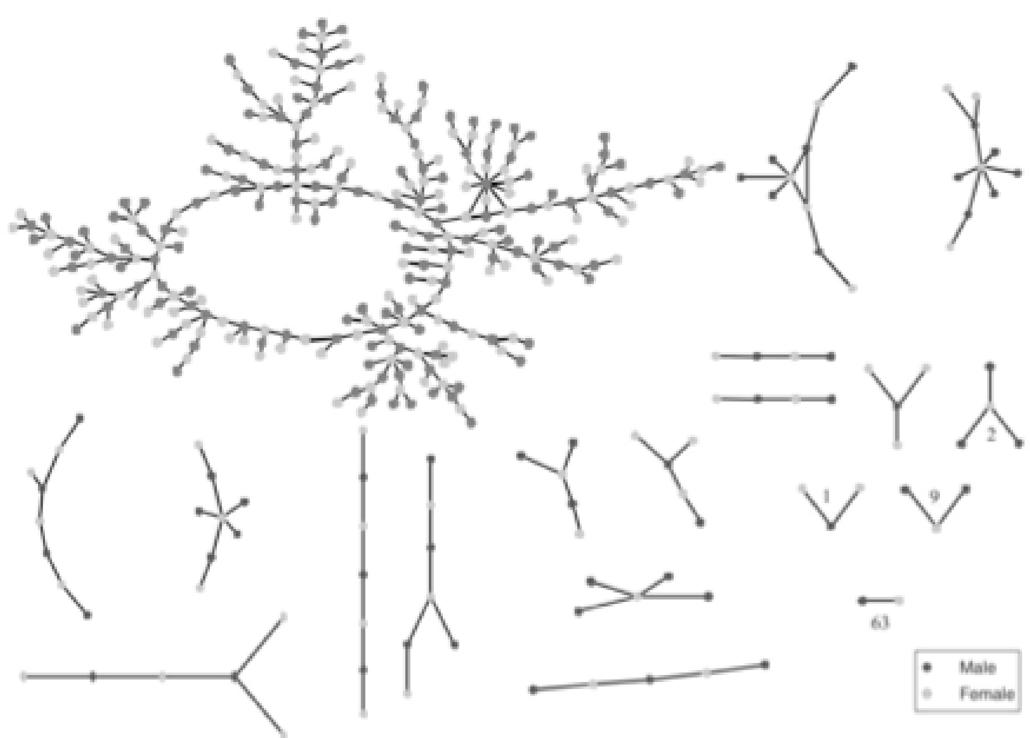
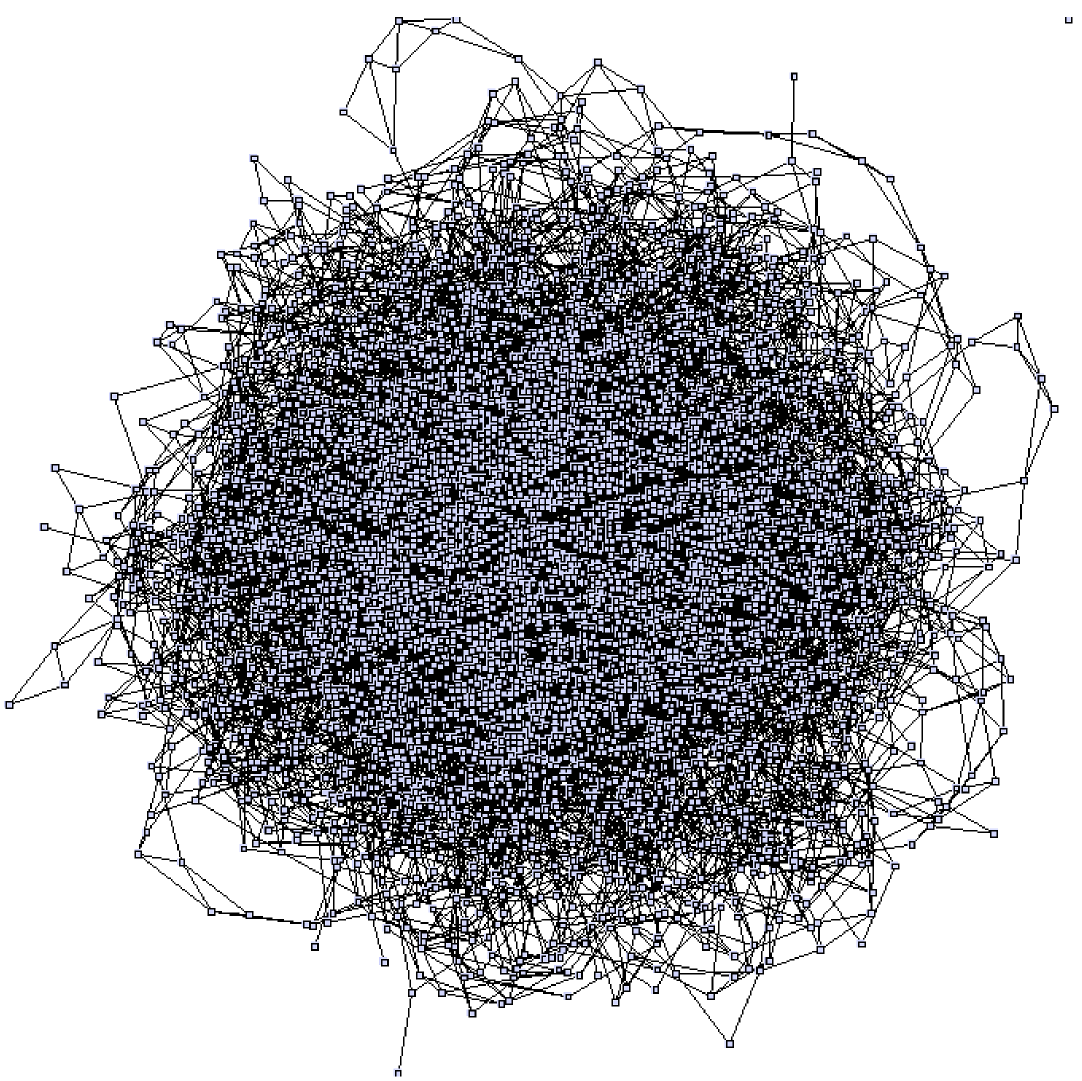


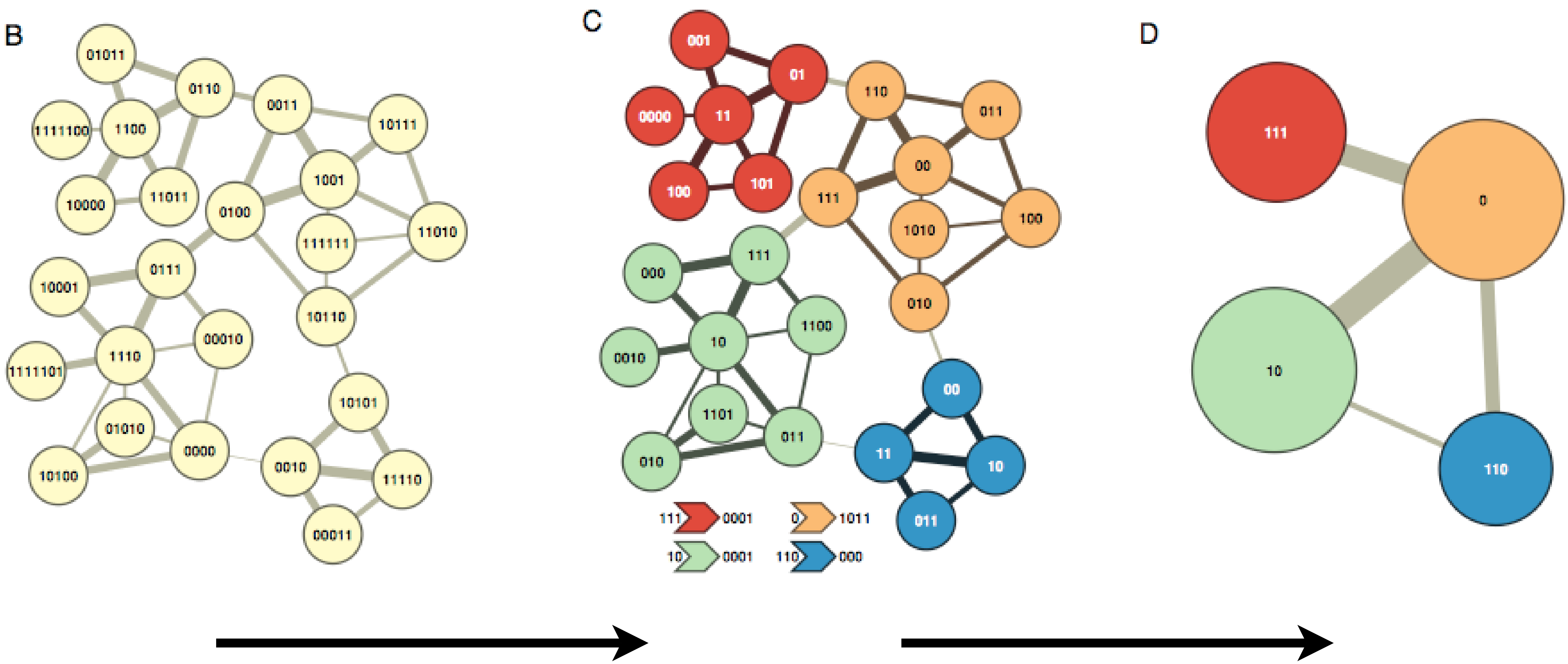

FIG. 2.—The direct relationship structure at Jefferson High



# Why looking for modules?

Is it possible to uncover modules/hierarchies in large networks?

Intermediate levels of organization of complex systems



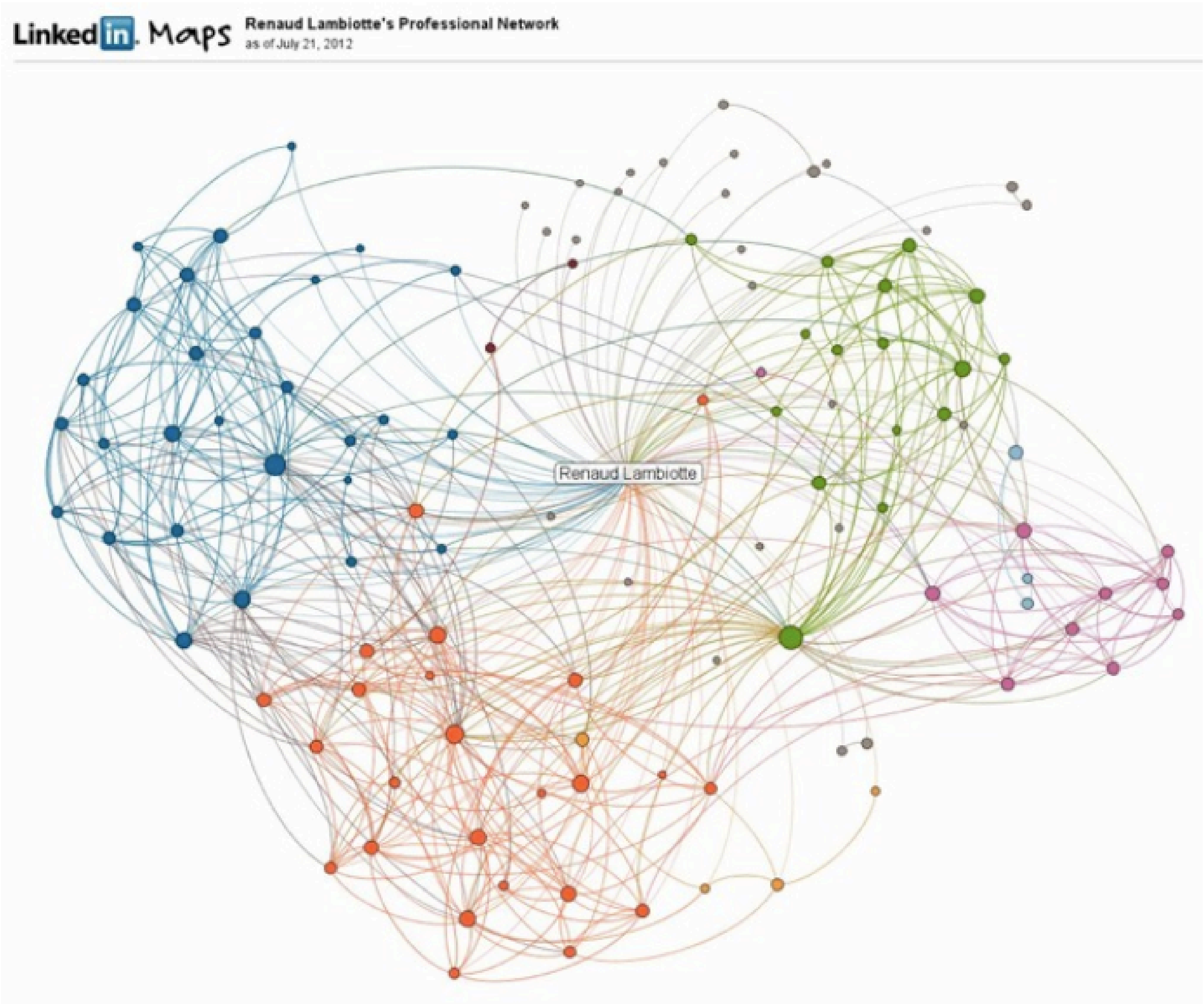

Find a partition of the network into communities

Coarse-grained description

Uncovering communities/modules helps to understand the structure of the network and to draw a readable map of the network (when N is large).

Martin Rosvall and Carl T. Bergstrom, PNAS 105, 1118 –1123 (2008)

# Why looking for modules?



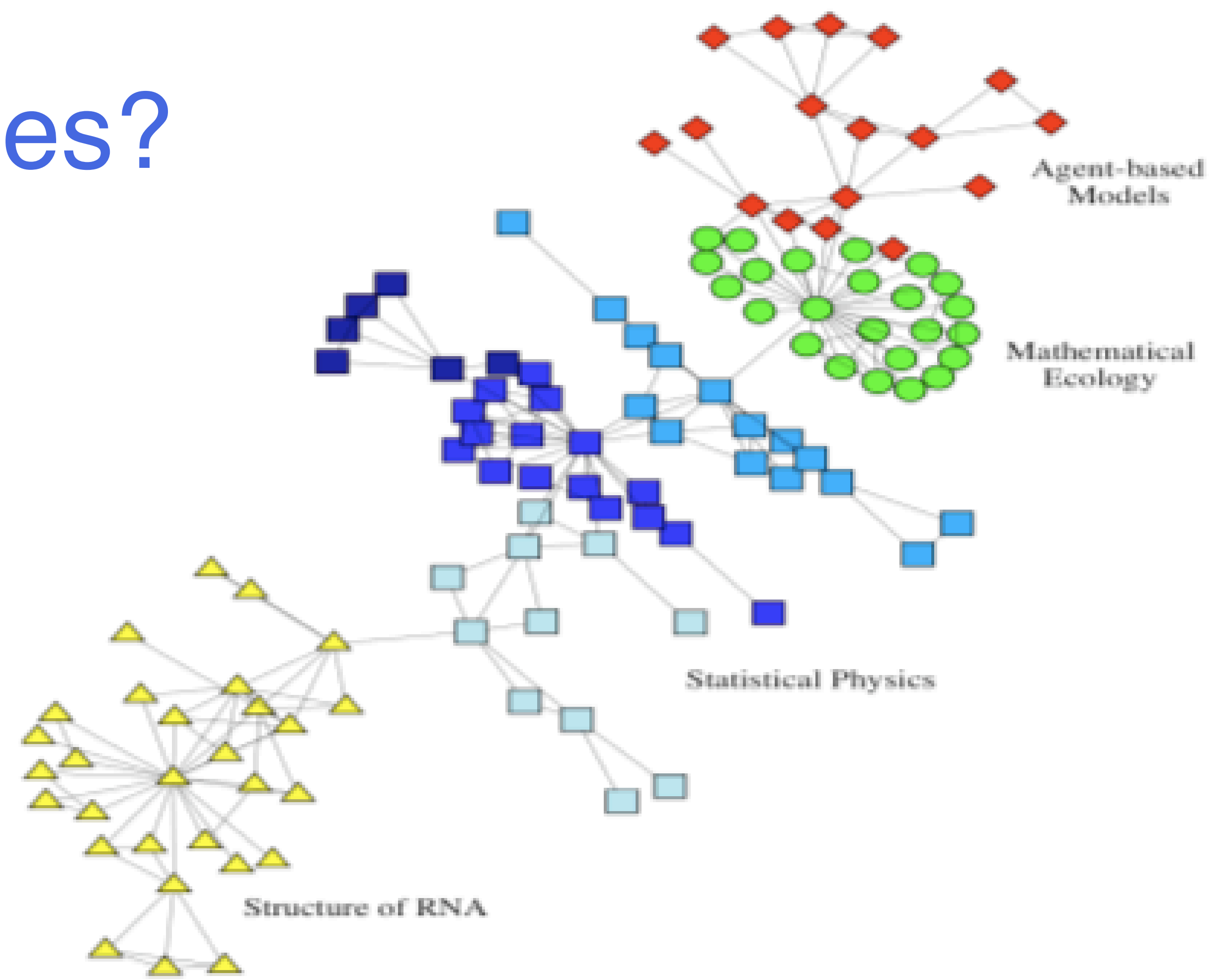
# Why looking for modules?

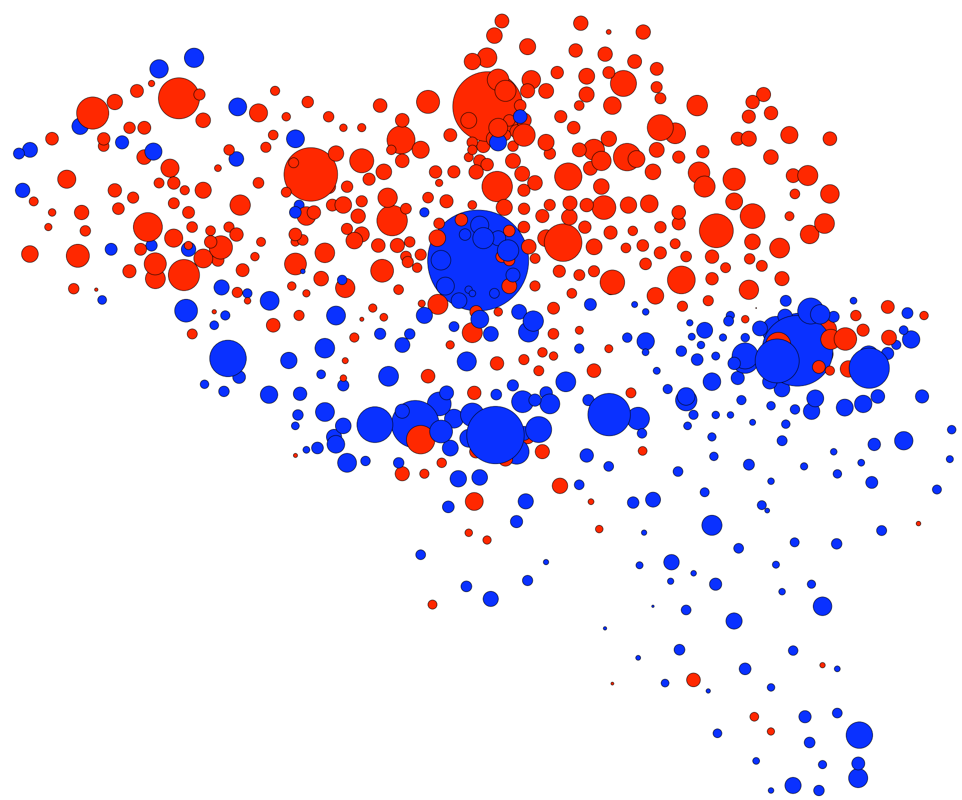
Modules often overlap with properties/functions of nodes

Data mining perspective:

Uncovering communities might help to uncover hidden properties between nodes

b



# Community detection

Hundreds of ways to uncover communities/modules in networks, often associated to different notions of communities:

- (very) fast vs (very) slow methods
- overlapping vs non-overlapping communities
- single-scale or multi-scale?

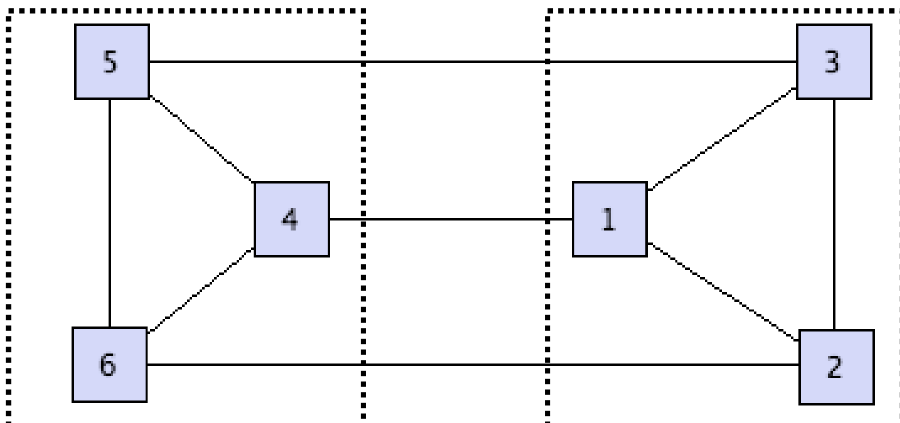
What does a good community mean?

S. Fortunato, Community detection in graph, Physics Reports, vol. 486, pp. 75-174 (2010)

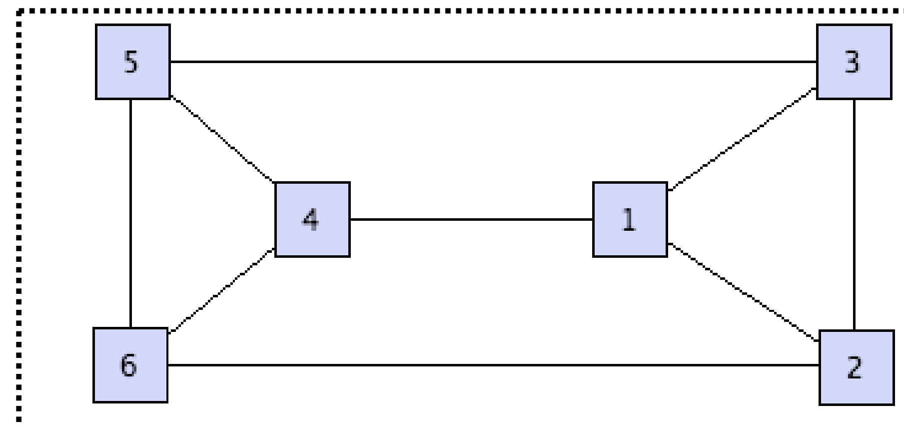
# Quality of a partition

What is the best partition of a network into modules?

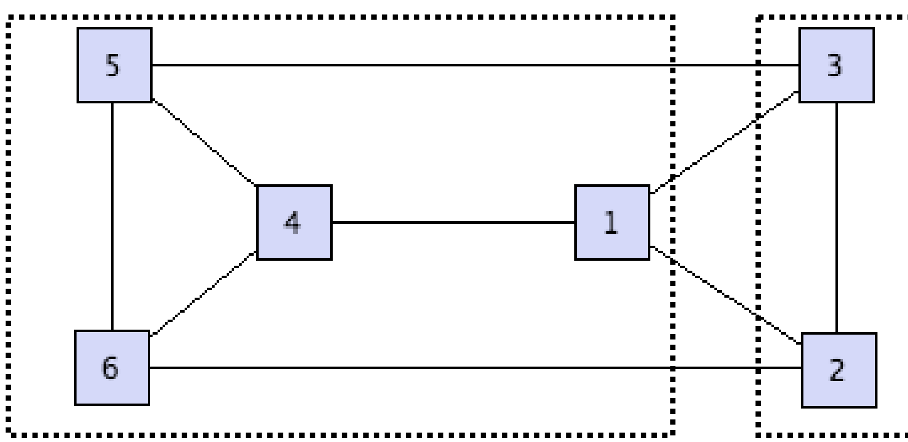
How do we rank the quality of partitions of different sizes?



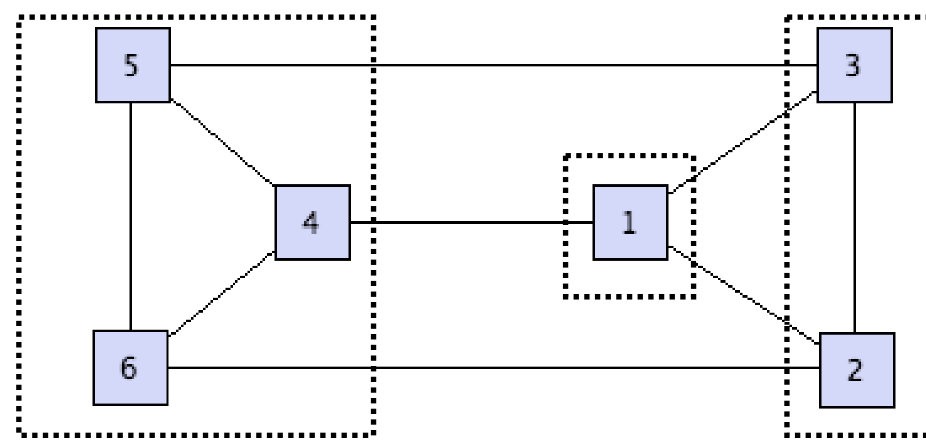

Q1




Q2




Q3




Q4

........

# Modularity

Q = fraction of edges within communities - expected fraction of such edges

Let us attribute each node i to a community ci

$$\boxed{Q = \frac{1}{2m} \sum_{i,j} \Big[ A_{ij} - P_{ij} \Big] \delta(c_i, c_j)} \qquad Q \in [-1, 1]$$

$$P_{ij} = \frac{k_i k_j}{2m}$$ expected number of links between i and j

$$\rightarrow \quad Q_C = \frac{1}{2m} \sum_{i,j} \Big[ A_{ij} - k_i k_j / 2m \Big] \delta(c_i, c_j)$$

Allows to compare partitions made of different numbers of modules

M.E.J. Newman and M. Girvan, Finding and evaluating community structure in networks, Phys. Rev. E, 69, 026113, 2004.

# Modularity optimisation

Different types of algorithms (many similar to or inspired by graph partioning methods) for different applications:

Small networks ($<10^2$): Simulated Annealing

Intermediate size ($10^2$ -$10^4$): Spectral methods, PL, etc.

Large size ($>10^4$): greedy algorithms

# Spectral optimization

Let us first focus on the best division of the network into 2 communities (of any size!).

Let us denote by $s_i = \pm 1$ the assignment of node i

$$Q = \frac{1}{2m} \sum_{ij} Q_{ij} \delta(c_i, c_j) = \frac{1}{4m} \sum_{ij} Q_{ij} s_i s_j$$

By performing a spectral decomposition of the modularity matrix, one finds:

$$Q_{ij} = \sum_{\alpha=1}^{N} \lambda_\alpha v_{\alpha,i} v_{\alpha,j}$$

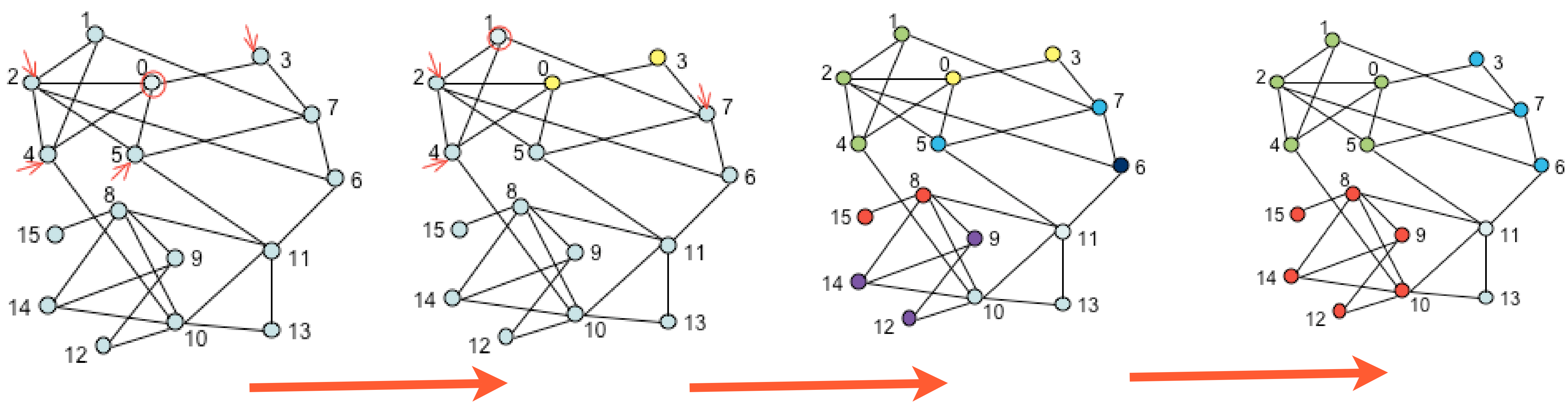
$s_i$ is chosen to be as similar to the dominant eigenvector of the modularity matrix

$$\boxed{\begin{array}{c} s_i = 1 \text{ if } v_{N,i} > 0 \\ s_i = -1 \text{ if } v_{N,i} < 0 \end{array}}$$

M.E.J. Newman, Finding community structure in networks using the eigenvectors of matrices, Phys. Rev. E, vol. 74, 036104, 2006.

# Greedy optimisation

The algorithm is based on two steps that are repeated iteratively.
First phase: Find a local maximum
1) Give an order to the nodes (0,1,2,3,...., N-1)
2) Initially, each node belongs to its own community (N nodes and N communities)
3) One looks through all the nodes (from 0 to N-1) in an ordered way. The selected node looks among its neighbours and adopt the community of the neighbour for which the increase of modularity is maximum (and positive).
4)This step is performed iteratively until a local maximum of modularity is reached (each node may be considered several times).




Node 0 moves to the community of Node 3

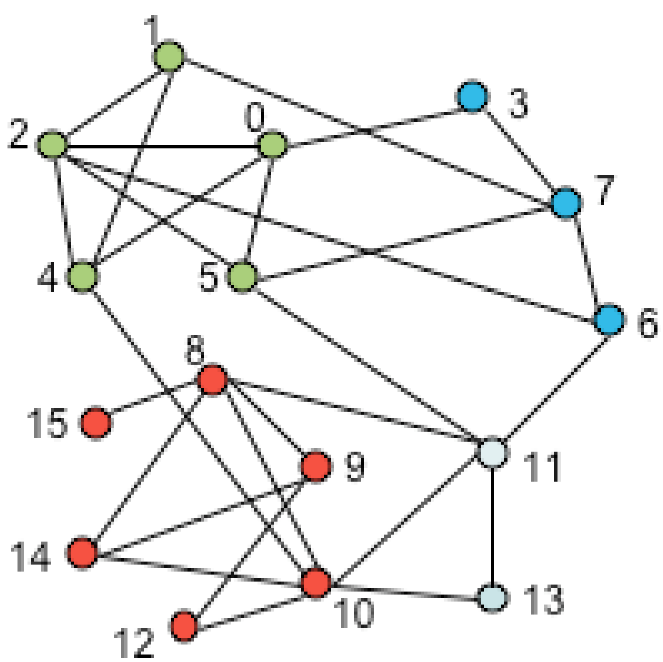
After N nodes have been considered

After each nodes has been considered 4 times

V.D. Blondel, J.-L. Guillaume, R. Lambiotte and E. Lefebvre, Fast unfolding of communities in large networks, J. Stat. Mech., P10008, 2008.

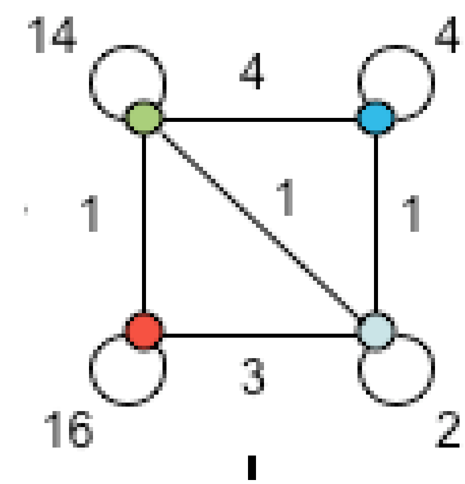
# Greedy optimisation

Once a local maximum has been attained, we build a new network whose nodes are the communities. The weight of the links between communities is the total weight of the links between the nodes of these communities.




New network of 4 nodes!

Note the self-loops

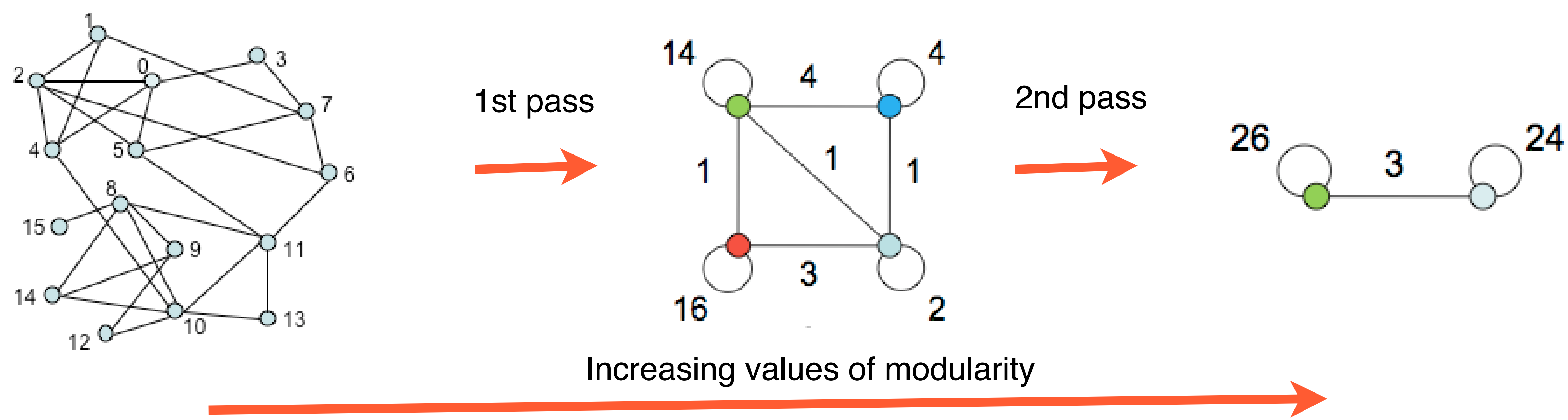



In typical realizations, the number of nodes diminishes drastically at this step.

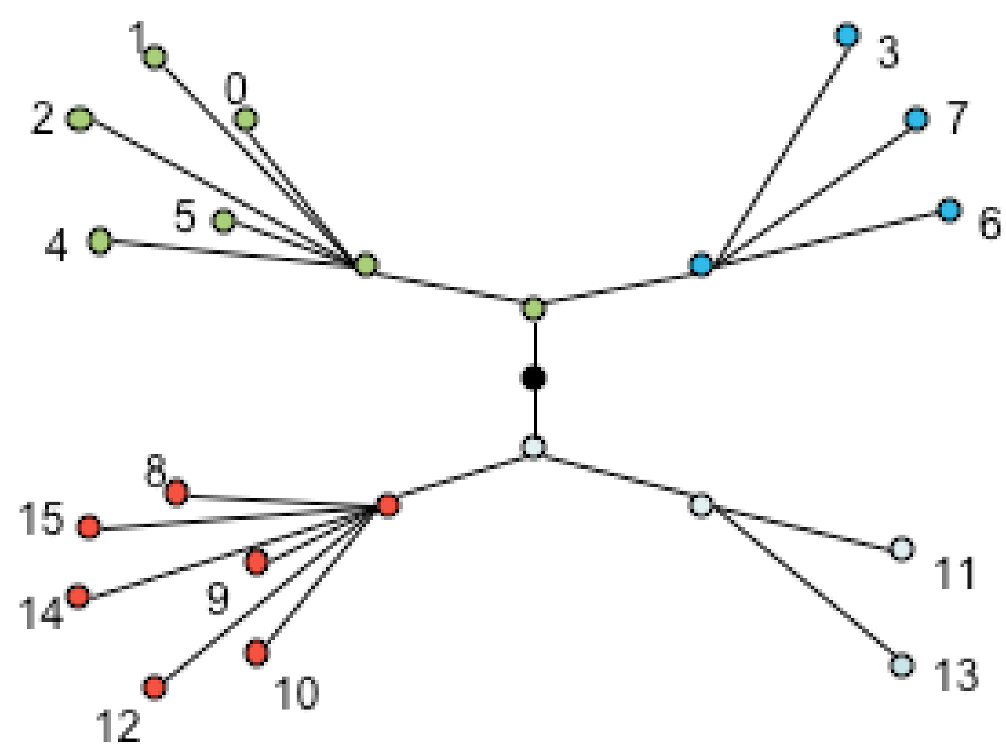
# Greedy optimisation

The two steps are repeated iteratively, thereby leading to a hierarchical decomposition of the network.

Multi-scale optimisation: local search first among neighbours, then among neighbouring communities, etc.




Hierarchical representation



Very fast: O(N) in practice. The only limitation being the storage of the network in main memory

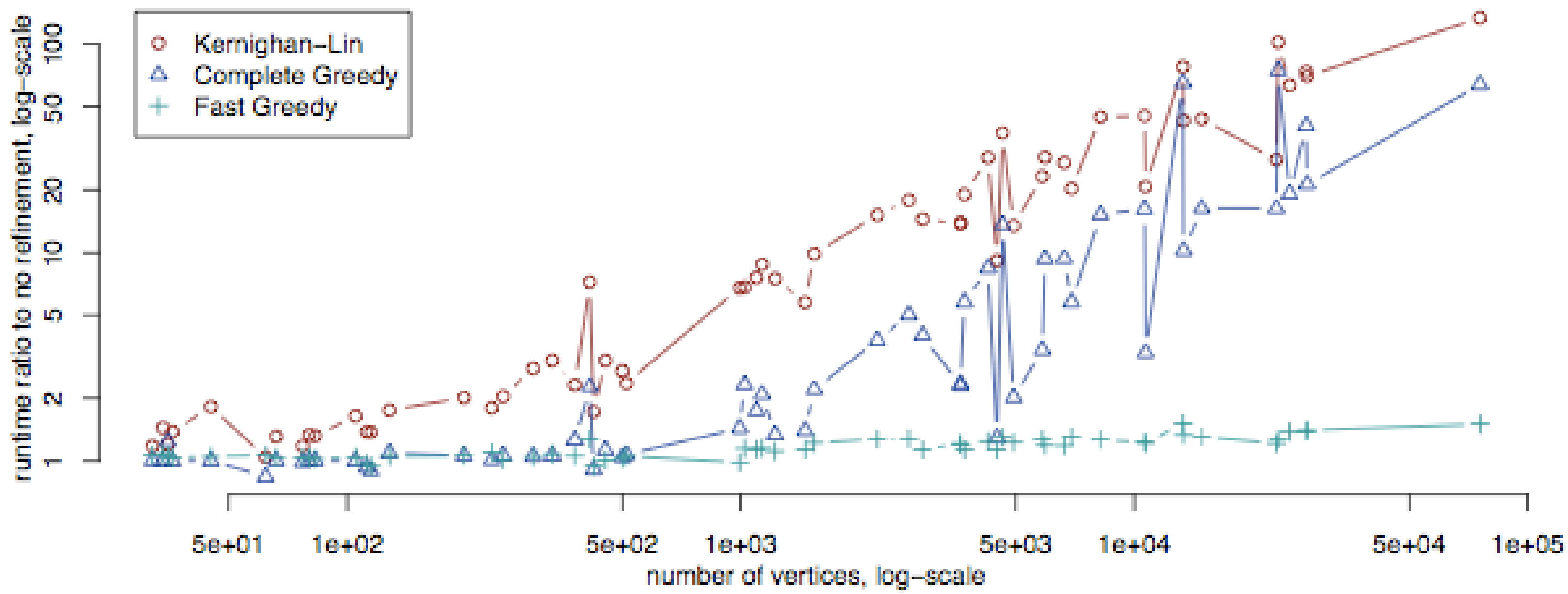
Good accuracy (among greedy methods)

| | Karate | Arxiv | Internet | Web nd.edu | Phone | Web uk-2005 | Web WebBase 2001 |
|---|---|---|---|---|---|---|---|
| Nodes/links | 34/77 | 9k/24k | 70k/351k | 325k/1M | 2.04M/5.4M | 39M/783M | 118M/1B |
| CNM | .38/0s | .772/3.6s | .692/799s | .927/5034s | -/- | -/- | -/- |
| PL | .42/0s | .757/3.3s | .729/575s | .895/6666s | -/- | -/- | -/- |
| WT | .42/0s | .761/0.7s | .667/62s | .898/248s | .553/367s | -/- | -/- |
| Our algorithm | .42/0s | .813/0s | .781/1s | .935/3s | .76/44s | .979/738s | .984/152mn |

Clauset A, Newman M E J and Moore C, 2004 Phys. Rev. E 70 066111.
Wakita K and Tsurumi T, 2007 Proceedings of IADIS international conference on WWW/Internet 2007 153.
Pons P and Latapy M, 2006 Journal of Graph Algorithms and Applications 10 191.

# How to test the methods?

Test the heuristics: what is the value of Q obtained for different algorithms? Time complexity?




| graph | size | subdivision | | coarsening | | local search | | math prog | | SS+ML |
|---|---|---|---|---|---|---|---|---|---|---|
| karate [42] | 34 | [29] | .419 | [41] | .4198 | [12] | .4188 | [1] | .4197 | .4197 |
| dolphins [22] | 62 | [29] | *.4893* | [31] | *.5171* | [33] | *.5285* | [40] | .5285 | .5276 |
| polBooks [21] | 105 | [29] | *.3992* | [37] | *.5269* | [4] | *.5204* | [1] | .5272 | .5269 |
| afootball [14] | 115 | [39] | .602 | [41] | .605 | [4] | *.6045* | [1] | .6046 | .6002 |
| jazz [15] | 198 | [29] | .442 | [9] | .4409 | [12] | .4452 | [1] | .445 | .4446 |
| celeg_metab [12] | 453 | [29] | .435 | [36] | .450 | [12] | .4342 | [1] | .450 | .4452 |
| email [17] | 1133 | [29] | .572 | [9] | .5569 | [12] | .5738 | [1] | .579 | .5774 |
| Erdos02 [16] | 6927 | [29] | *.5969* | [32] | .6817 | [33] | *.7094* | | | .7162 |
| PGP_main [5] | 11k | [29] | .855 | [9] | .7462 | [12] | .8459 | | | .8841 |
| cmat03_main [25] | 28k | [29] | .723 | [41] | .761 | [12] | .6790 | | | .8146 |
| ND_edu [2] | 325k | | | [7] | .927 | [4] | .935 | | | .9509 |

# How to test the methods?

Comparison with real-world data: do modules reveal nodes having similar meta-data?

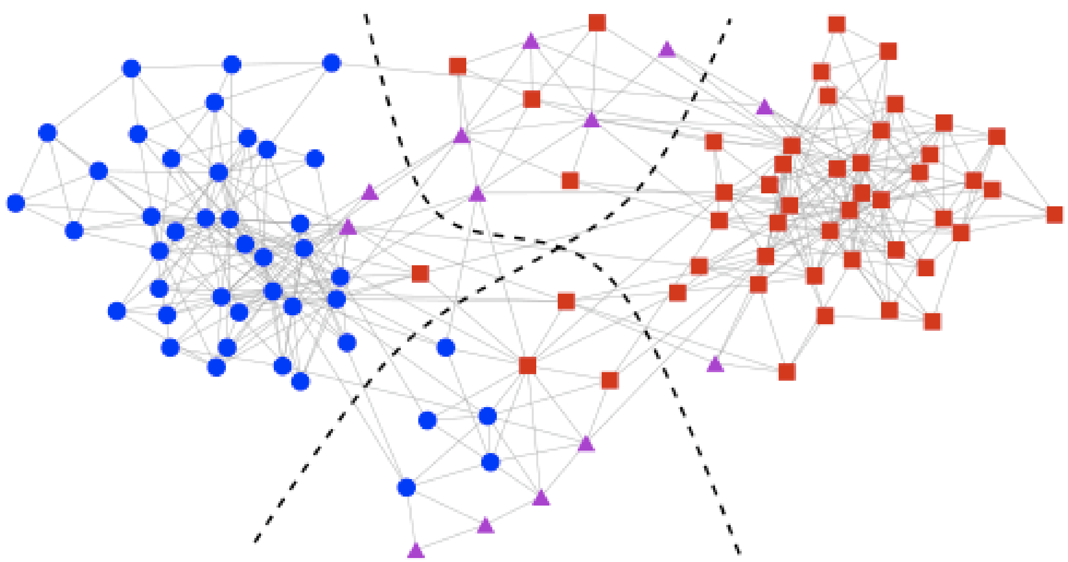
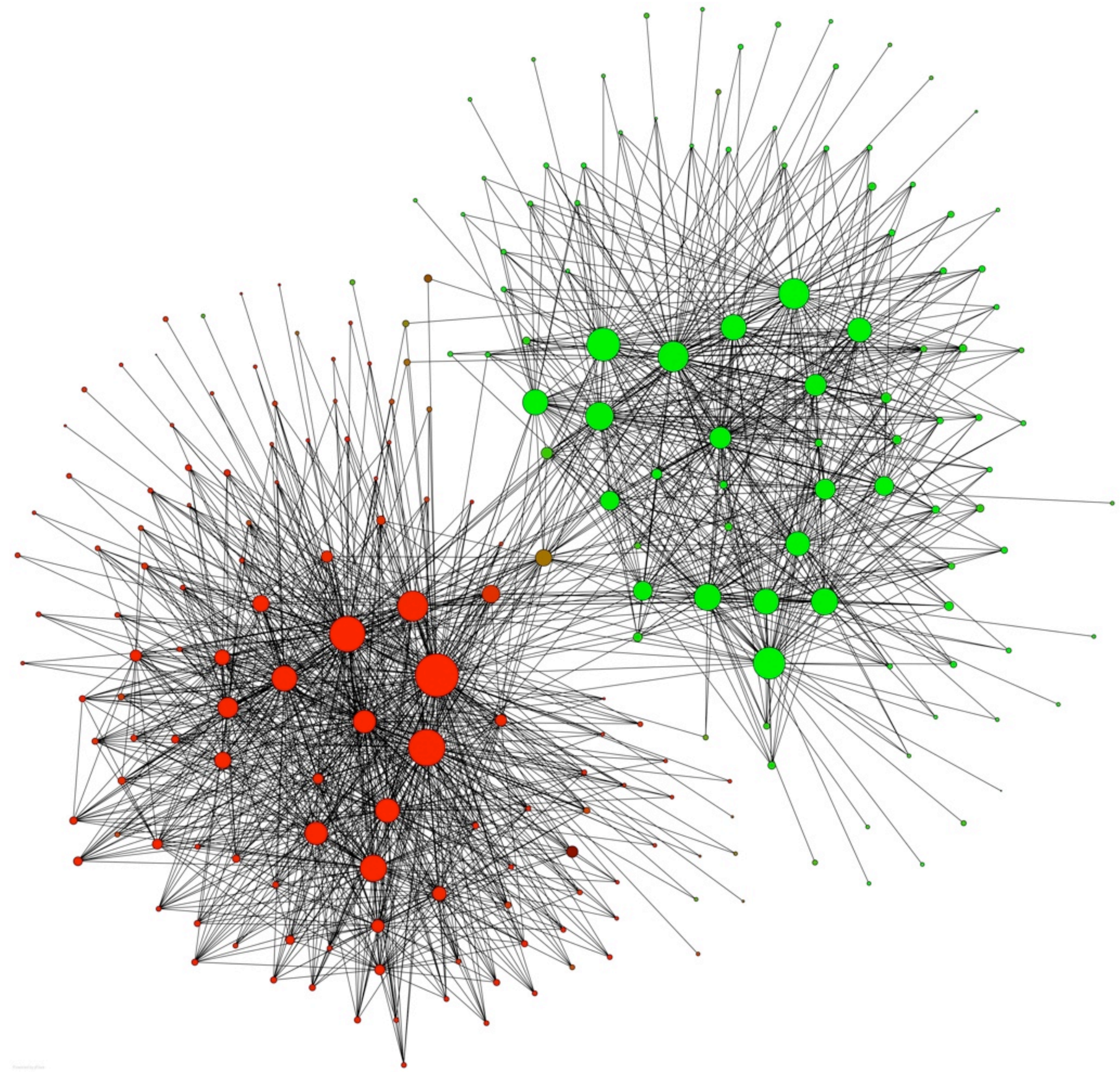


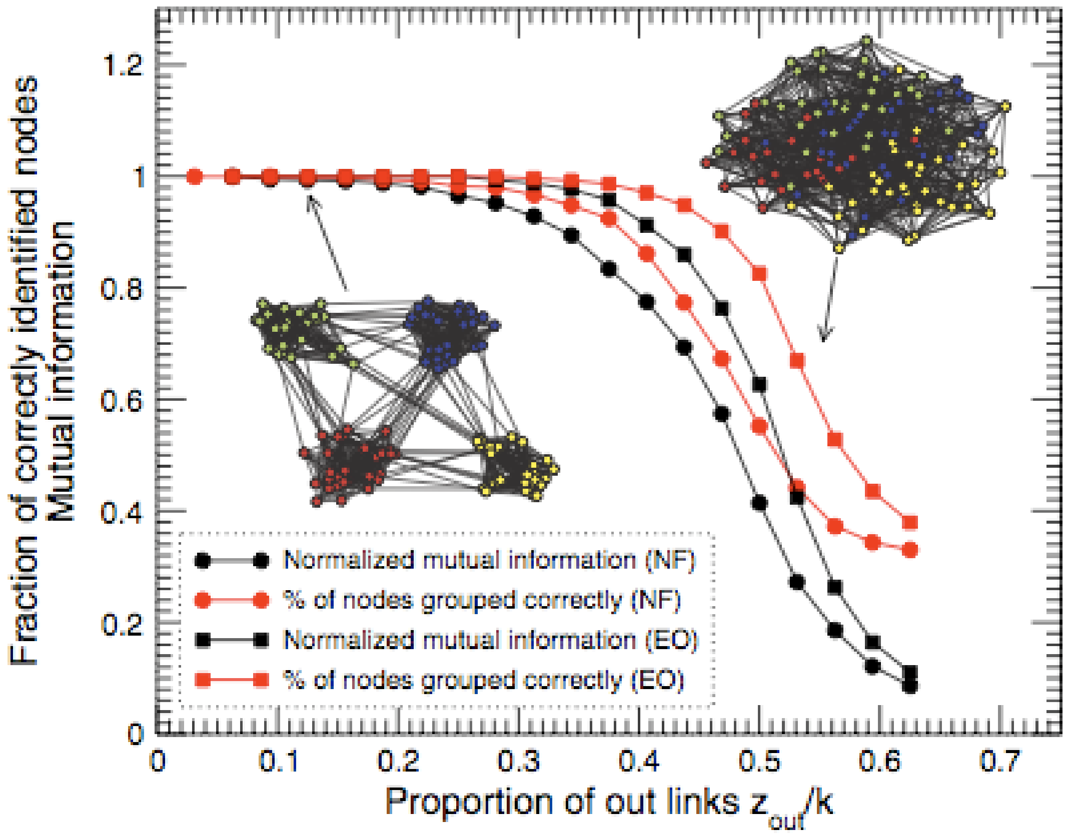
FIG. 3: Krebs' network of books on American politics. Vertices represent books and edges join books frequently purchased by the same readers. Dotted lines divide the four communities found by our algorithm and shapes represent the political alignment of the books: circles (blue) are liberal, squares (red) are conservative, triangles (purple) are centrist or unaligned.

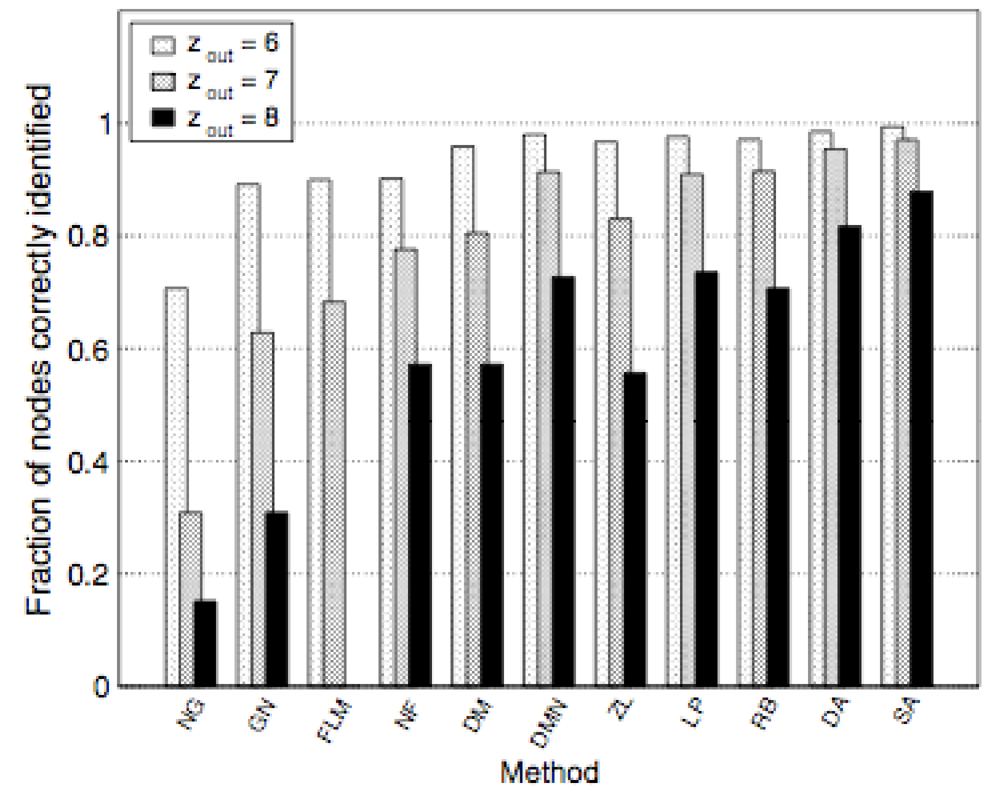


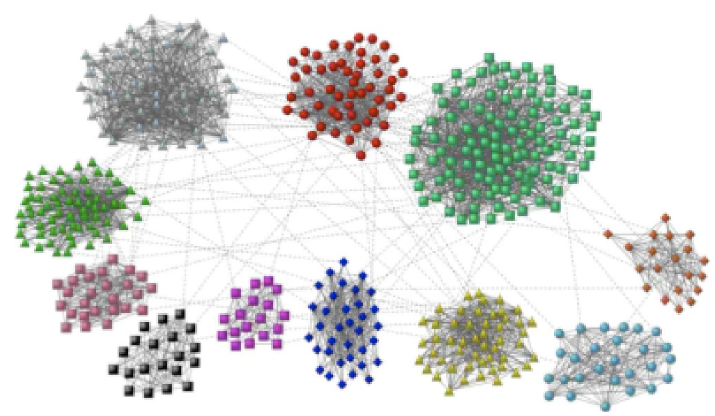
But: meta-data are often unknown. No insurance that modular organization coincides with semantic/cultural organisation

# How to test the methods?

Benchmarks: artificial networks with known community structure.



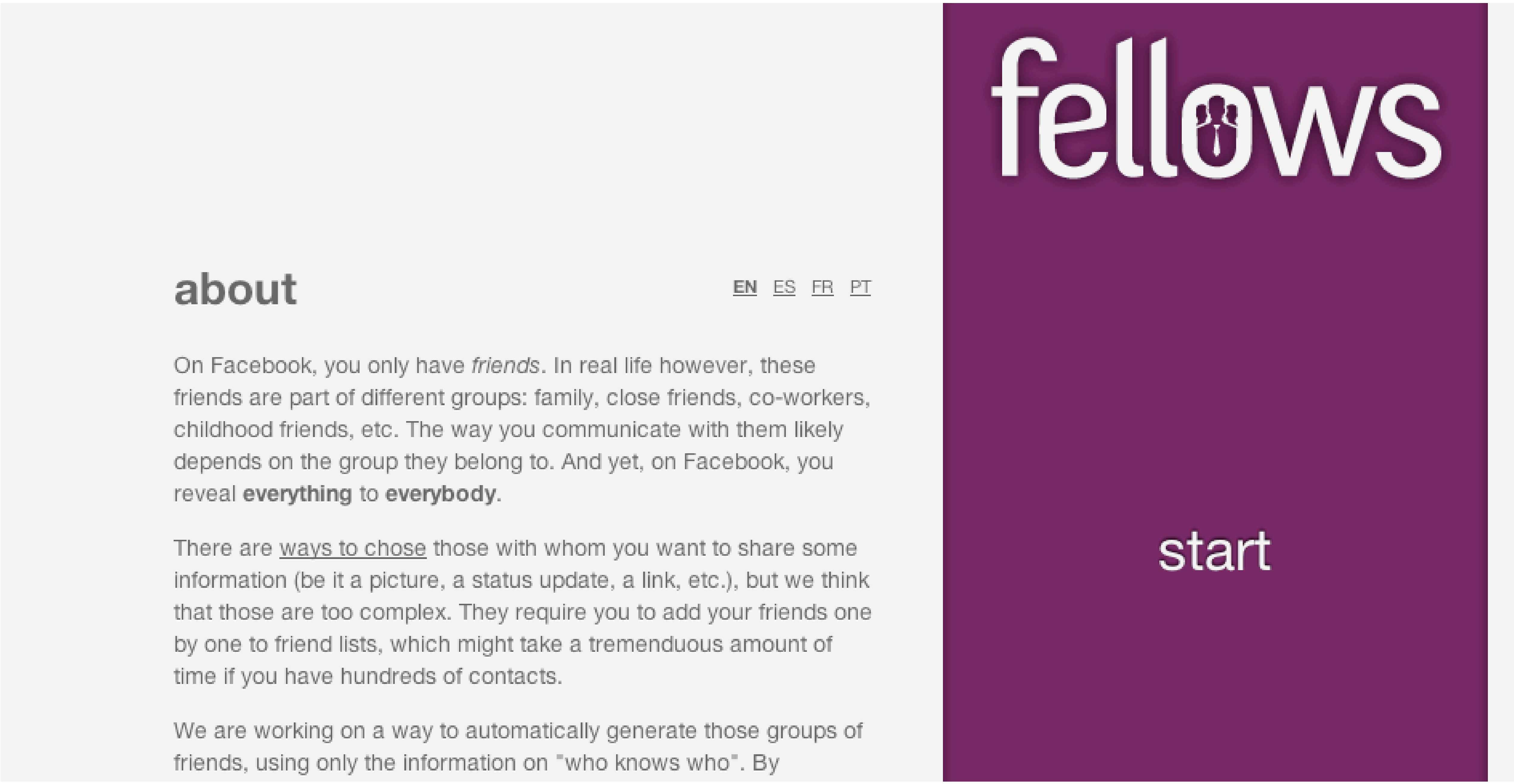



But: random networks (their structure is quite different from real-world networks); in the way the benchmark is built, there is a (hidden) choice for what good partitions should be

Leon Danon, Jordi Duch, Albert Diaz-Guilera, Alex Arenas, J. Stat. Mech. (2005) P09008
Andrea Lancichinetti, Santo Fortunato, and Filippo Radicchi, Phys. Rev. E 78, 046110 (2008)

# How to test the methods?

Benchmarks: ask the people!

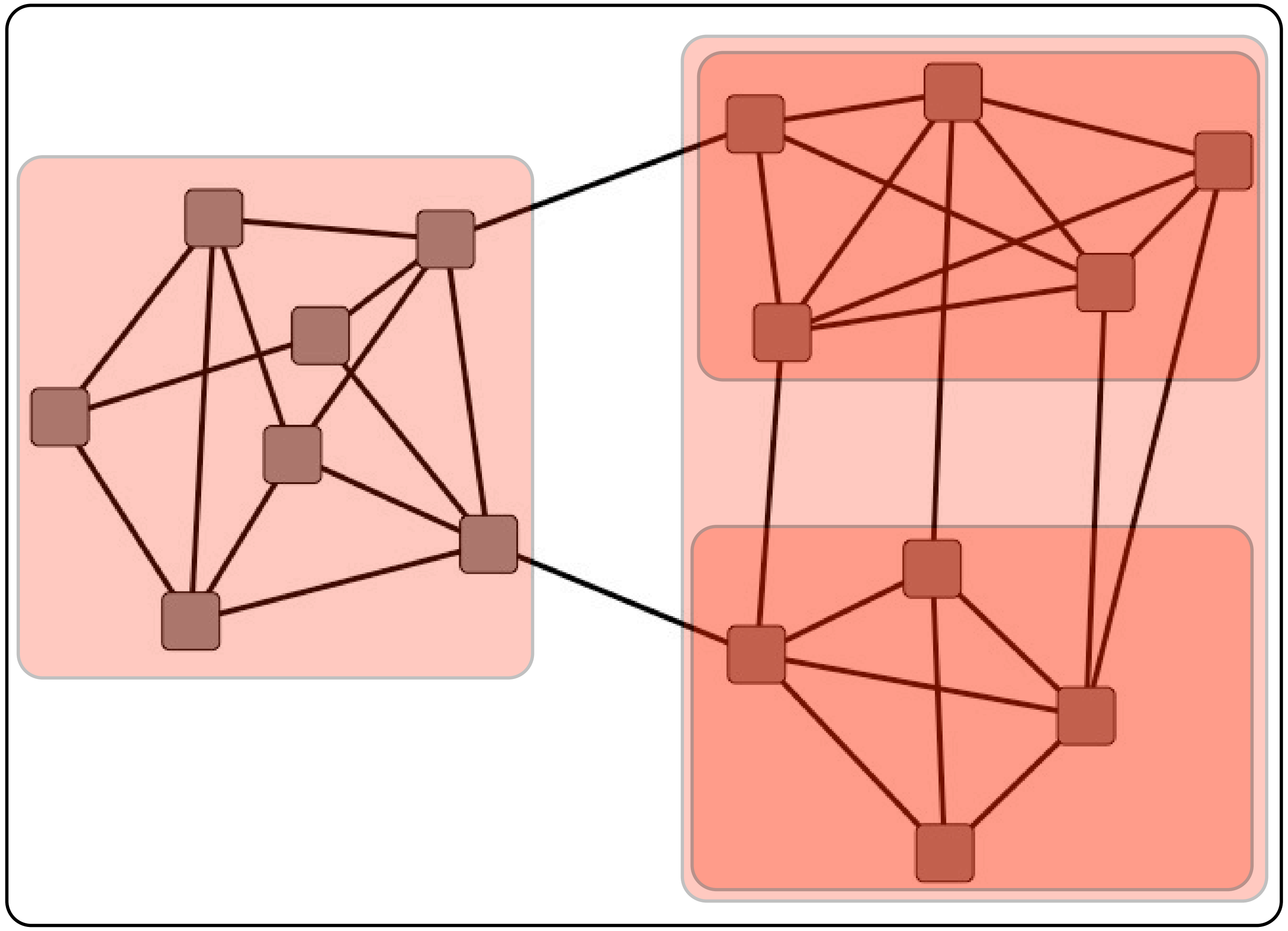


# Beyond modularity: Hierarchy

Resolution limit

Optimising modularity uncovers one partition

What about sub (or hyper)-communities in a hierarchical network?



# Hierarchical Modularity

Resolution limit

Optimising modularity uncovers one partition

What about sub (or hyper)-communities in a hierarchical network?

Reichardt & Bornholdt

$$\boxed{Q_\gamma = \frac{1}{2m} \sum_{i,j} \left[ A_{ij} - \gamma P_{ij} \right] \delta(c_i, c_j)}$$

Arenas et al.

$$\boxed{Q(A_{ij} + r I_{ij})}$$

Tuning parameters allow to uncover communities of different sizes

Reichardt & Bornholdt different of Arenas, except in the case of a regular graph where

$$\boxed{\gamma = 1 + r/\langle k \rangle}$$


*J. Reichardt and S. Bornholdt, Phys. Rev. E **74**, 016110 (2006). Statistical mechanics of community detection*

*A Arenas, A Fernandez, S Gomez, New J. Phys. **10**, 053039 (2008). Analysis of the structure of complex networks at different resolution levels*

# Hierarchical Modularity

Resolution limit

Optimising modularity uncovers one partition

What about sub (or hyper)-communities in a hierarchical network?

Reichardt & Bornholdt

$$\boxed{Q_\gamma = \frac{1}{2m}\sum_{i,j}\Big[A_{ij} - \gamma P_{ij}\Big]\delta(c_i, c_j)}$$

Corrected Arenas et al.

$$\boxed{Q(A_{ij} + r\frac{k_i}{\langle k\rangle}\delta_{ij})}$$

Preserves the eigenvectors of Laplacian (not A) and has a nice dynamical interpretation

Reichardt & Bornholdt = corrected Arenas

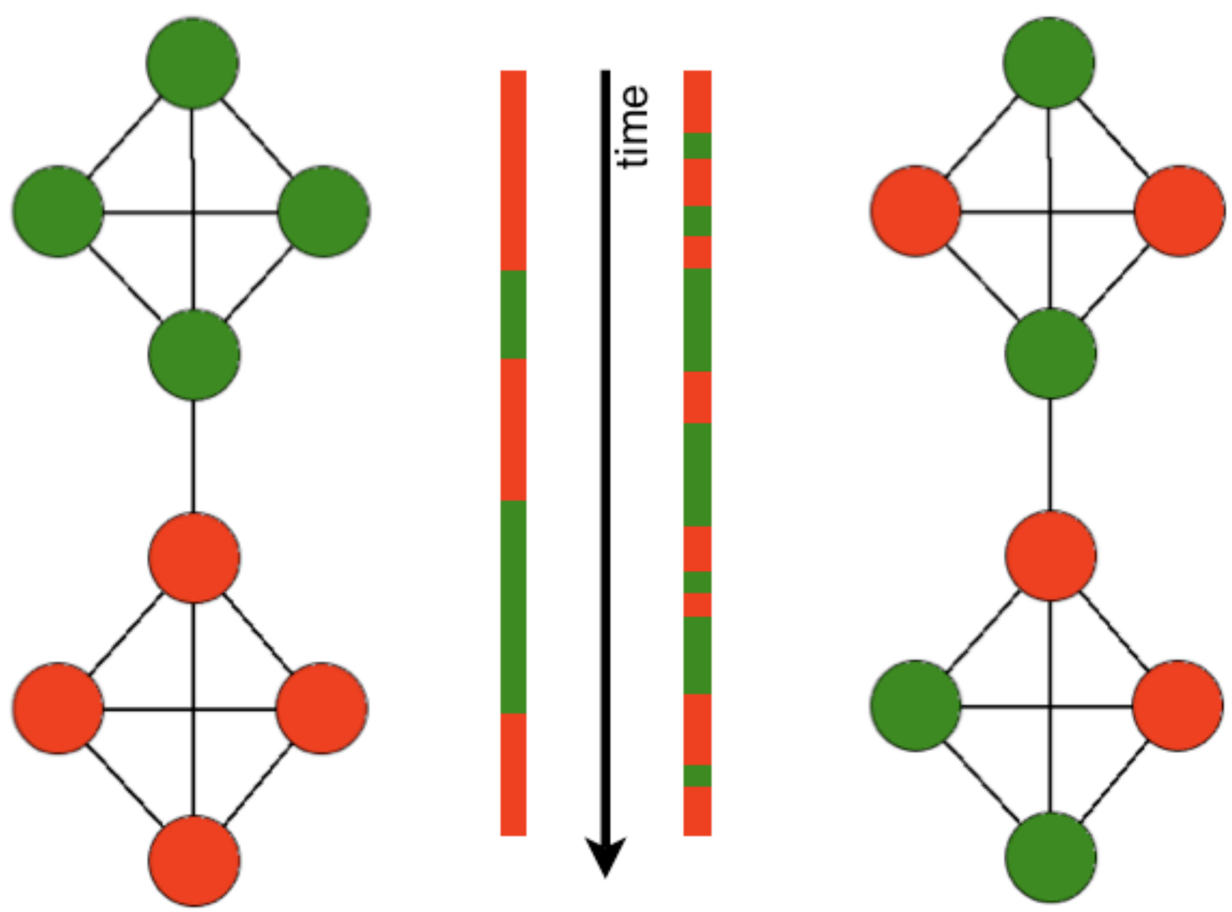
$$\boxed{\gamma = 1 + r/\langle k\rangle}$$

R. Lambiotte, Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt), 2010 Proceedings of the 8th International Symposium on, 546-553 (2010)

# Combinatorial versus flow-based

The quality of a partition is determined by the patterns of a flow within the network: a flow should be trapped for long time periods within a community before escaping it.

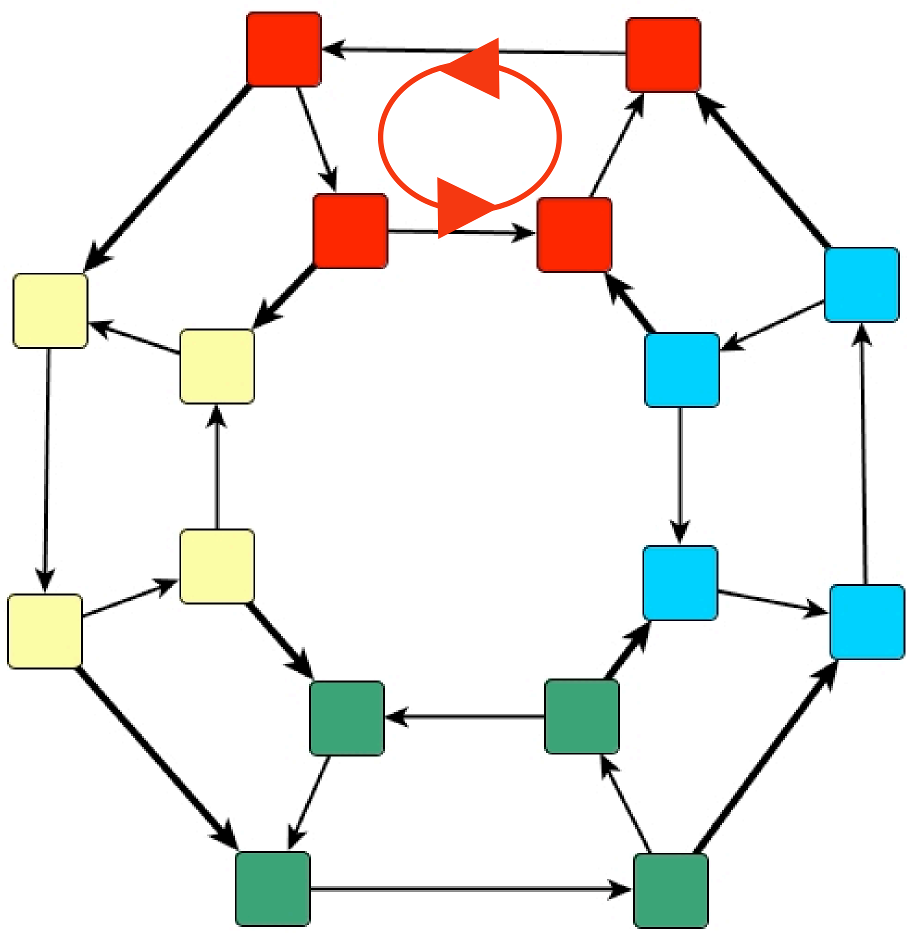
The stability of a partition is defined by the statistical properties of a random walker moving on the graph:



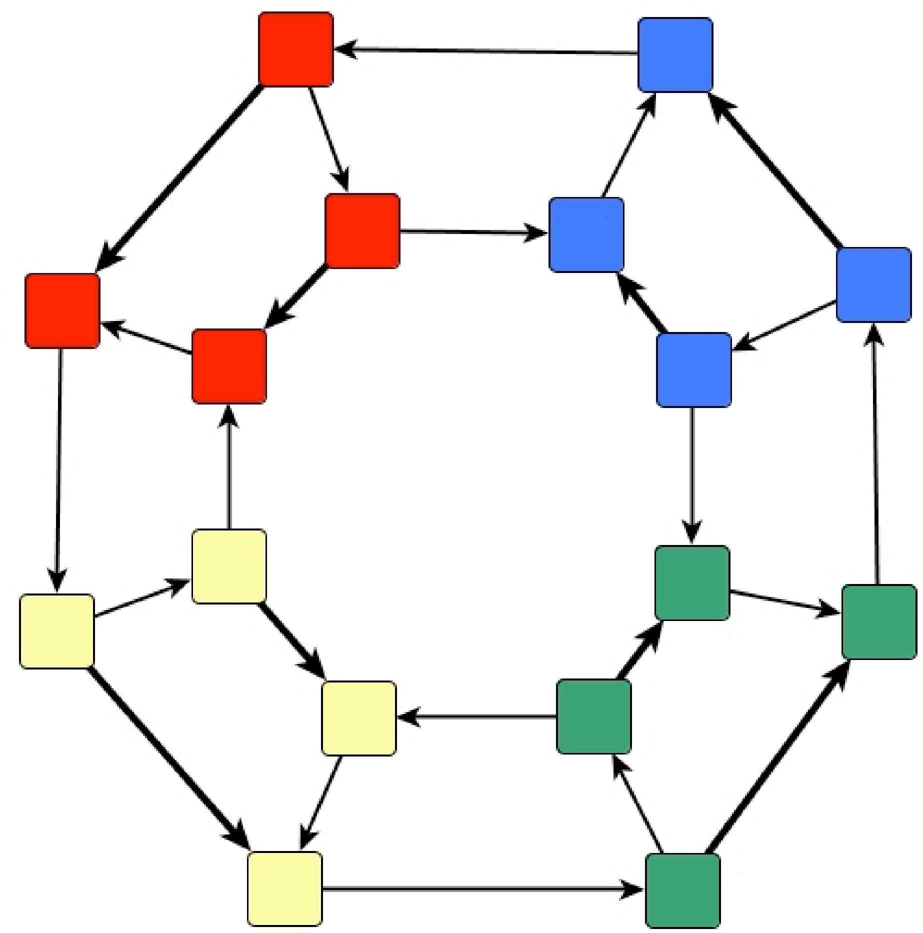

*M. Rosvall and C. T. Bergstrom, PNAS 105, 1118 –1123 (2008)*

*J.-C. Delvenne, S. Yaliraki & M. Barahona, Stability of graph communities across time scales. arXiv:0812.1811.*

# Combinatorial versus flow-based



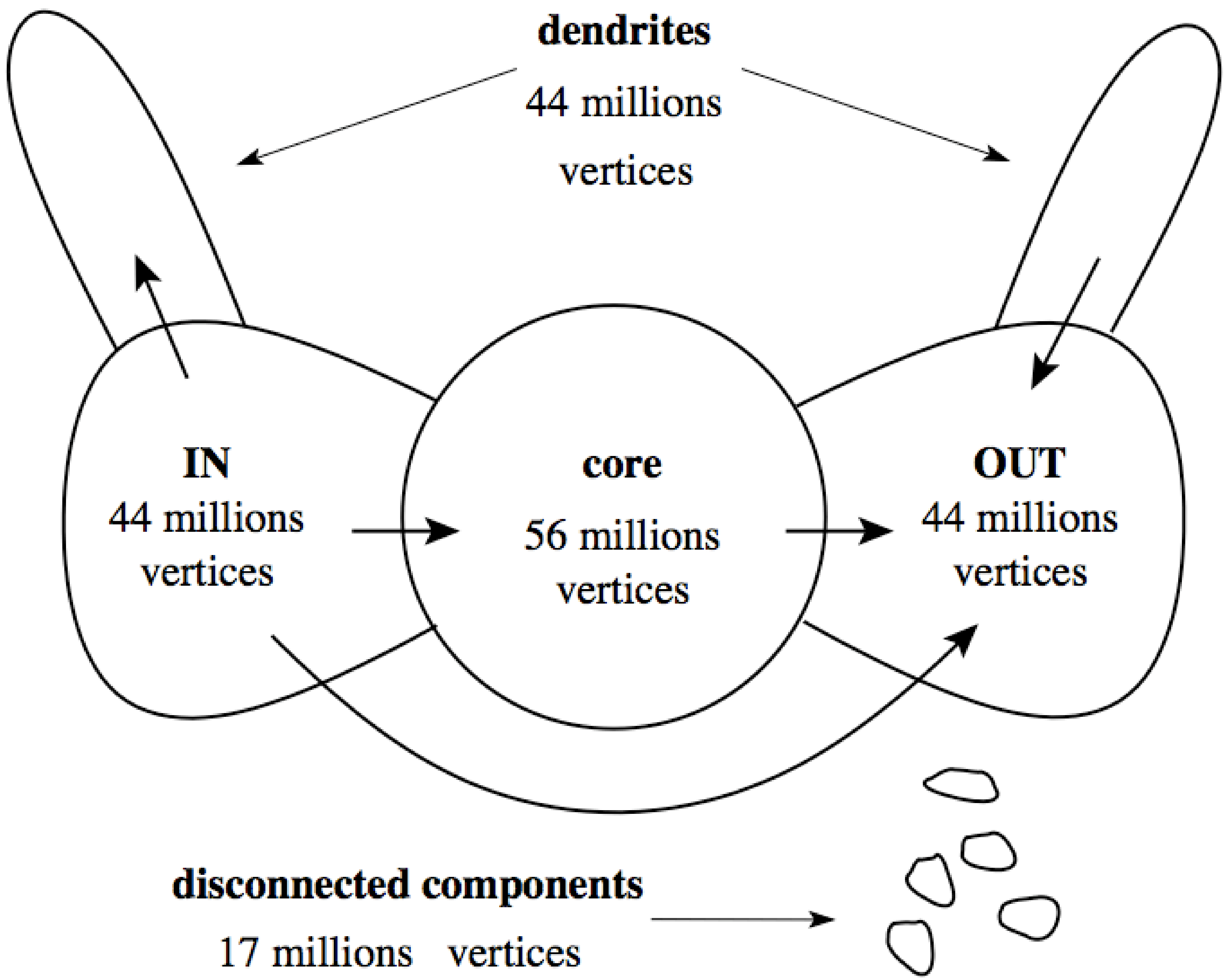
Flow-based modules



Combinatorial modules

*M. Rosvall and C. T. Bergstrom, PNAS 105, 1118 –1123 (2008)*

# Combinatorial versus flow-based

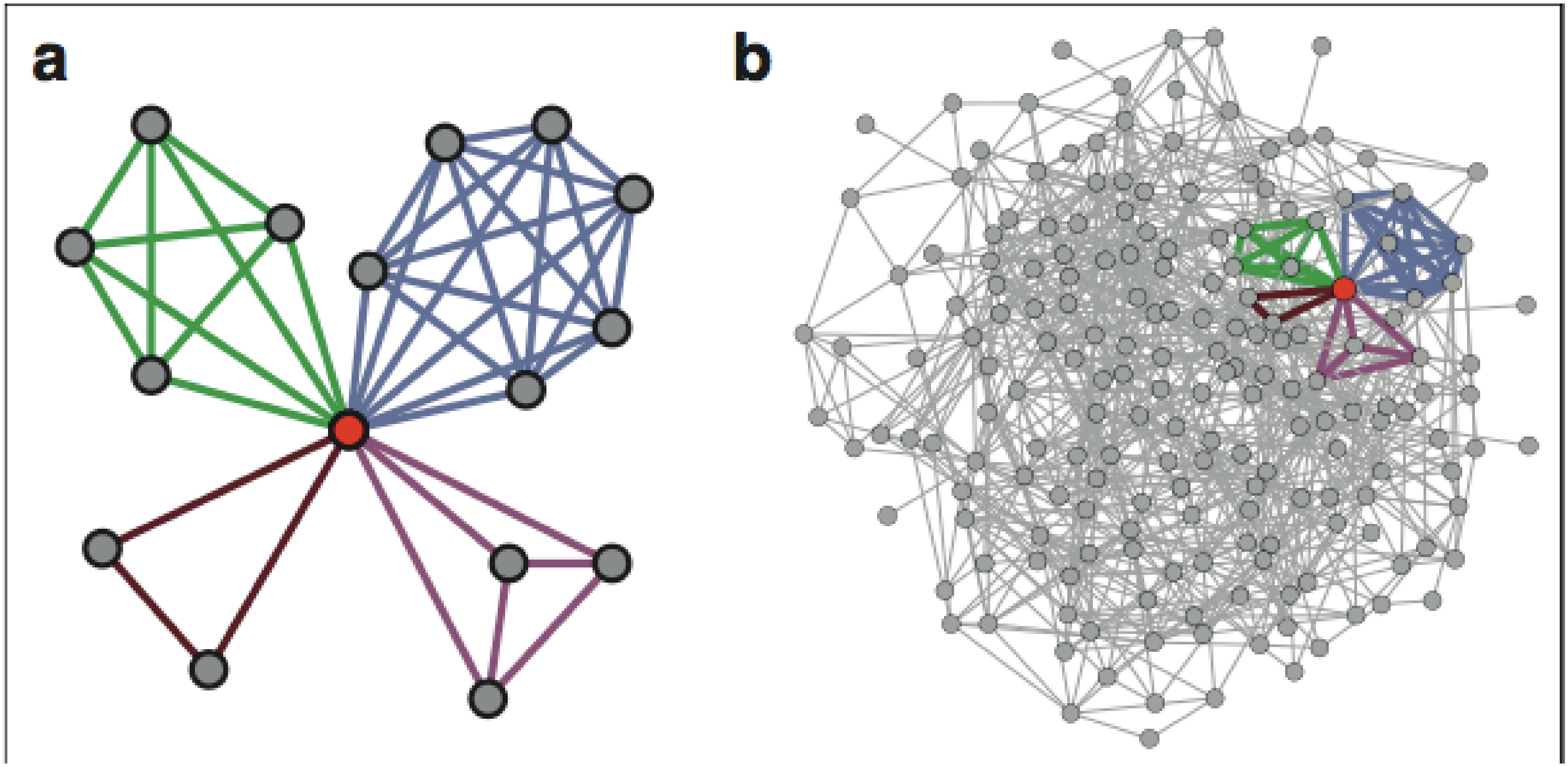


# Beyond partitioning: Overlapping communities

Node partitions are of limited use in systems where communities are overlapping, especially when the overlap is pervasive

We all have different types of friends:

- Family
- Friends
- Work Colleagues

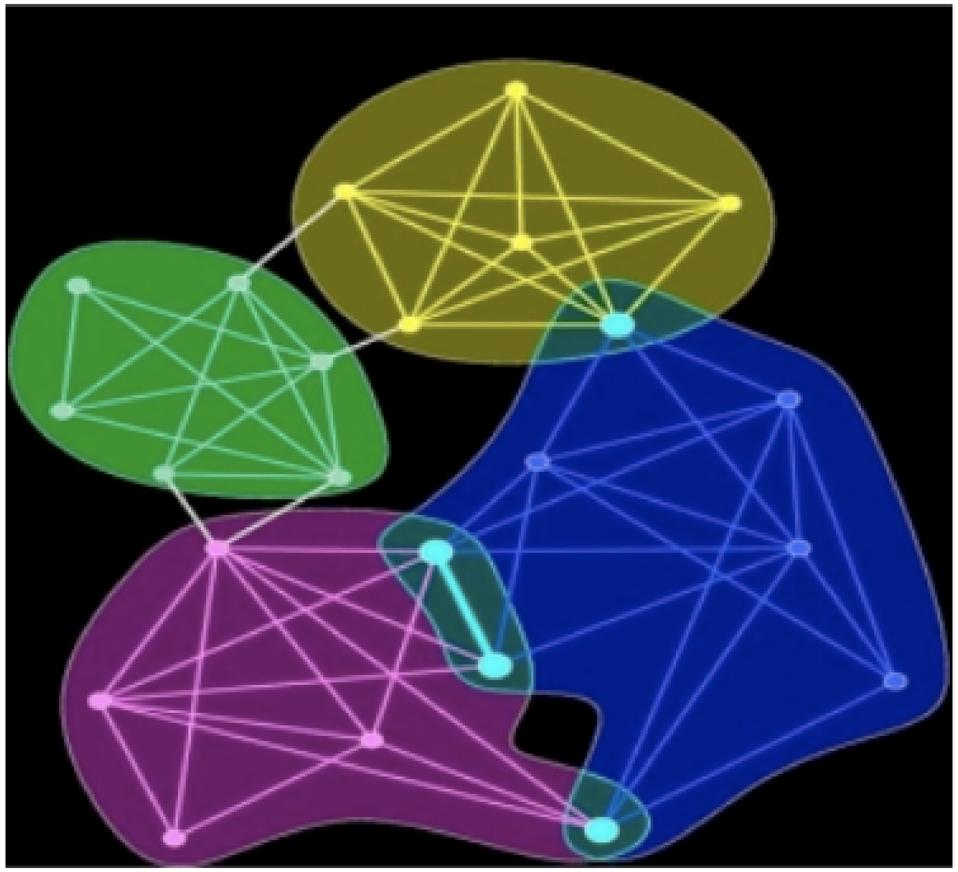


# Overlapping communities

Clique Percolation Method

Principle: looking at connectivity in terms of cliques

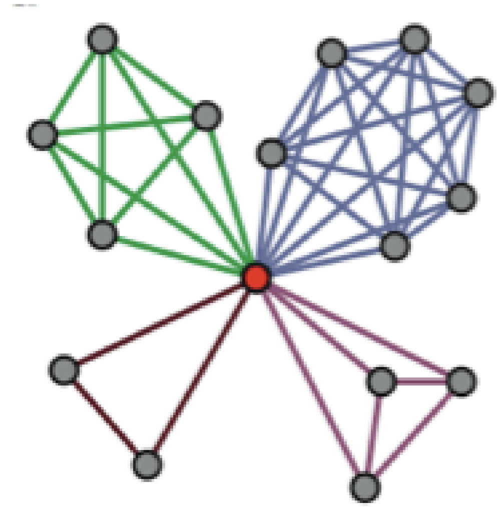
1) Two k-cliques are neighbours if they have a (k-1)-clique in common
2) Connected components = modules



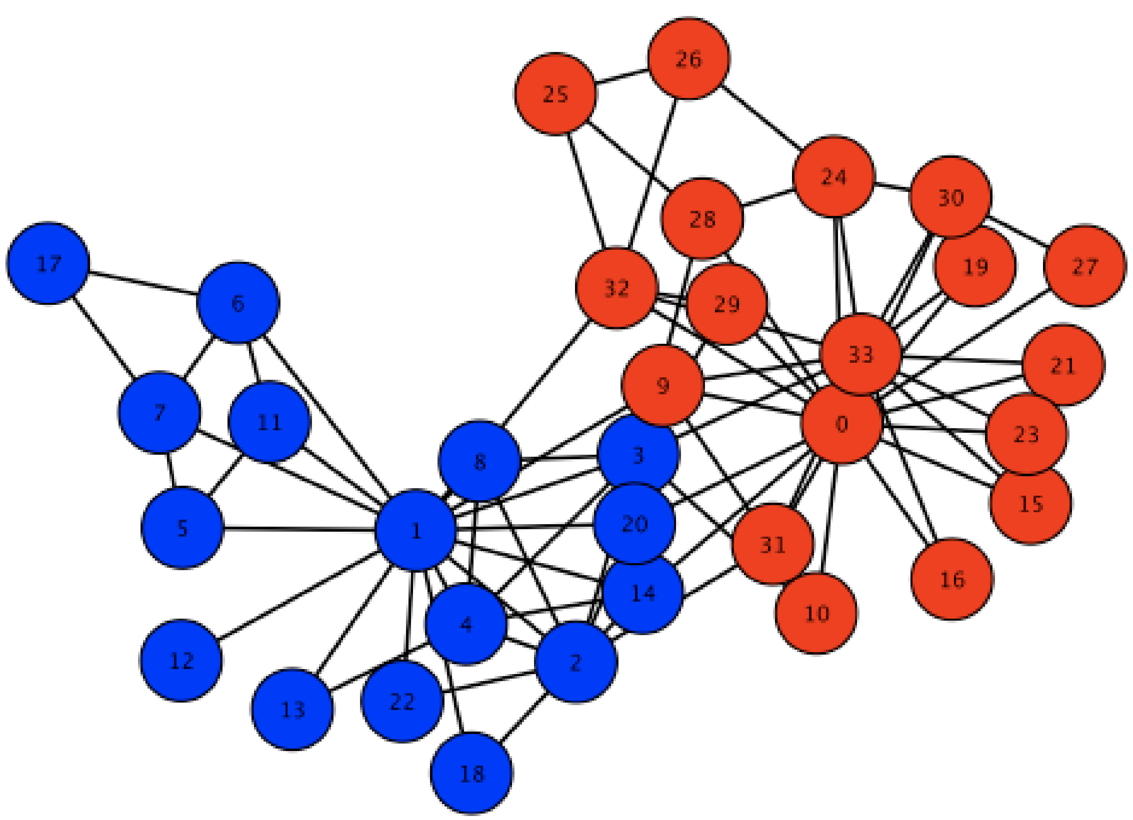
Palla et al., Nature 2005

# Overlapping communities

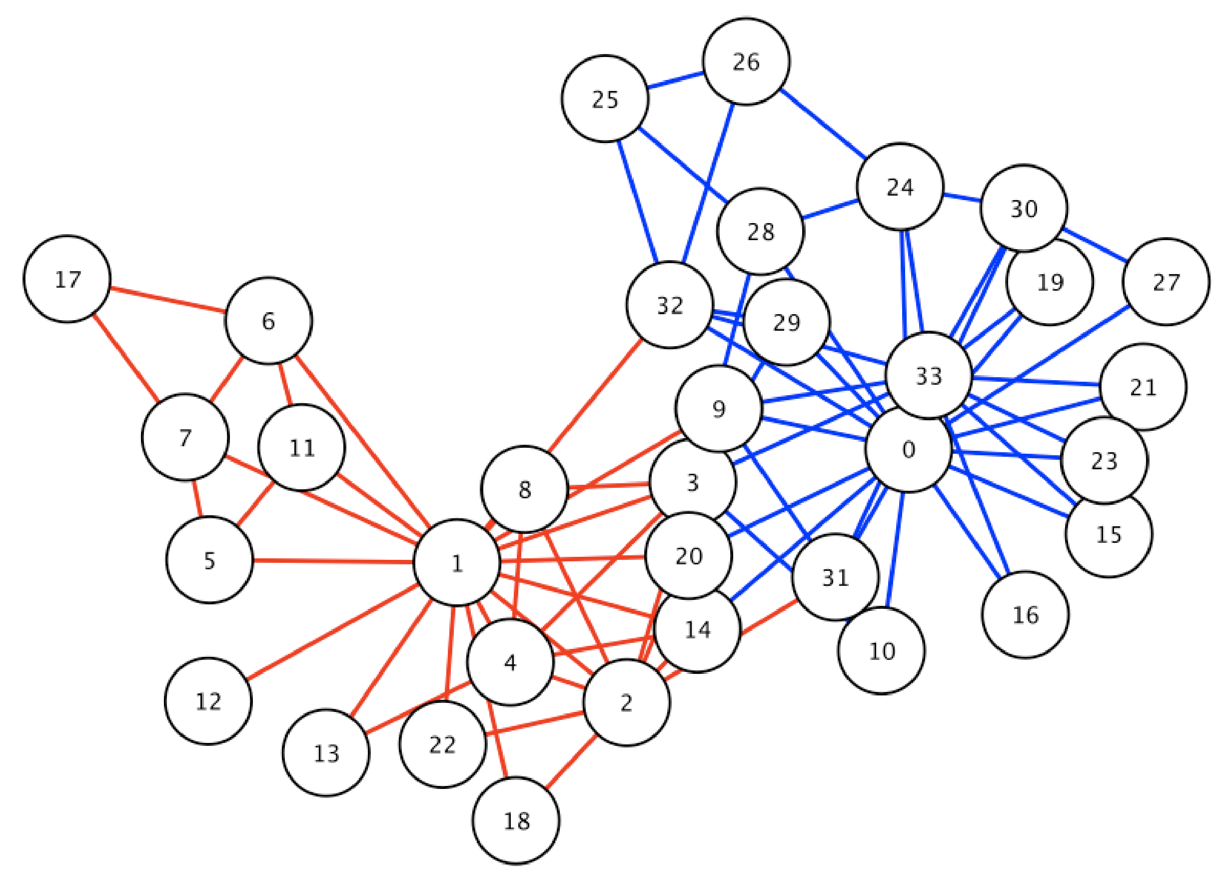
The connection between two persons usually exists for one dominant reason => links therefore typically belong to one single module



Node partition



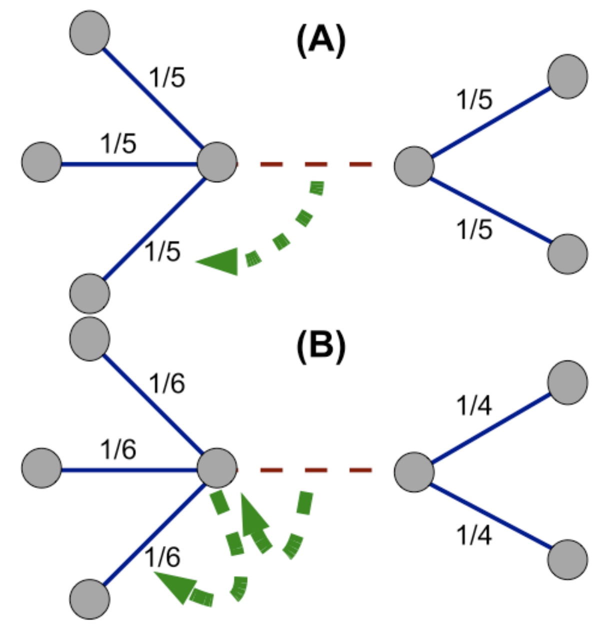

Edge partition



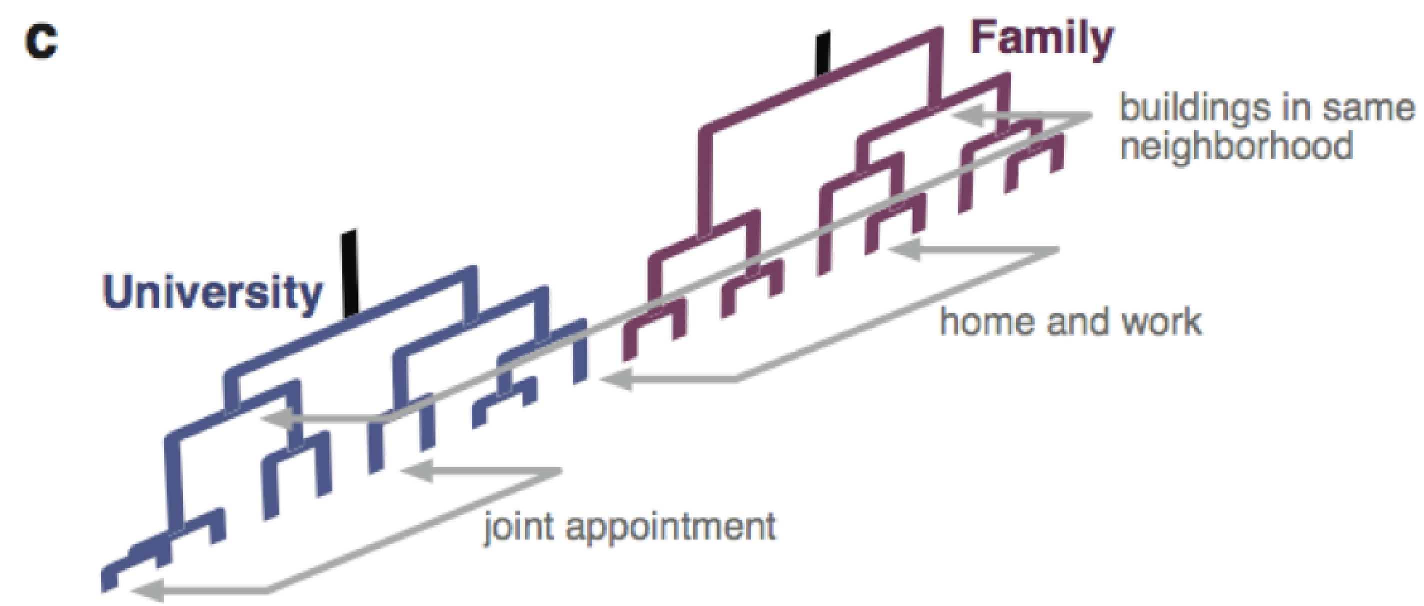

T. Evans and R. Lambiotte, *Phys. Rev. E*, **80** (2009) 016105
Y.-Y. Ahn, J.P. Bagrow, S. Lehmann, Science 2010

# Overlapping communities

Modularity optimization on the Line Graph




Hierarchical clustering on a matrix of proximity between links




T. Evans and R. Lambiotte, *Phys. Rev. E*, **80** (2009) 016105
Y.-Y. Ahn, J.P. Bagrow, S. Lehmann, Science 2010

# Overlapping communities

Local definitions of communities instead of global approaches

Goodness of communities instead of partitions

Allows for overlapping communities

Triangles to Capture Social Cohesion, and $C^3$ method for optimal covering, Friggeri et al.

$$\mathcal{C}(S) = \underbrace{\frac{\triangle(S)}{\binom{|S|}{3}}}_{\text{transitivity}} \times \underbrace{\frac{\triangle(S)}{\triangle(S) + \bar{\triangle}(S)}}_{\text{isolation}}$$

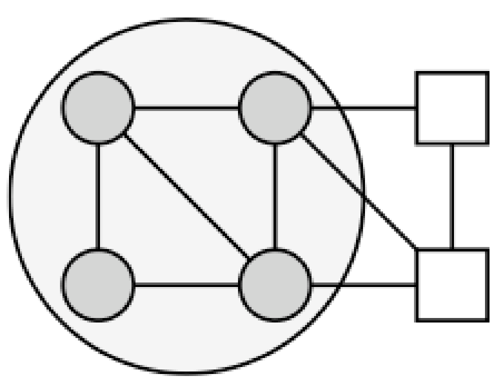
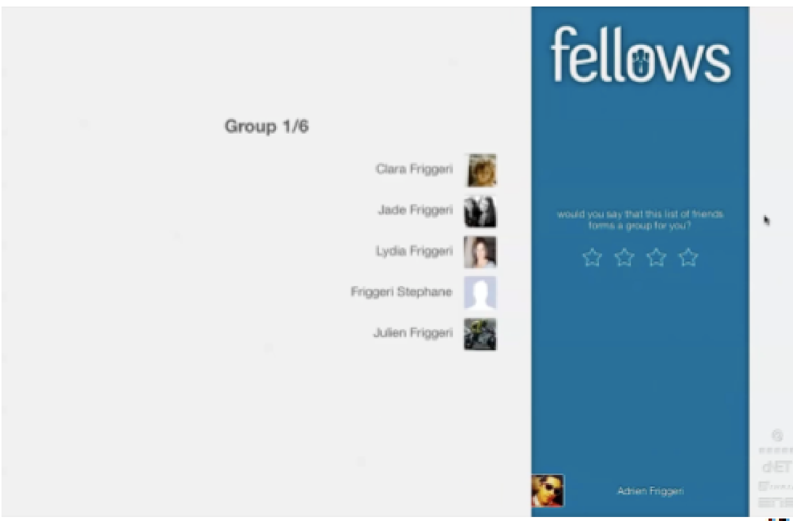
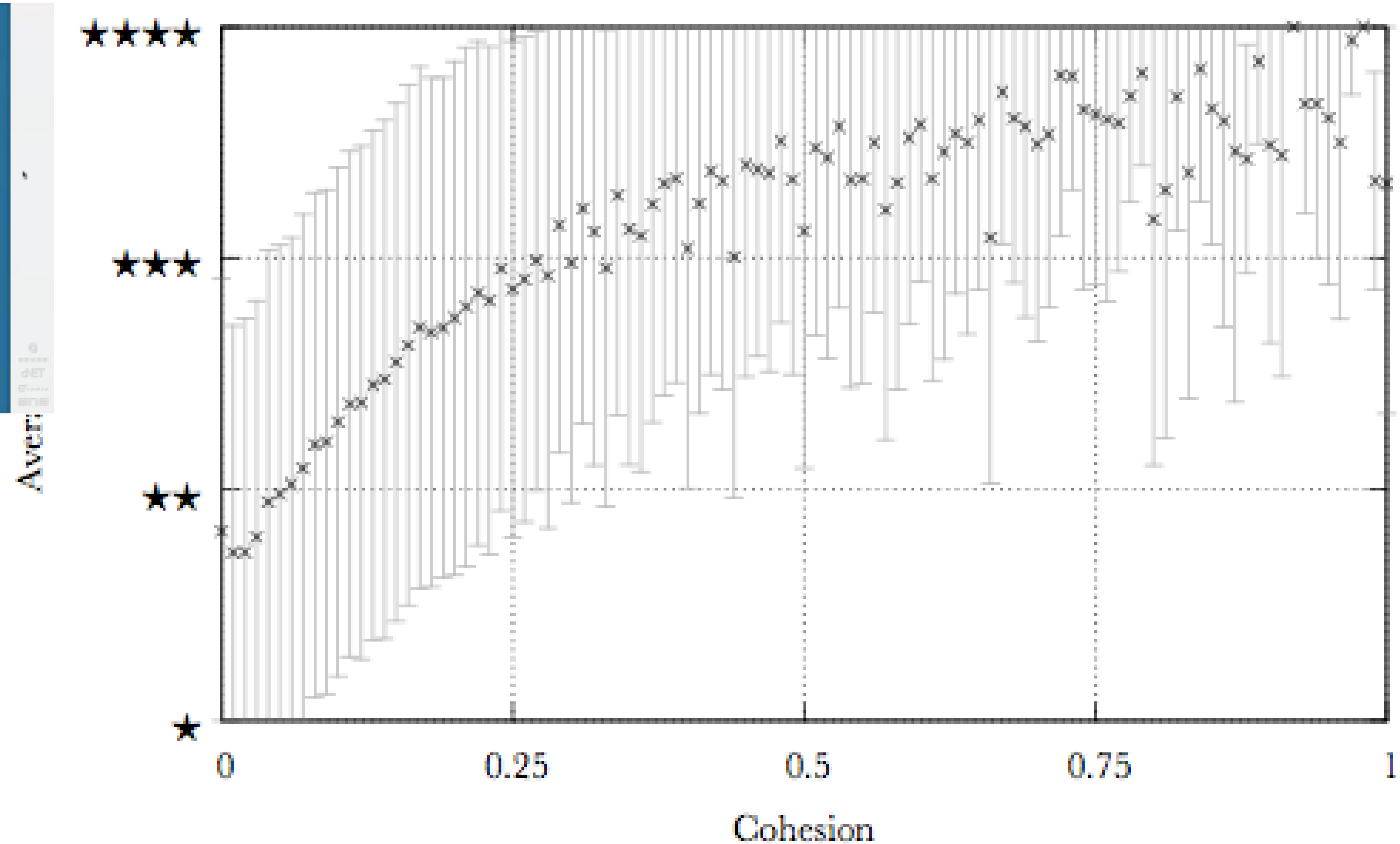


Fig. 3. In this example, the set of circle nodes contains 4 nodes, features 2 inbound triangles and only 1 outbound triangles, leading to a cohesion $\mathcal{C} = \frac{1}{3}$.

# Overlapping communities

Validated on a large-scale experiment on Facebook (*Fellows*): algorithmic communities strongly correlated to the users' perception of the quality of social communities.




*Triangles to Capture Social Cohesion*, A. Friggeri, G. Chelius and E. Fleury, SocialCom (2011)

# Overlapping communities: an Application

Is the community structure of our networks the reflect of our individual psychological traits?

# Overlapping communities: an Application

Is the community structure of our networks the reflect of our individual psychological traits?

The five-factor model of personality, or the big five, is the most comprehensive, reliable and useful set of personality concepts. The idea is that an individual can be associated with 5 scores that correspond to 5 main personality traits.

Personality traits predict a number of real-world behaviors. They, for example, are strong predictors of how marriages turn out: if one of the partner is high in Neuroticism, then divorce is more likely.

| **Dimension** | **High scorers** | **Low scorers** |
|---|---|---|
| **O**penness | Imaginative | Conventional |
| **C**onscientiousness | Organized | Spontaneous |
| **E**xtraversion | Outgoing | Solitary |
| **A**greeableness | Trusting | Competitive |
| **N**euroticism | Prone to stress and worry | Emotionally stable |

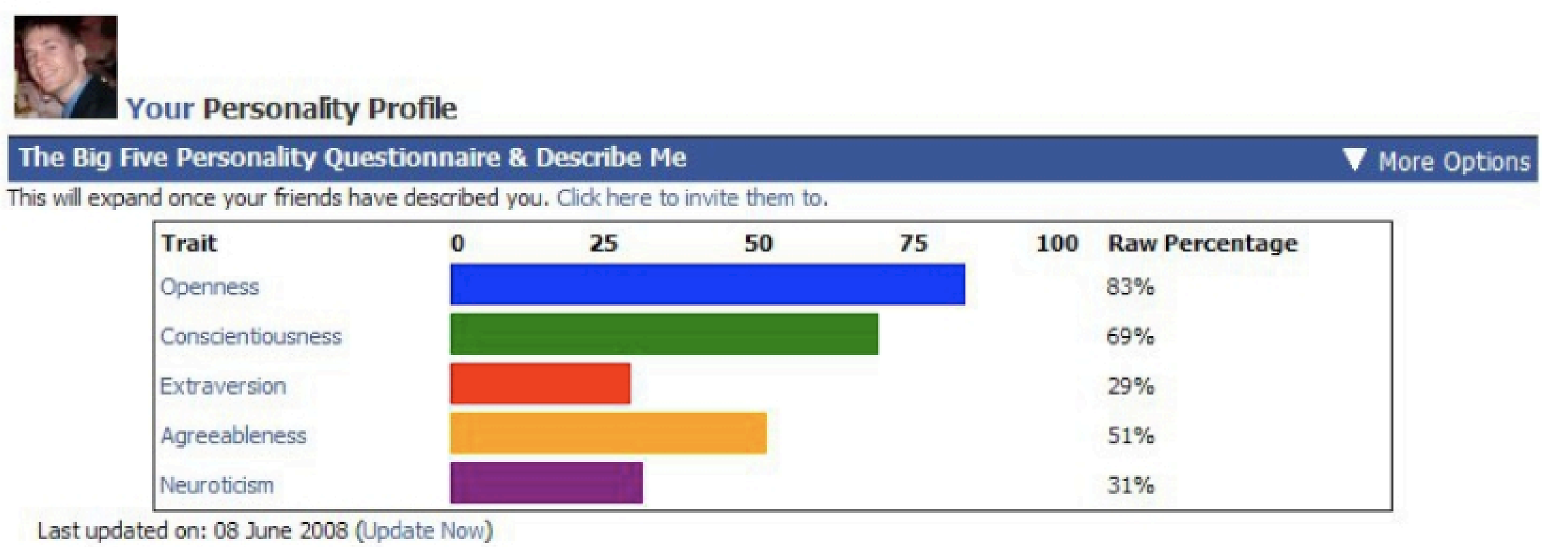
**Table 1: The big five personality dimensions.**

*The Revised NEO Personality Inventory*, P. Costa and R. Mccrae, SAGE Publications (2005)

# Overlapping communities: an Application

Is the community structure of our networks the reflect of our individual psychological traits?

- Facebook application: 5.5 million users
- Users can opt in and give their consent to share their profile information (40%)
- Right incentives: subjects are not paid nor receive college credits. myPersonality users are solely motivated by the prospect of receiving reliable feedback and test results that accurately describe their personalities.
- Unreliable results are removed. Numerous validity tests




- myPersonality is able to obtain test results that are more reliable than those in pen-and-paper studies.
- myPersonality users are far less biased than those studies' subjects for gender, age, and geography.
- VERY large scale data

# Overlapping communities: an Application

Is the community structure of our networks the reflect of our individual psychological traits?

Ego-network of person *John*: friends of *John* and connections between them (*John* does not belong to his ego-network).

Number of friends = size of the ego-network

In the social science, long tradition in analysing and theorising ego-networks, e.g. their connection to social capital: dense ego-networks favor trust and facilitate information flow // open ego-networks indicate bridging capital as individuals bridge structural holes between disconnected others.

~ 50k users with number of friends comprised between 50 and 2000

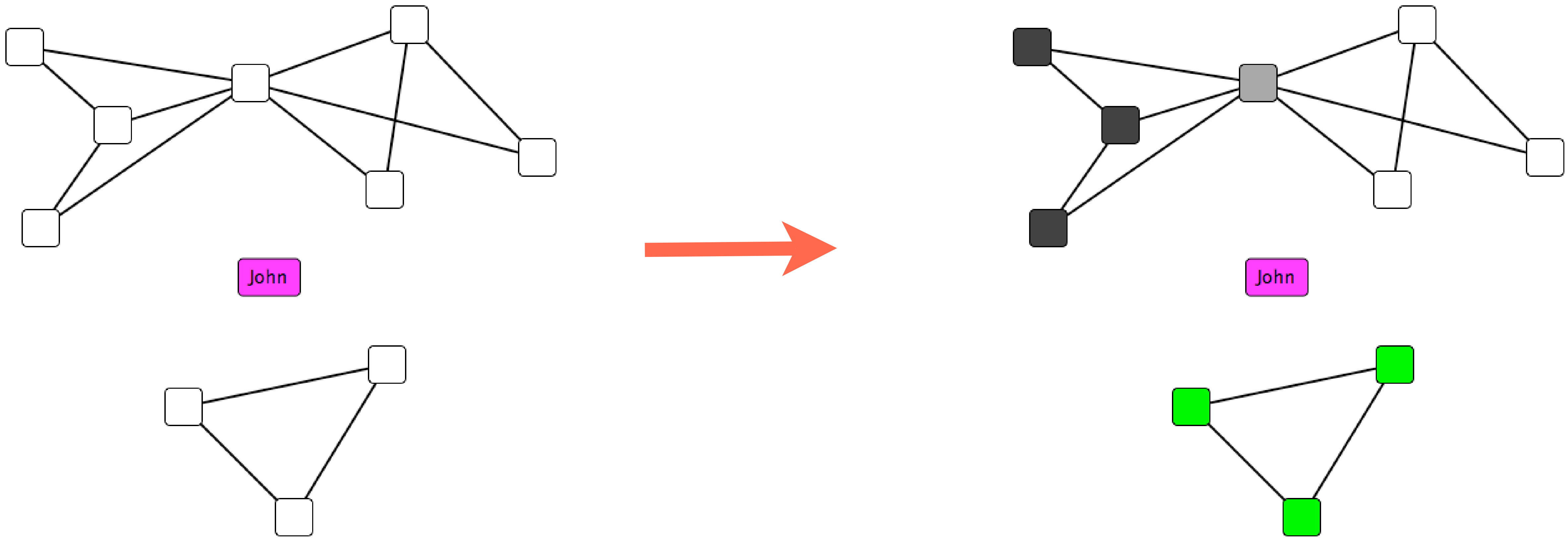
PS: Local network analysis because of our very incomplete knowledge of the whole Facebook network (thousands vs billions)

Extraversion is positively correlated to the size of the ego-network, but what about its organization?

*Psychological Aspects of Social Communities*, A. Friggeri, R. Lambiotte, M. Kosinski and E. Fleury, SocialCom (2012)

# Overlapping communities: an Application

Is the community structure of our networks the reflect of our individual psychological traits?

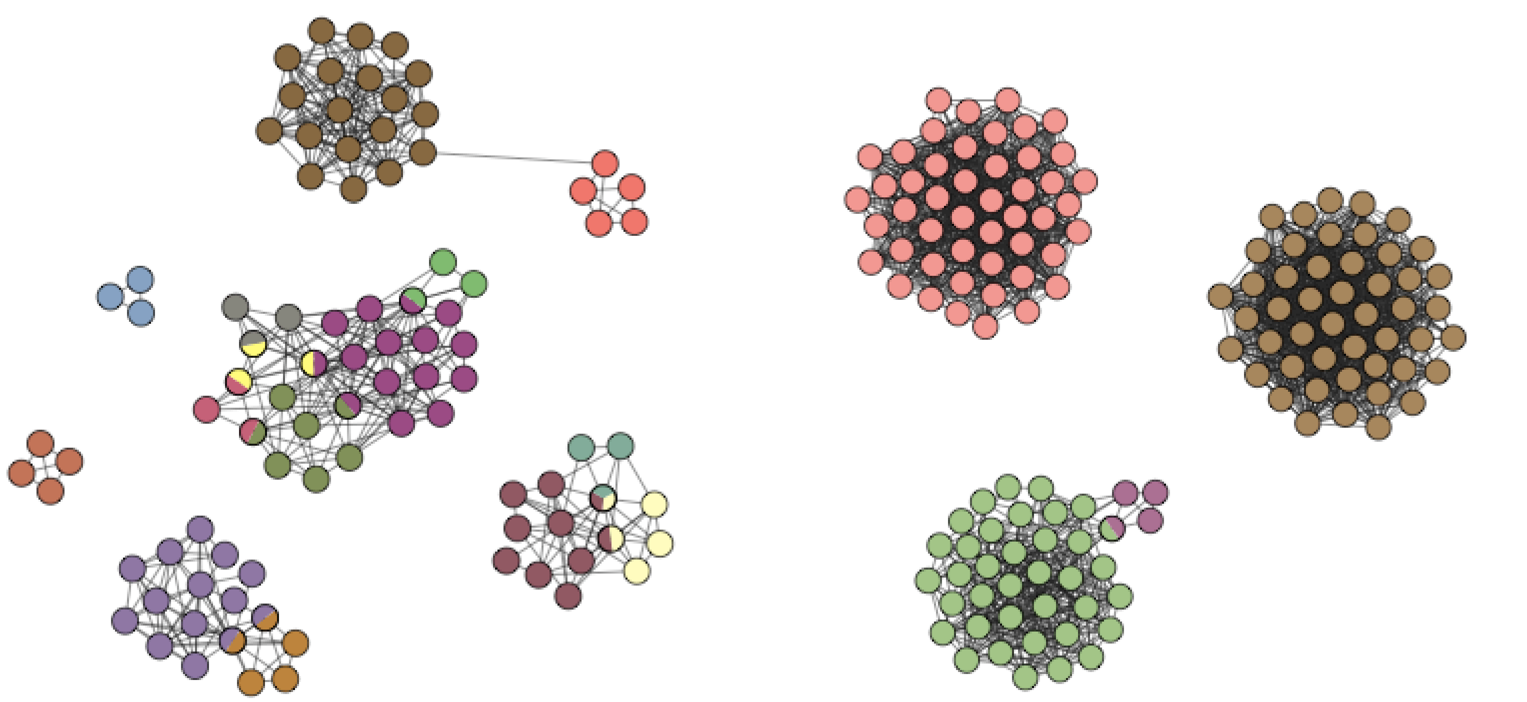



*Psychological Aspects of Social Communities*, A. Friggeri, R. Lambiotte, M. Kosinski and E. Fleury, SocialCom (2012)

# Overlapping communities: an Application

Is the community structure of our networks the reflect of our individual psychological traits?

Introverts tend to have less, larger communities: they *hide* into large communities.
Extroverts exhibit a higher overlap of the communities: they act as *bridges* between communities
No significant difference in the average value of cohesion.



(a) User A, 26 years old: high extraversion (ext = 1.33), 101 friends of which 91 are split across 15 communities of size varying between 3 and 19, and average cohesion $\bar{C} = 0.46$.

(b) User B, 19 years old: low extraversion (ext = −1.21), 145 friends of which 136 are split across 4 communities of size 4, 37, 48 and 48, and average cohesion $\bar{C} = 0.31$.

Fig. 8. Examples of two ego-networks of subjects with different psychological traits and structural features.

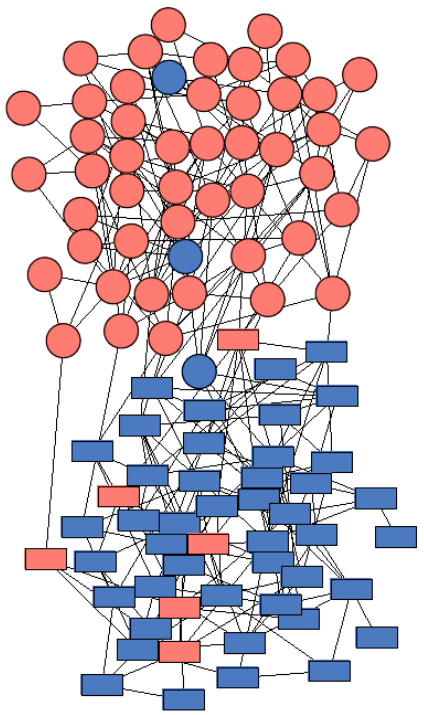
*Psychological Aspects of Social Communities*, A. Friggeri, R. Lambiotte, M. Kosinski and E. Fleury, SocialCom (2012)

# Question: Modularity and Dynamics

Non-overlapping modules naturally produce small-world networks, with the additional, crucial property of time-scale separation corresponding to fast intra-modular processes and slow inter-modular processes.

Near-decomposability:
- the short-time behaviour within a module is approximately independent of the short-time behaviour of the other components
- in the long-run, the behaviour of a module depends in only an aggregate way on the behaviour of other modules (i.e. not on the detailed state of their components).

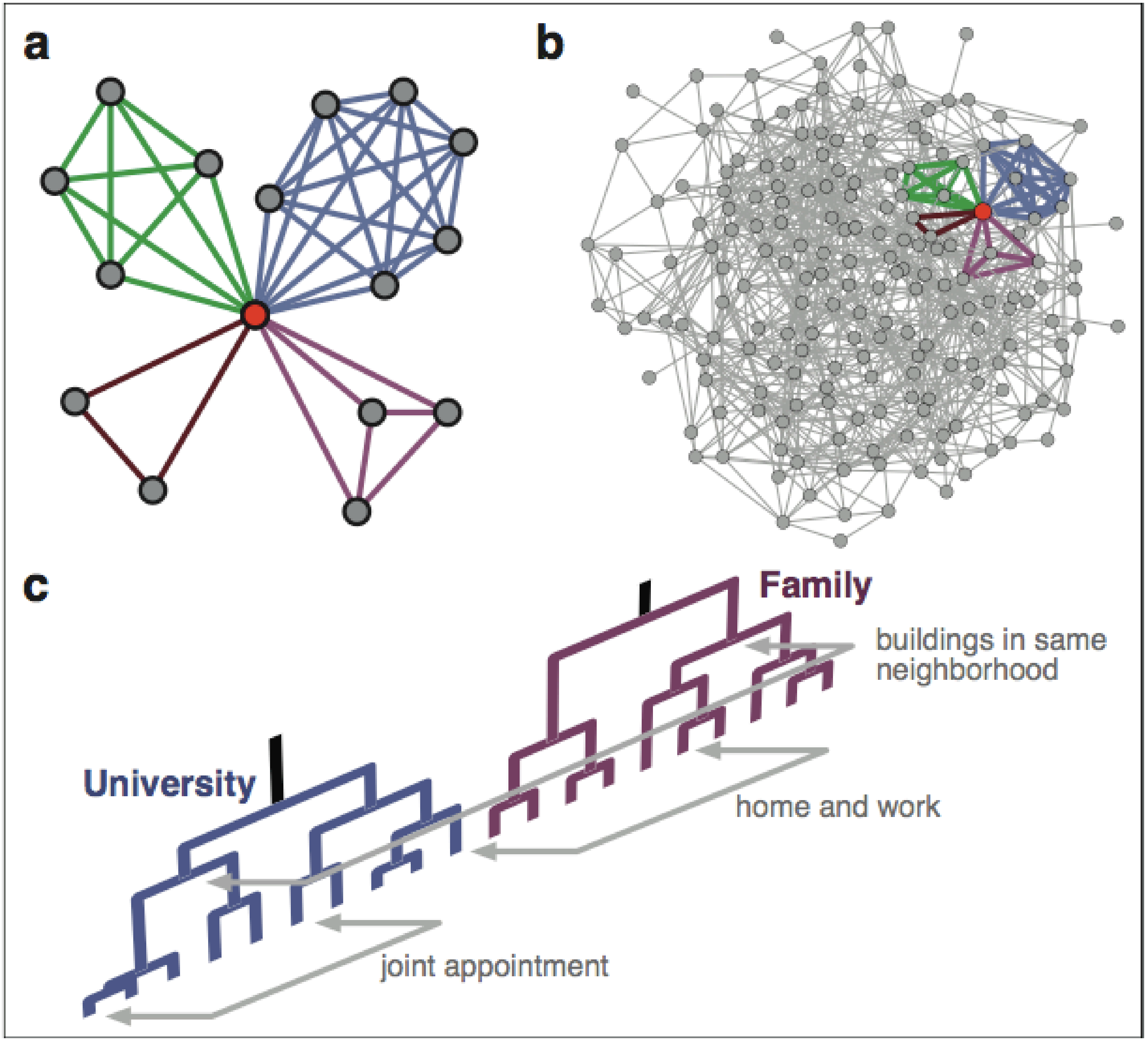
Spectral gap: encapsulation of local activation: local consensus, local synchronization, local ordering, etc.



Aggregation of Variables in Dynamic Systems by HA Simon and A Ando (1961)

# Question: Modularity and Dynamics

What if the networks has pervasive overlaps?



# Why communities?

Many complex systems are modular/hierarchical:

Generic mechanisms driving the emergence of modularity?

D. Meunier, R. Lambiotte and E.T. Bullmore, “Modular and hierarchical organisation in complex brain networks”, to appear in Frontiers in NeuroScience (2010) - 7 pages

# Faster evolution

Simple systems evolve more rapidly if there are stable intermediate forms (modules) than if there are not present.

Among possible complex organizations, hierarchies are observed because they are the ones that have had the time to evolve

Simon, H. (1962). The architecture of complexity. Proceedings of the American Philosophical Society, 106, 467–482.

# Faster evolution

Watchmaker parable:

"There once were two watchmakers, named Hora and Tempus, who made very fine watches. The phones in their workshops rang frequently; new customers were constantly calling them. However, Hora prospered while Tempus became poorer and poorer. In the end, Tempus lost his shop. What was the reason behind this?
The watches consisted of about 1000 parts each.

The watches that Tempus made were designed such that, when he had to put down a partly assembled watch (for instance, to answer the phone), it immediately fell into pieces and had to be reassembled from the basic elements.

Hora had designed his watches so that he could put together subassemblies of about ten components each. Ten of these subassemblies could be put together to make a larger sub- assembly. Finally, ten of the larger subassemblies constituted the whole watch. Each subassembly could be put down without falling apart."

Simon, H. (1962). The architecture of complexity. Proceedings of the American Philosophical Society, 106, 467–482.

# Faster evolution when modular

Probability that an interruption occurs while a piece is added, say p=0.01

**Tempus**

- must complete 1 assembly of 1000 elements
- loses on average 100 pieces (1/0.01)
- finishes an assembly with probability (1-0.01)^1000 ~ 0.000004

**Hora**

- must complete 111 assemblies of 10 elements
- loses on average 5 pieces
- finishes an assembly with probability (1-0.01)^10 ~ 0.9

Time to finish a watch: 100 /(1-0.01)^1000

Time to finish a watch: 111 * 5 /(1-0.01)^10

It will take Tempus 4000 times as long to assemble a swatch

Importance of disturbance due to the environment

Robust intermediate steps during evolution: if the system breaks down (whatever the reason), evolution does not restart from scratch, but from intermediate, stable solutions (back-up!).

# Why communities?

## Generic mechanisms driving the emergence of modularity?

- Watchmaker: intermediate states facilitates the emergence of complex organisation from elementary subsystems

- Separation of time scales: enhances diversity, locally synchronised states

- locally dense but globally sparse: advantages of dense structures while minimising the wiring cost

- in social systems, offer the right balance between dense networks (foster trust, facilitate diffusion of complex knowledge), and open networks (small diameter, ensures connectivity, facilitates diffusion of “simple” knowledge)

- naturally emerges from co-evolution and duplication processes (see Modularity “for free” in genome architecture? Ricard V. Sole and Pau Fernandez)

- Optimality of modular networks at performing tasks in a changing environment (Kashtan, N. and Alon, U. (2005) Spontaneous evolution of modularity and network motifs. Proc. Natl. Acad. Sci. USA, 102:13773--13778.)

- enhanced adaptivity and dynamical complexity, e.g. transient “chimera” states

- delivers highly adaptive processing systems and to solve the dynamical demands imposed by global integration and functional segregation (brain organisation)

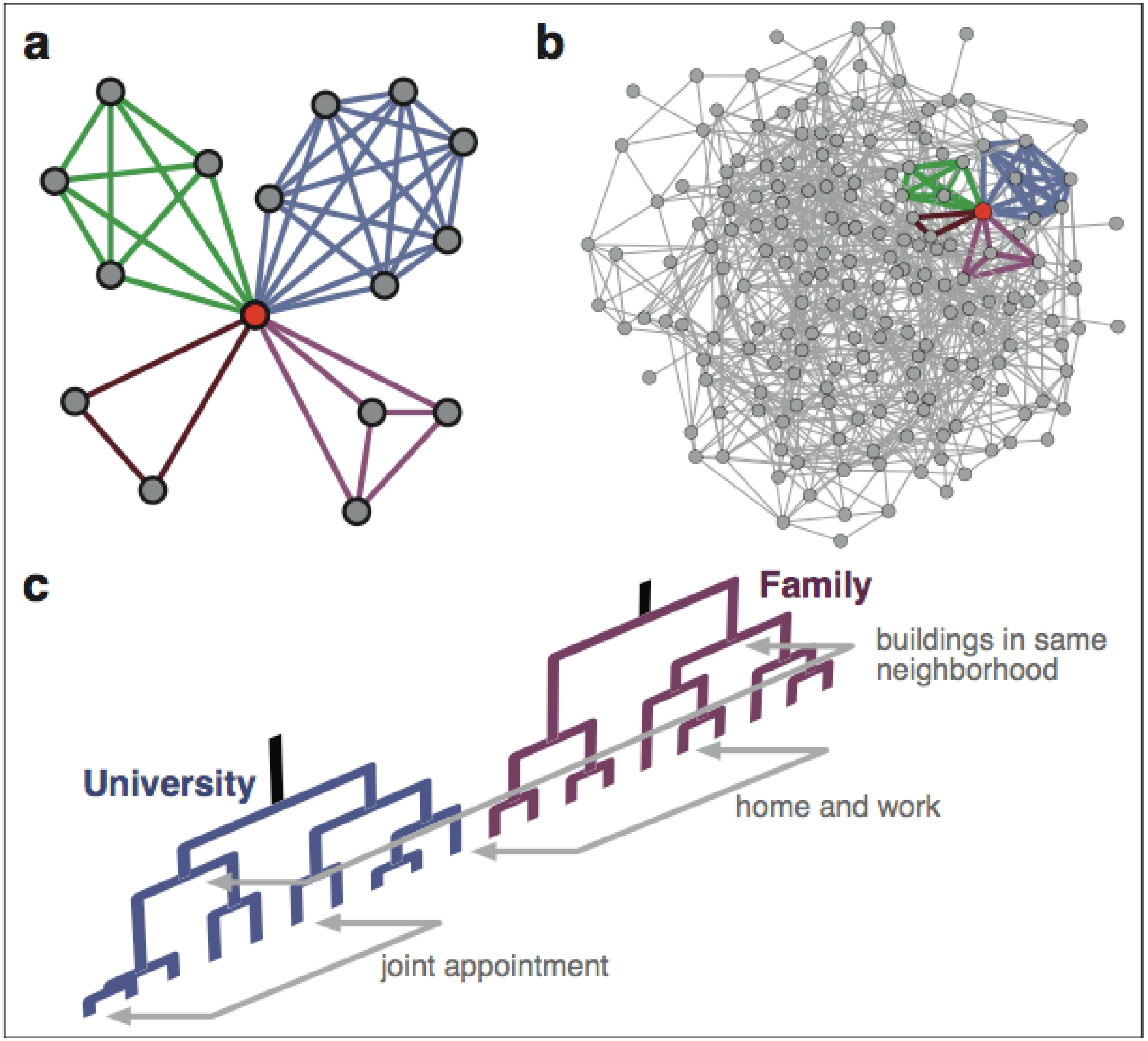
D. Meunier, R. Lambiotte and E.T. Bullmore, “Modular and hierarchical organisation in complex brain networks”, to appear in Frontiers in NeuroScience (2010) - 7 pages

# Overlapping modules and Multiplexity

Non-overlapping modules => different types of nodes

Overlapping modules => different types of edges

What is the meaning of edges? What type of interaction do they represent?



# Network science

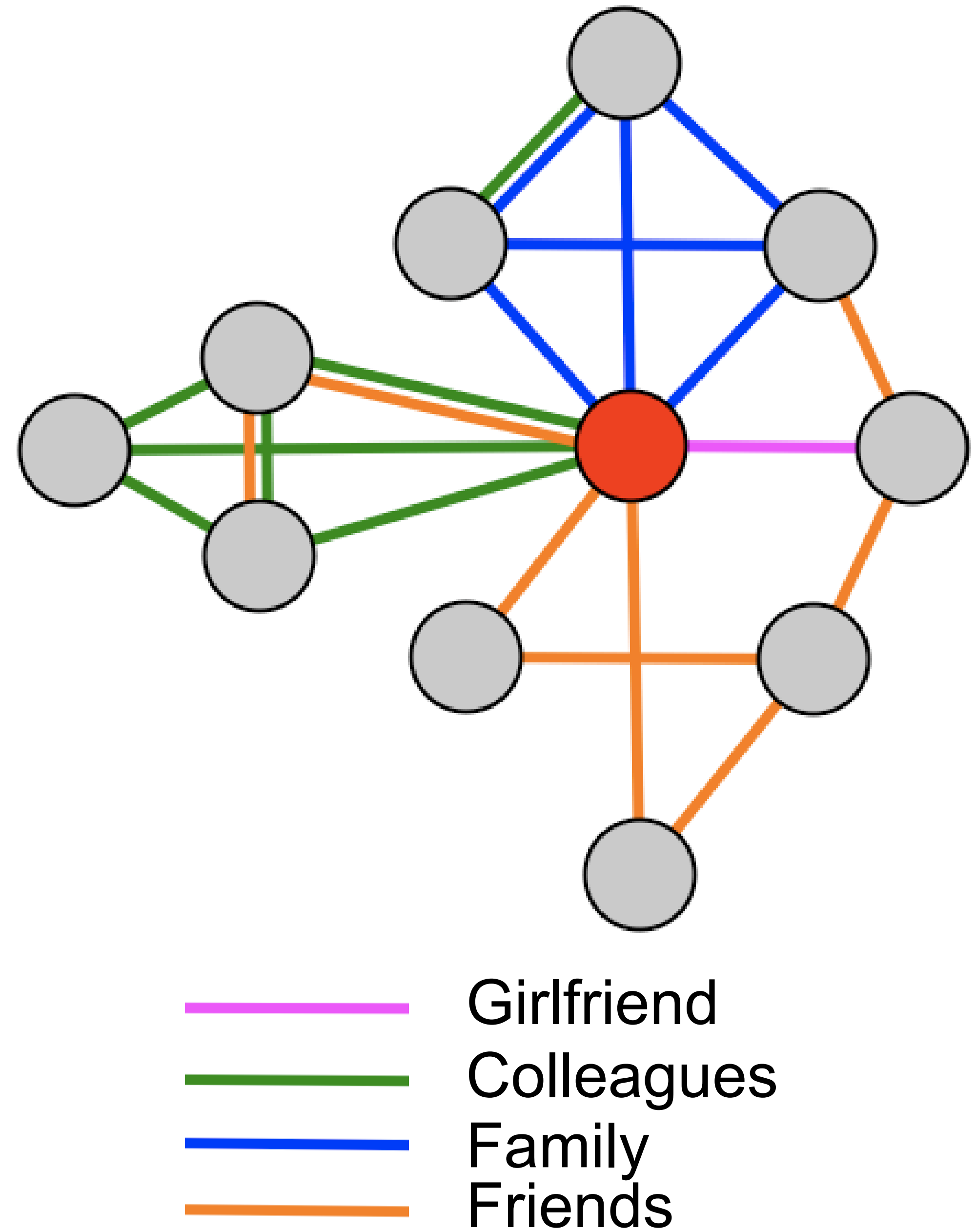
... is blind to the existence of several types of social interactions between individuals

Relational ties are highly diverse and can represent a feeling, communication, exchange of goods (trade) or behavioural interactions
BUT electronic logs typically capture one channel of communication

Contrary to nodes (characterised by their age, sex, location, etc), the nature of interaction (family or work?) is usually unavailable in electronic data-sets

A society is characterised by the superposition of its constitutive socio-economic networks, all defined on the same set of nodes (multiplex networks)

A systemic understanding of a whole society can only be achieved by understanding these individual networks and how they influence and co-construct each other.




M. McPherson, L. Smith-Lovin and J.M. Cook (2001) Annu. Rev. Sociol. 27, 415.
K. Lewis, J. Kaufman, M. Gonzalez, A. Wimmer, and N. Christakis (2008) Social Networks 30, pp. 330-342.
S Wuchty, PNAS 2009 106 (36) 15099-15100
N Eagle, A Pentland and D Lazer, PNAS 2009 106 (36) 15274-15278

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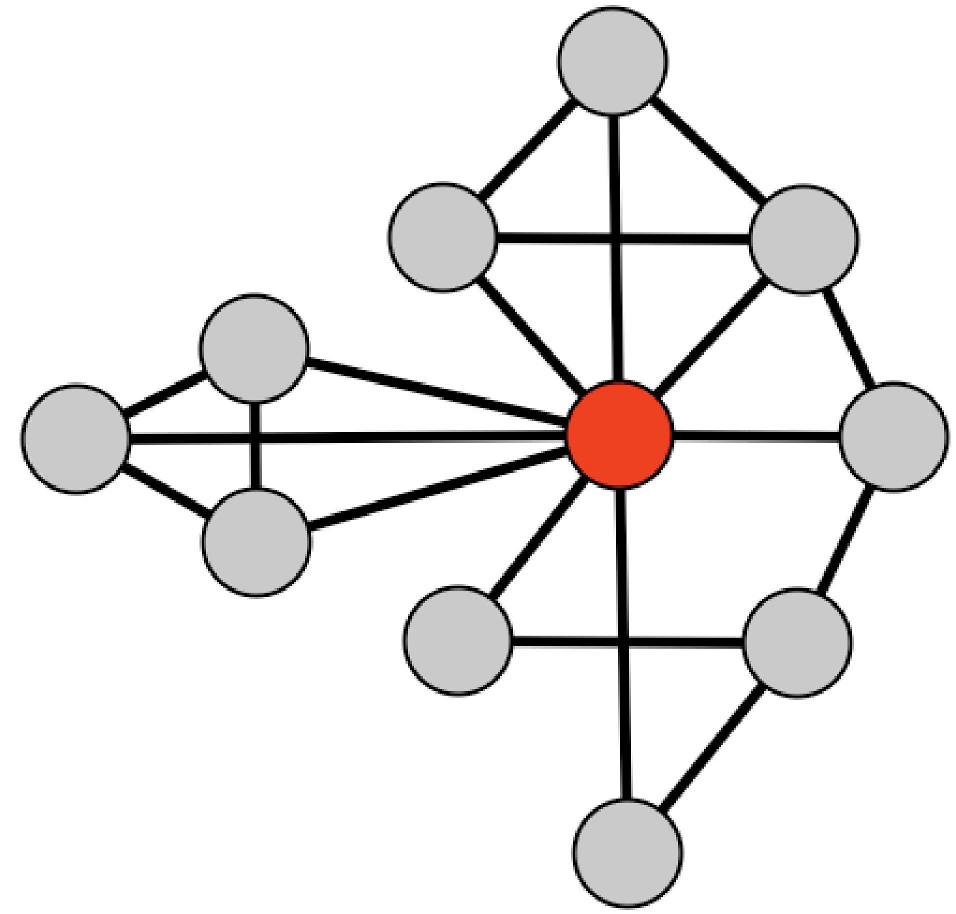

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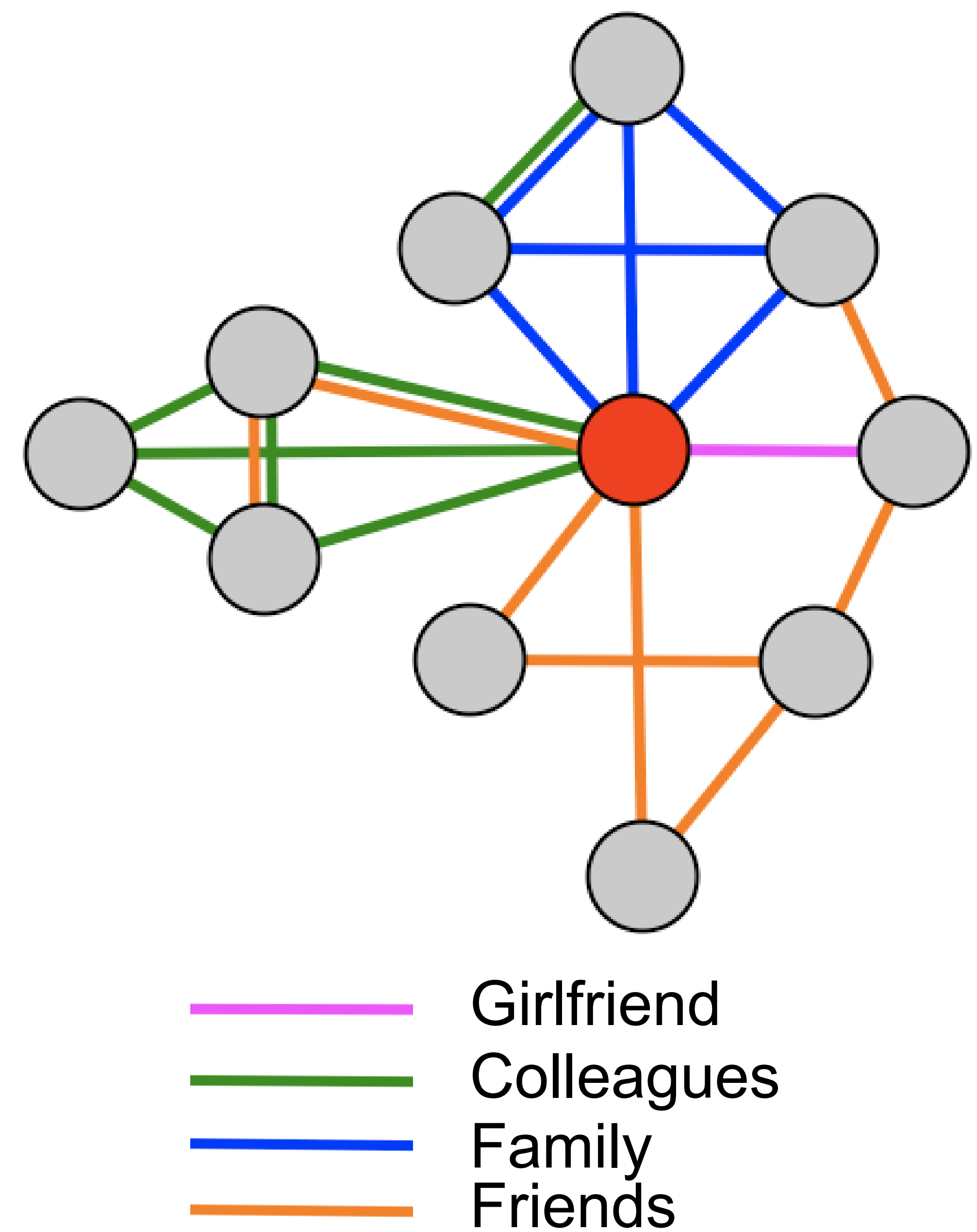


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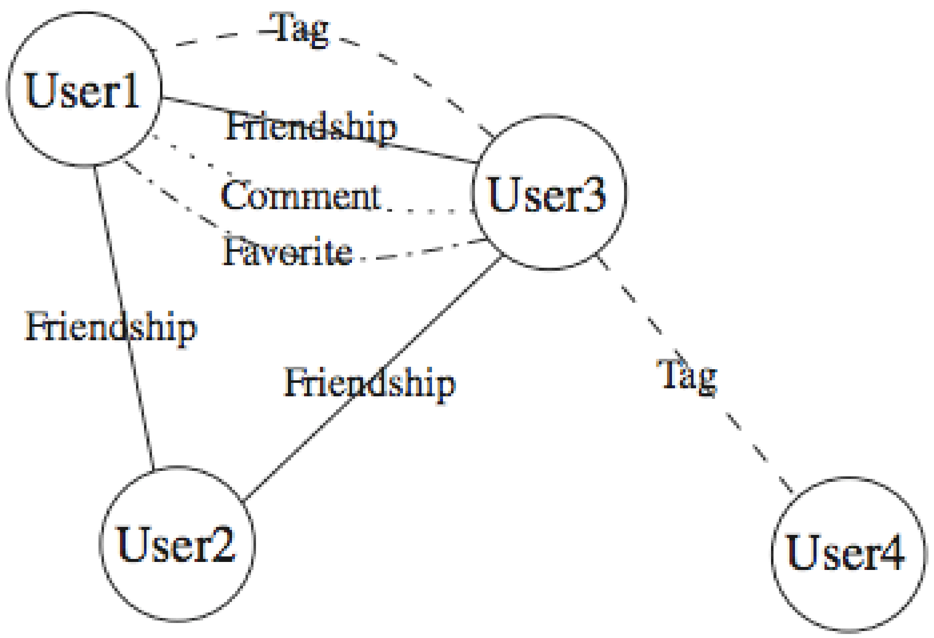
N Eagle, A Pentland and D Lazer, PNAS 2009 106 (36) 15274-15278

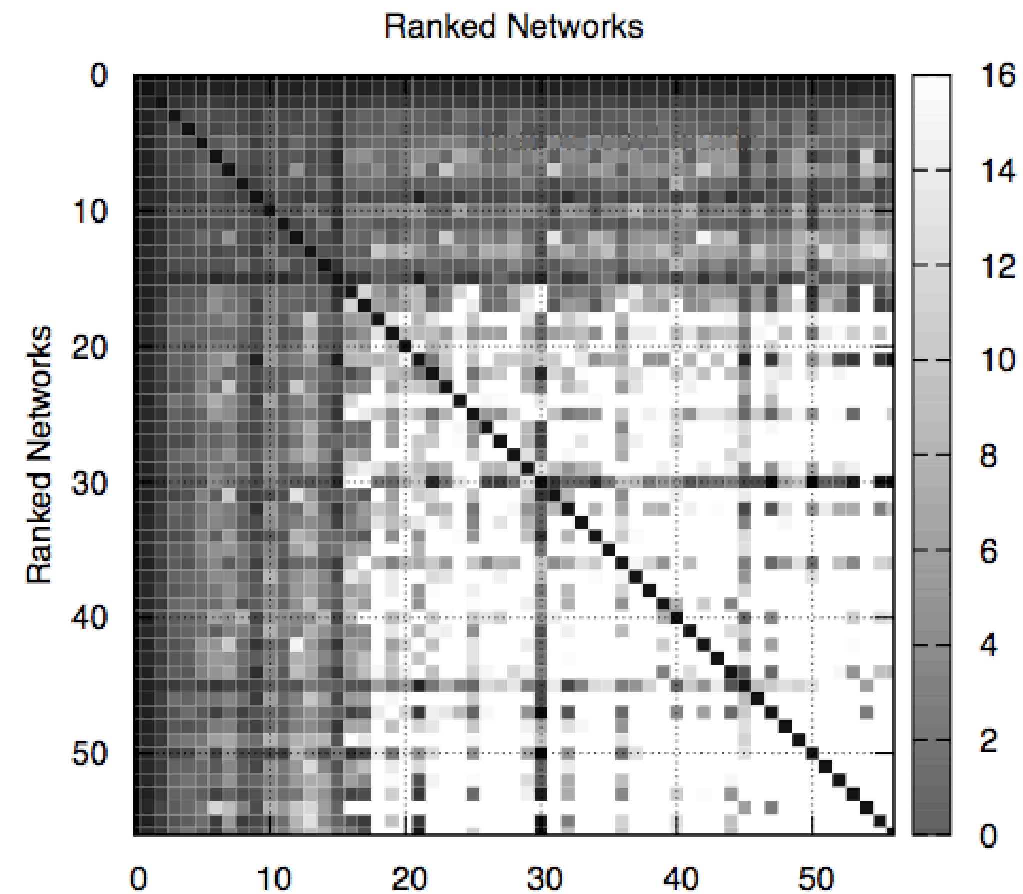
# Plethora of new services - new opportunities

Interweaving the social and the physical world
Offering more and more refined data about individuals and their interactions

# Plethora of new services - new opportunities




(d) Small example of Flickr




Different types of relations within one service

Different types of relations in different services

Multidimensional networks: foundations of structural analysis, Michele Berlingerio ·Michele Coscia ·, Fosca Giannotti·Anna Monreale ·Dino Pedreschi
Analyzing the Multigraph of Online Social Networks , Liam McNamara, private communication

# Plethora of new services - new challenges - new theoretical questions

**Mathematical framework for multiplex networks**

**Start date:2012-11-01**

**End date:2015-10-31**

**Project Acronym:**PLEXMATH

**Project status:**Execution

| Coordinator | |
|---|---|
| **Organization name:**UNIVERSITAT ROVIRA I VIRGILI | |
| **Administrative contact** | **Address** |
| **Name:**Alex ARENAS (Professor) | DEPARTAMENT D´ENGINYERIA INFORMATICA I MATEMATIQUES<br>AVDA PAISOS CATALANS 26<br>43007<br>TARRAGONA<br>ESPAÑA |
| **Tel:**+34-977559687 | |
| **Fax:**+34-977559710 | |

# Plethora of new services - new challenges - new theoretical questions

Phys. Rev. Lett. 110, 028701 (2013) [5 pages]

## Diffusion Dynamics on Multiplex Networks

Abstract | References | Citing Articles (1) | Supplemental Material

Download: PDF (189 kB) Buy this article Export: BibTeX or EndNote (RIS)

S. Gómez[1], A. Díaz-Guilera[2,3], J. Gómez-Gardeñes[3,4], C. J. Pérez-Vicente[2], Y. Moreno[3,5], and A. Arenas[1,3]

AIP Chaos
An Interdisciplinary Journal of Nonlinear Science

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Chaos / Volume 21 / Issue 1 / FOCUS ISSUE: MESOSCALES IN COMPLEX NETWORKS

Chaos **21**, 016104 (2011); http://dx.doi.org/10.1063/1.3534792 (*4 pages*)

## The interaction between multiplex community networks

Junjun Hao[1], Shuiming Cai[2,1], Qinbin He[1], and Zengrong Liu[1,2]
[1]*Institute of System Biology, Shanghai University, Shanghai 200444, China*
[2]*Department of Mathematics, Shanghai University, Shanghai 200444, China*

ELSEVIER
Social Networks
Available online 20 December 2012
In Press, Corrected Proof — Note to users

## Multiplex networks and interest group influence reputation: An exponential random graph model

Michael T. Heaney
Organizational Studies Program and Department of Political Science, University of Michigan, 722 Dennison Building, 500 Church Street, Ann Arbor, MI 48109, United States

http://dx.doi.org/10.1016/j.socnet.2012.11.003, How to Cite or Link Using DOI
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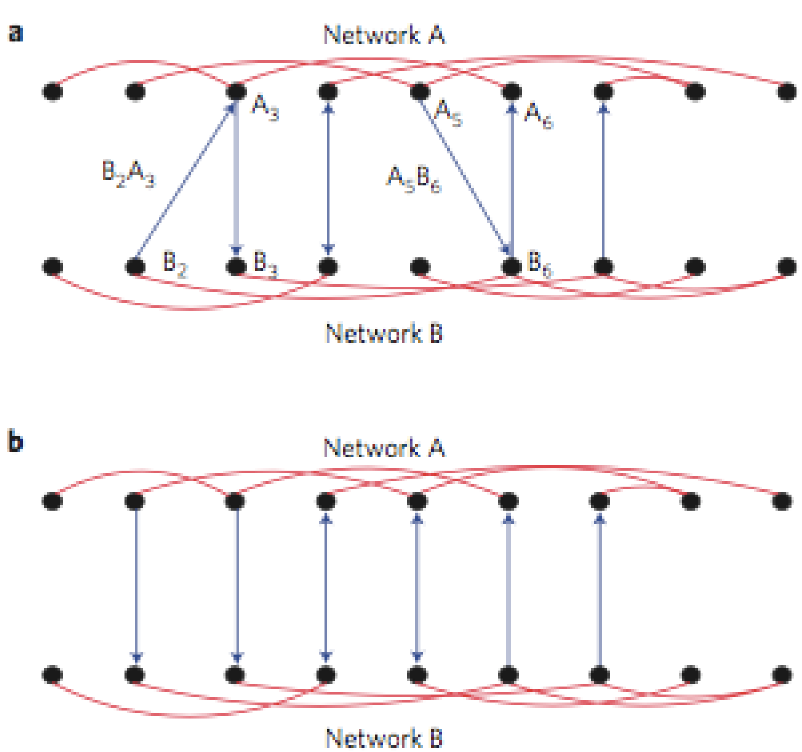
# Plethora of new services - new challenges - new theoretical questions

Vol 464 | 15 April 2010 | doi:10.1038/nature08932 nature

LETTERS

## Catastrophic cascade of failures in interdependent networks

Sergey V. Buldyrev[1,2], Roni Parshani[3], Gerald Paul[2], H. Eugene Stanley[2] & Shlomo Havlin[3]




> Current Issue > vol. 109 no. 12 > Charles D. Brummitt, E680–E689

## Suppressing cascades of load in interdependent networks

Charles D. Brummitt[a,b,1], Raissa M. D'Souza[b,c,d,e], and E. A. Leicht[f]

Author Affiliations

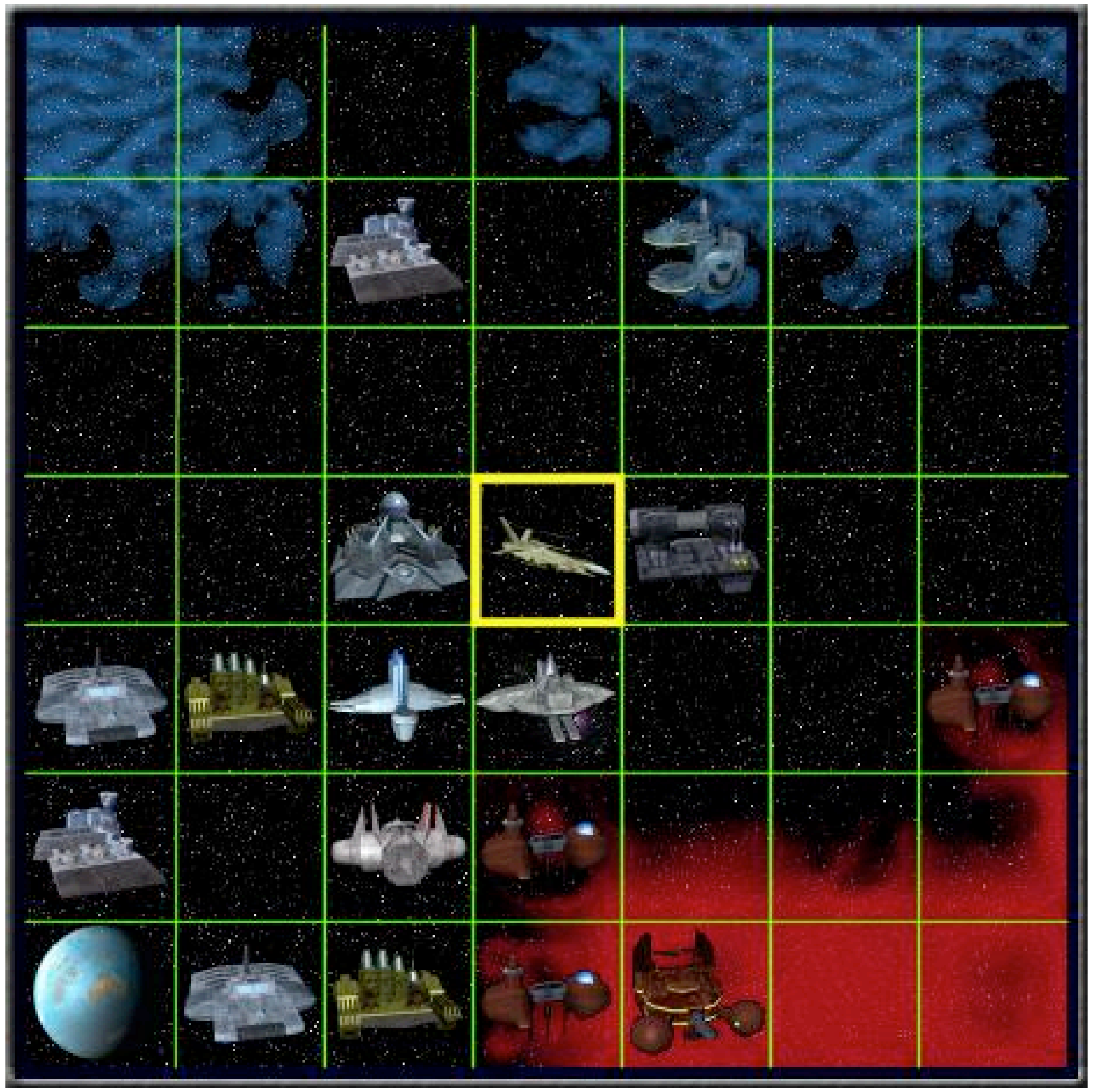
Edited by H. Eugene Stanley, Boston University, Boston, MA, and approved December 5, 2011 (received for review July 5, 2011)

# Massive Online Games

Players are immersed in a virtual world where they experience an *alternative* life with a variety of possible social interactions among players.

Motivation: establish friendships, gain respect and status in the virtual community.

All information about all actions taken by all players is stored in log-files



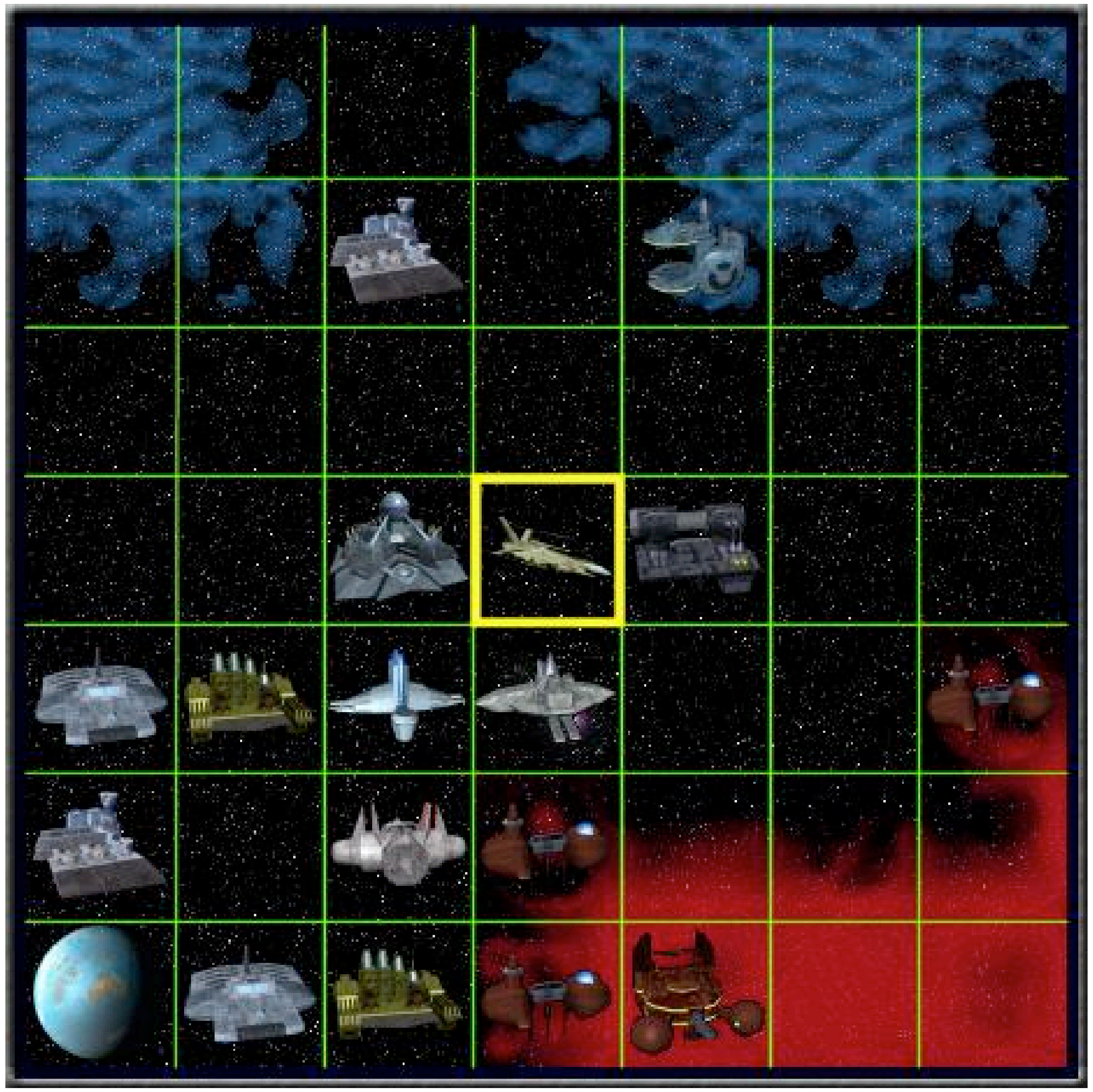
W.S. Bainbridge, The Scientific Research Potential of Virtual Worlds, Science 317 472 (2007);

# Massive Online Games

Pardus.at: Massive multiplayer browser game
330,000 registered, 13,000 active players
Played since 2004 (Free, optional 5$/month)

Open-ended game (no winner)
Players self-organise within groups and subgroups, claim territories, decide to go to war, etc., completely on their own account.

Economic life: Trade, production
Social life: Chats, forums, private messages
Exploratory life: explore of an unknown universe



M. Szell and S.Thurner, arXiv:0911.1084.
M. Szell, R. Lambiotte and S. Thurner, arXiv:1003.5137

# Massive Online Games

Multiplexity: 6 types of directed, one-to-one interactions

Communication network: personal messages (similar to email)

Trade network: exchange of money for commodity

Friendship network: players can mark others as friends. Only the marker and the marked player know this information

Attack network: attacks performed by one player on the spaceshift of another player

Bounty network: money promised for the destruction of a certain player

Enmity network: players can mark others as friends. Only the marker and the marked player know this information

M. Szell and S.Thurner, arXiv:0911.1084.
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# Massive Online Games

Multiplexity: 6 types of directed, one-to-one interactions

Communication network: personal messages (similar to email)
Trade network: exchange of money for commodity
Friendship network: players can mark others as friends. Only the marker and the marked player know this information

positive interactions

Attack network: attacks performed by one player on the spaceshift of another player
Bounty network: money promised for the destruction of a certain player
Enmity network: players can mark others as enemies. Only the marker and the marked player know this information

antagonistic interactions

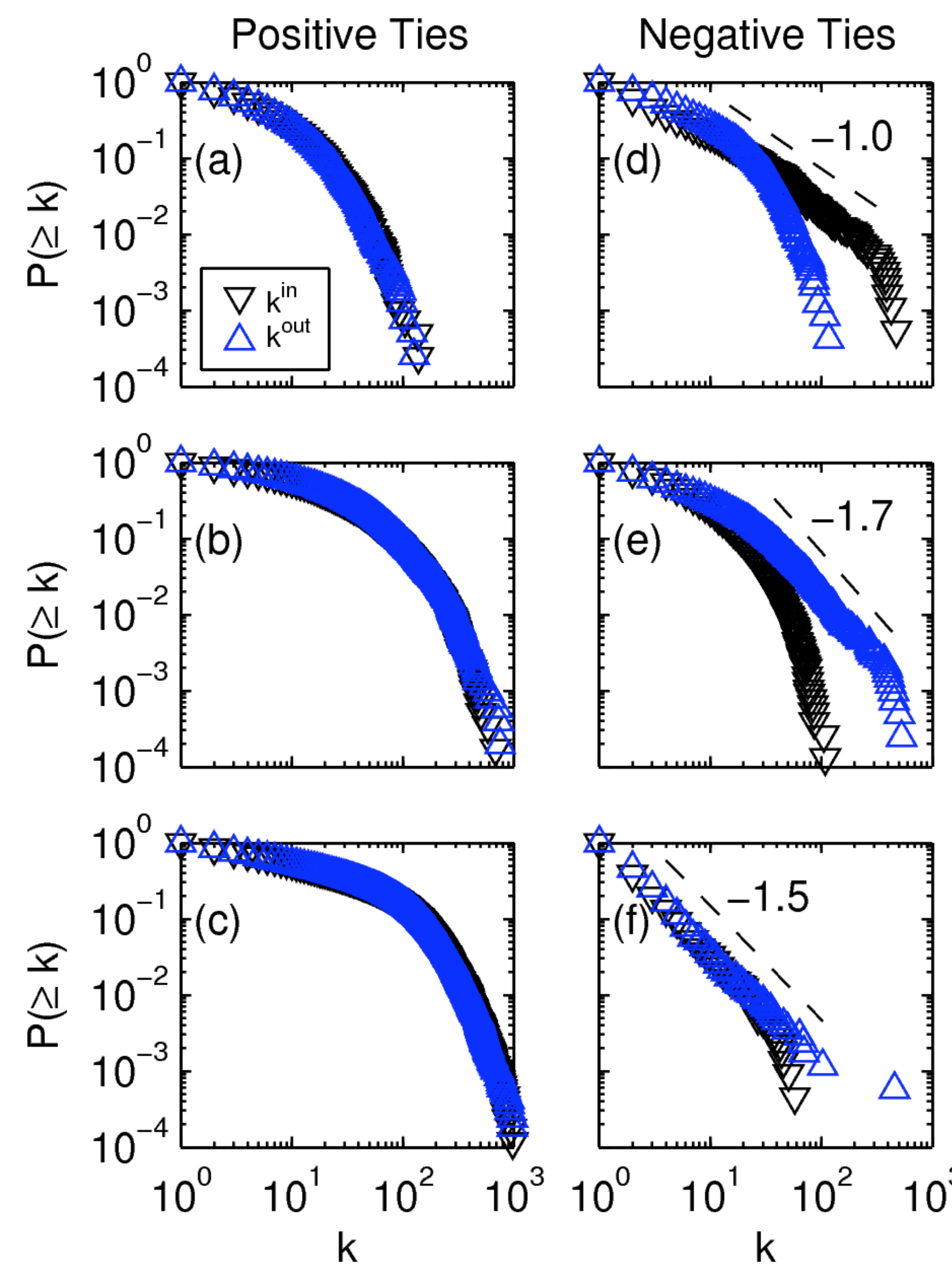
Static networks: Friendship and enmity networks are taken as snapshots at the last available day. All other networks are aggregated over time. For simplicity, we use unweighted, directed networks.
Undirected networks are also constructed: a link exist between i and j if there exists at least one directional edge between those nodes

M. Szell and S.Thurner, arXiv:0911.1084.
M. Szell, R. Lambiotte and S. Thurner, arXiv:1003.5137

# 1) Structural difference between “positive” and “negative” interactions

| | Friends | PMs | Trades | Enemies | Attacks | Bounties |
|---|---|---|---|---|---|---|
| $N$ | 4,313 | 5,877 | 18,589 | 2,906 | 7,992 | 2,980 |
| $L_{\mathrm{dir}}$ | 31,929 | 185,908 | 796,733 | 21,183 | 57,479 | 5,096 |
| $\rho$ | 0.68 | 0.84 | 0.57 | 0.11 | 0.13 | 0.20 |
| $\rho(k^{\mathrm{in}}, k^{\mathrm{out}})$ | 0.88 | 0.98 | 0.93 | 0.64 | 0.11 | 0.31 |
| $L_{\mathrm{undir}}$ | 21,118 | 107,448 | 568,923 | 20,008 | 53,603 | 4,593 |
| $\bar{k}$ | 9.79 | 36.57 | 61.21 | 13.77 | 13.41 | 3.08 |
| $C$ | 0.25 | 0.28 | 0.43 | 0.03 | 0.06 | 0.01 |
| $C/C_{\mathrm{r}}$ | 109.52 | 45.71 | 131.95 | 6.13 | 37.27 | 13.88 |

| | |
|---|---|
| High reciprocity | Low reciprocity |
| High cohesion | Low cohesion |
| No power-law | “Power-law” |




$r$ reciprocity coefficient: tendency for directed links to be reciprocal

D. Garlaschelli and M.I. Loffredo (2004) Phys. Rev. Lett. 93, 268701

# 1) Structural difference between "positive" and "negative" interactions

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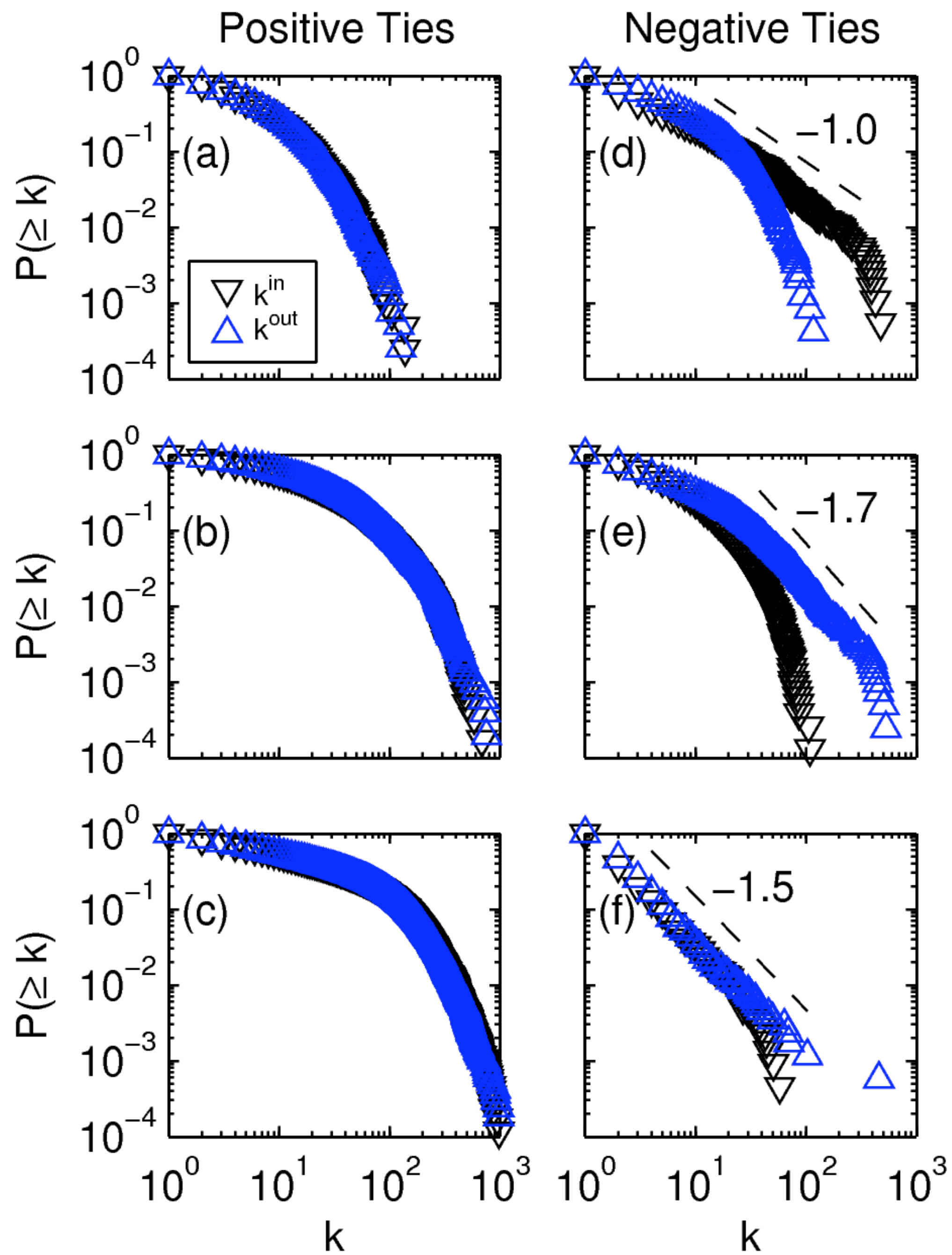
| $\rho(k^{\mathrm{in}}, k^{\mathrm{out}})$ | 0.88 | 0.98 | 0.93 | 0.64 | 0.11 | 0.31 |
| $L_{\mathrm{undir}}$ | 21,118 | 107,448 | 568,923 | 20,008 | 53,603 | 4,593 |
| $\bar{k}$ | 9.79 | 36.57 | 61.21 | 13.77 | 13.41 | 3.08 |
| $C$ | 0.25 | 0.28 | 0.43 | 0.03 | 0.06 | 0.01 |
| $C/C_{\mathrm{r}}$ | 109.52 | 45.71 | 131.95 | 6.13 | 37.27 | 13.88 |

| | |
|---|---|
| High reciprocity | Low reciprocity |
| High cohesion | Low cohesion |
| No power-law | "Power-law" |




$C$ clustering coefficient

# 1) Structural difference between "positive" and "negative" interactions

| | Friends | PMs | Trades | Enemies | Attacks | Bounties |
|---|---|---|---|---|---|---|
| $N$ | 4,313 | 5,877 | 18,589 | 2,906 | 7,992 | 2,980 |
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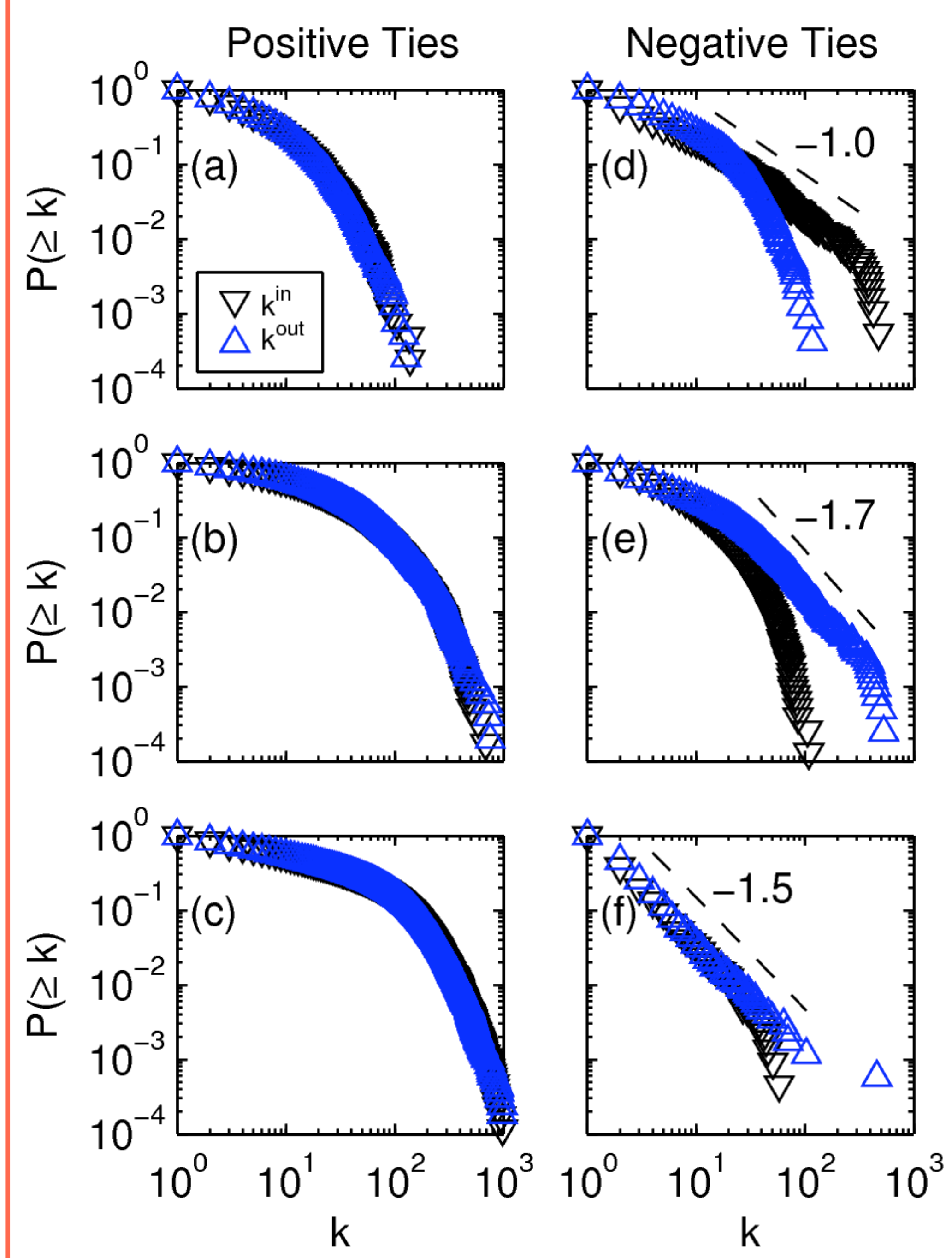


(a) friendship; (b) PM; (c) trade; (d) enmity; (e) attack; (f) bounty

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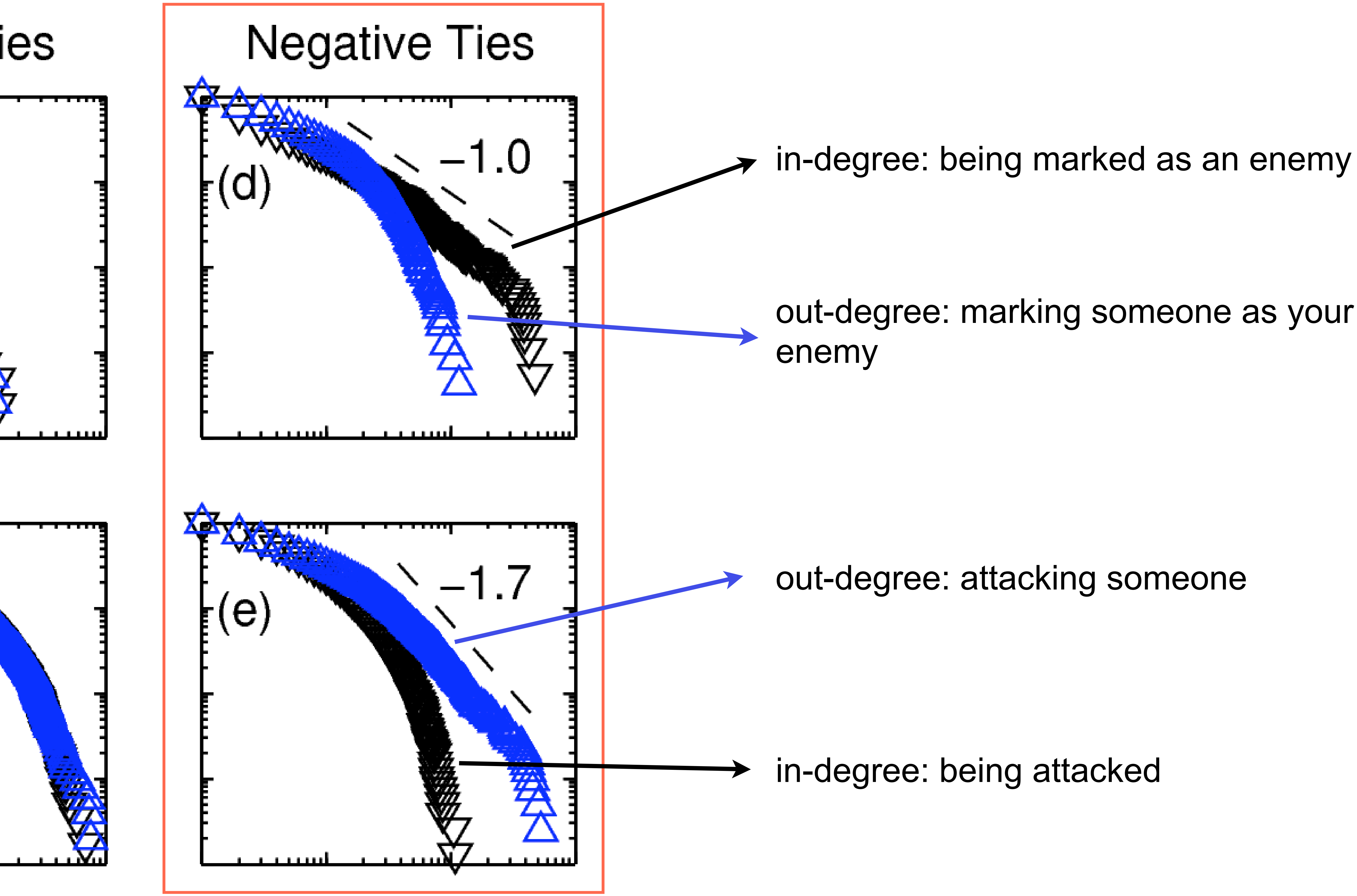

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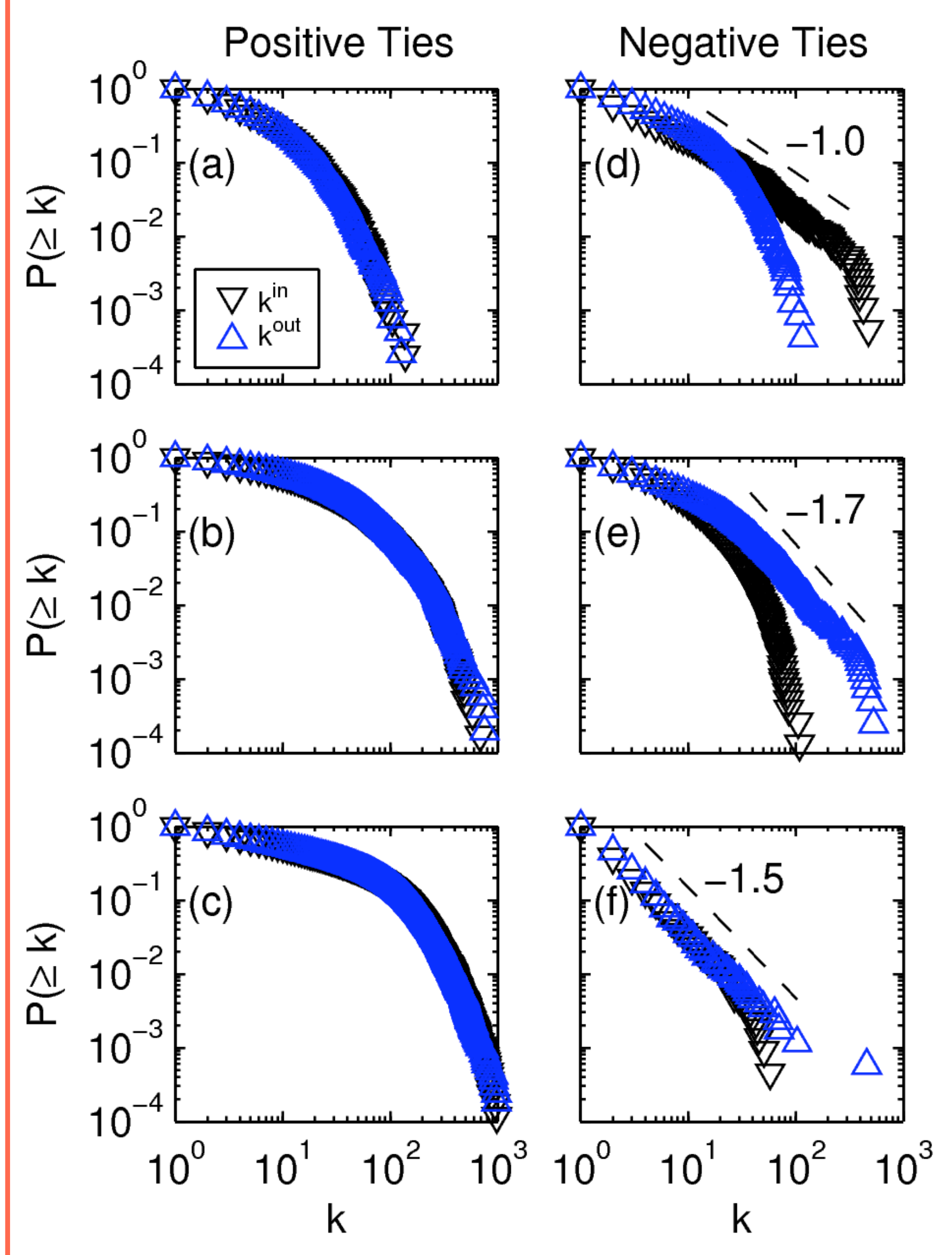



$\rho(k_\alpha^{\mathrm{in}}, k_\alpha^{\mathrm{out}})$ Pearson's correlation of in- vs out-degree

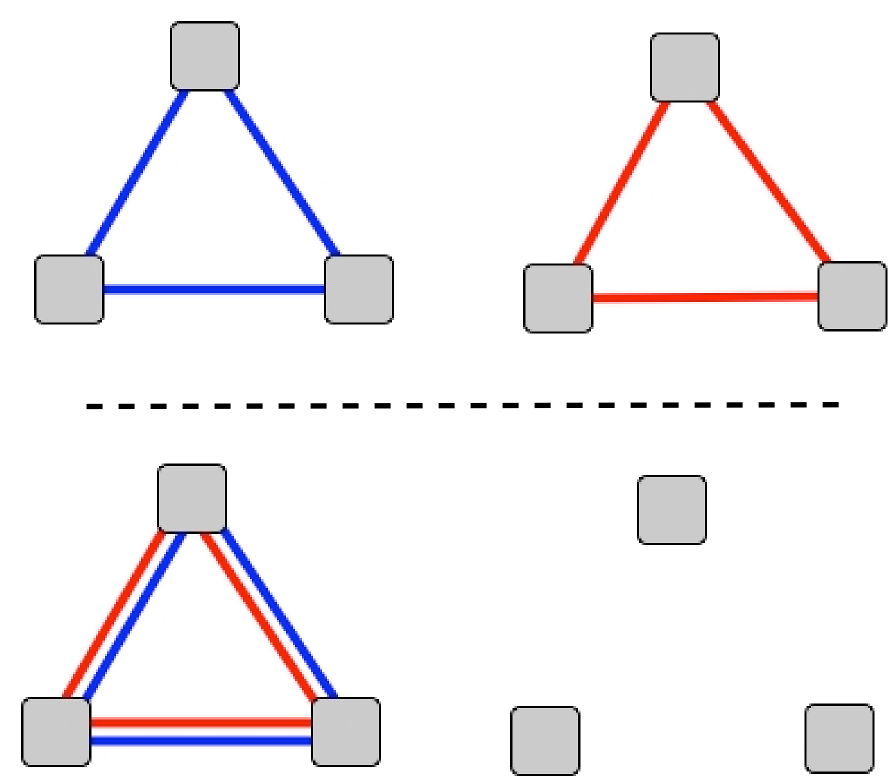
# 2) Interaction between networks

Interactions between different social relations (positive or negative feed-backs), e.g. network of communications poses constraints on the network of friendships, which itself reinforces communication

Description of the co-existence of different types of links.

To quantify the resulting inter-dependencies between pairs of networks, we follow two approaches:

a) Jaccard coefficient between two different sets of links measures the tendency that links simultaneously are present in both networks => Network overlap



b) Correlations between node degrees in different networks (and between rankings of node degrees). These coefficients measure to which extent degrees of agents in one type of network correlate with degrees of the same agents in another one. Do players who have many (few) links in a network have many (few) links in another network?

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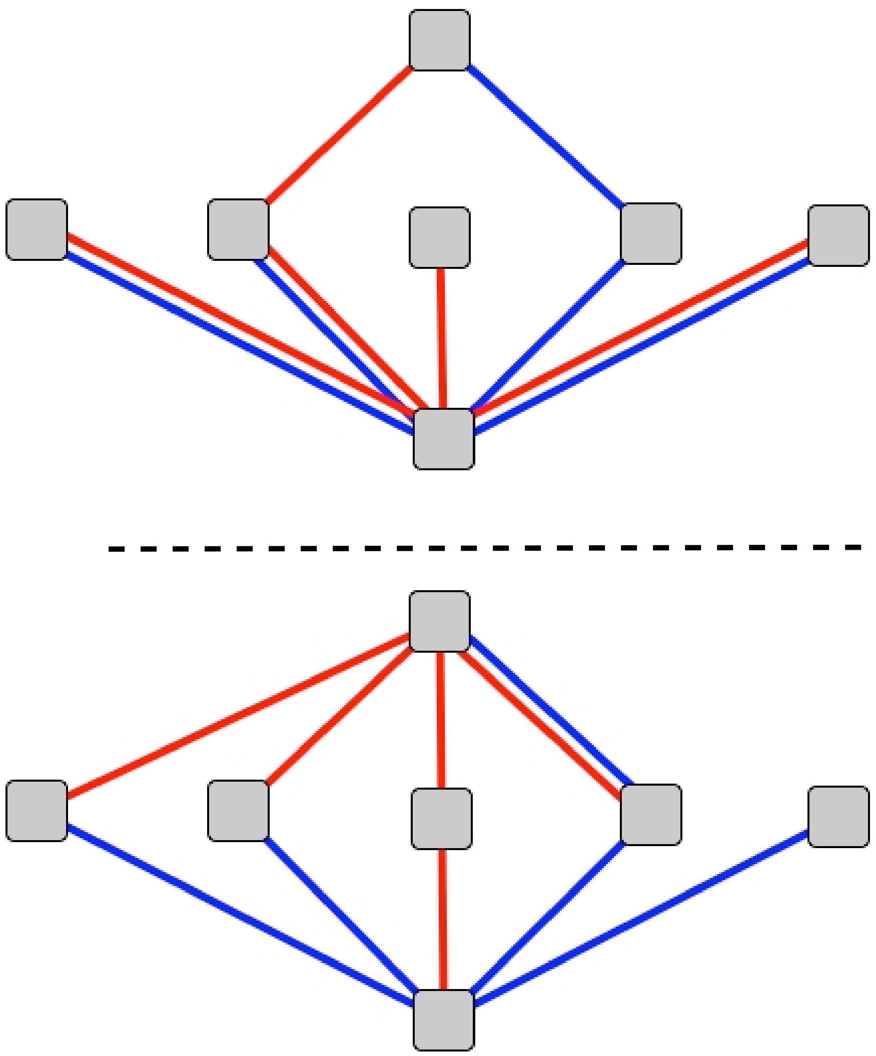

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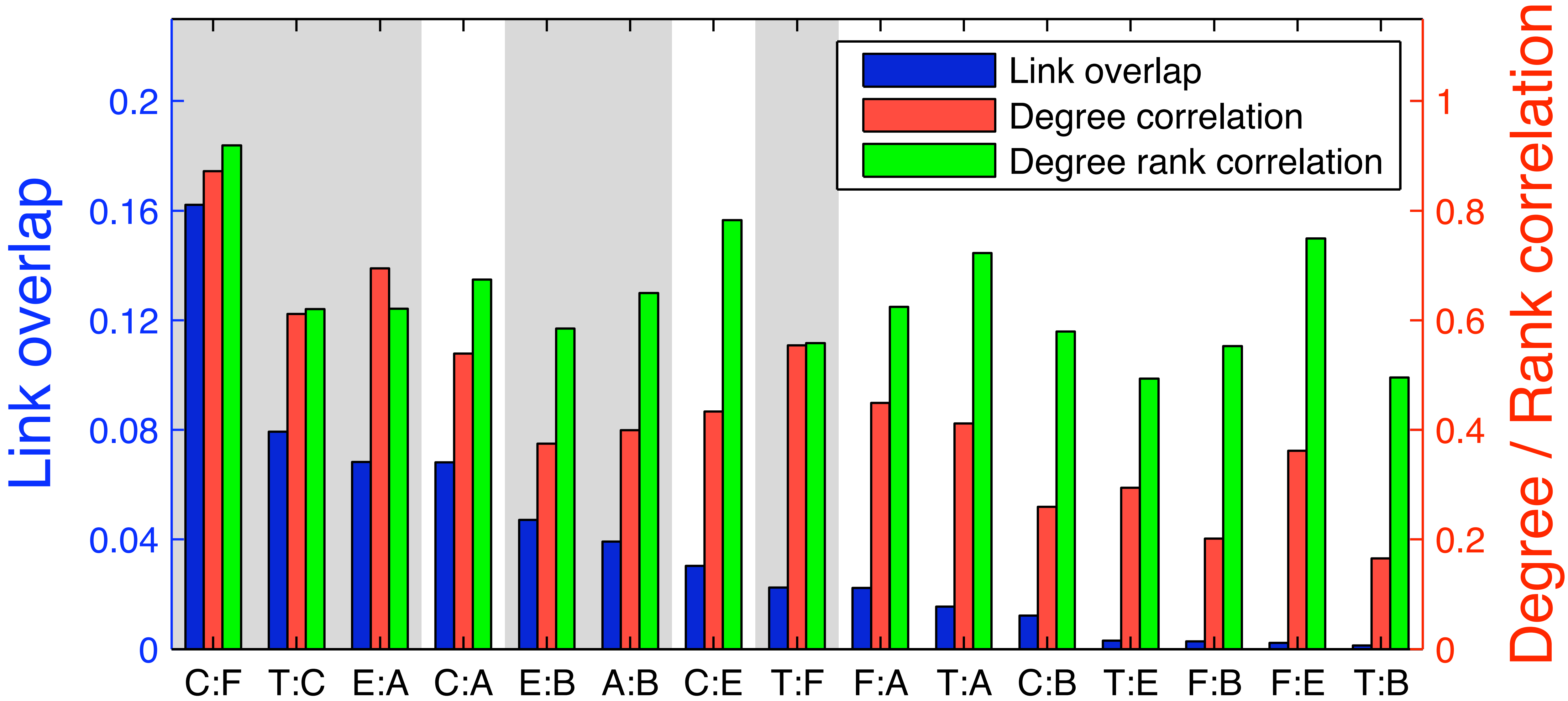
Different roles in different relational networks?

# 2) Interaction between networks




Communication, Friendship, Trade, Attack, Enmity, Bounty

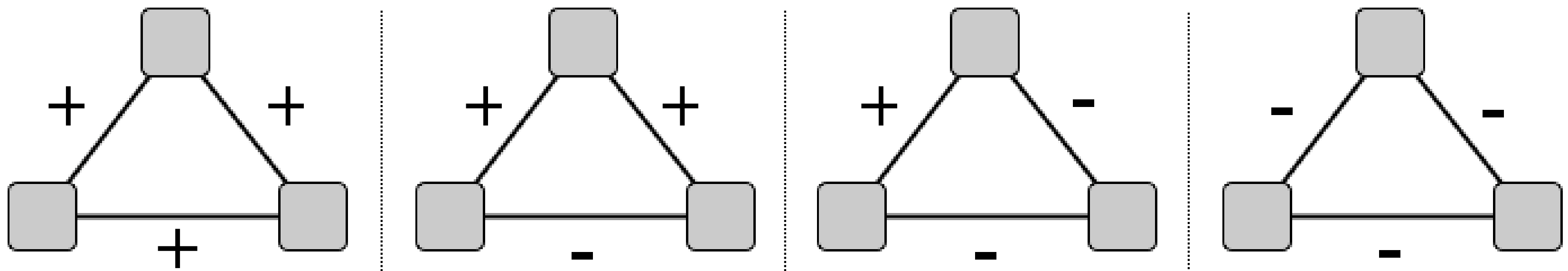
Exclusion of some networks (e.g. F/E, T/E and T/A) vs high overlap for others (e.g. C/F, E/A)

Low degree correlation for some networks: different roles/strategies in different networks (e.g. T/A, T/E and F/E)

# 3) Empirical verification of structural balance

Some configurations of signed motifs are socially and psychologically more likely than others

Unbalanced triads are sources of stress and therefore tend to be avoided by actors when they adapt their personal relationships

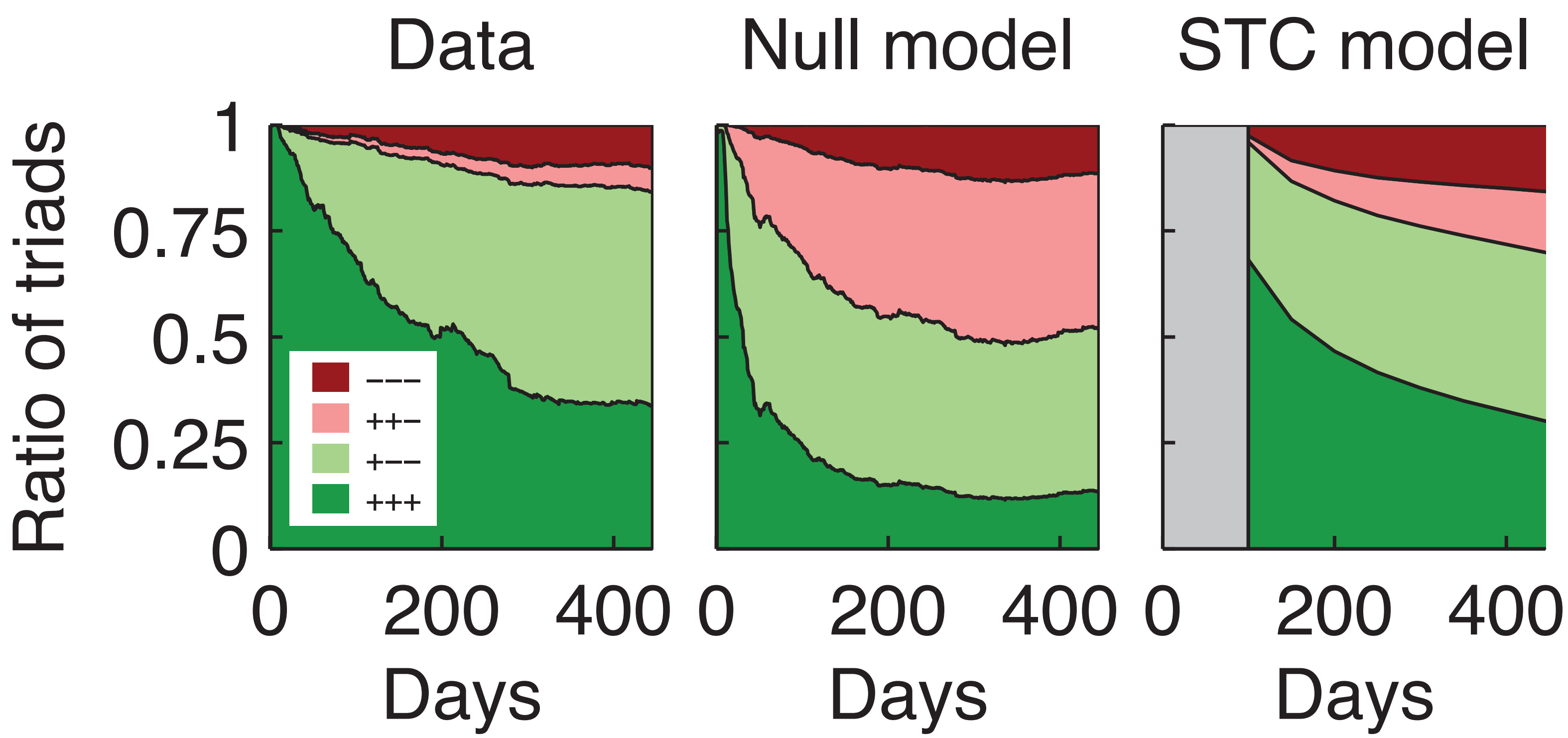



| | + + + | + + - | + - - | - - - | |
|---|---|---|---|---|---|
| Strong formulation of balance | B | U | B | U | Cartwright (after Heider) |
| Weak formulation of balance | B | U | B | B | Davis |
| $N_\Delta$ | 26,329 | 4,428 | 39,519 | 8,032 | |
| $N_{\Delta,r}$ | 10,608 | 30,145 | 28,545 | 9,009 | |
| $z$ | 71 | -112 | 47 | -5 | |

Leskovec J, Huttenlocher D, Kleinberg J (2010) Predicting positive and negative links in online social networks. ACM WWW Int Conf on World Wide Web 2010.

# 3) Empirical verification of structural balance

Dynamical re-organisation of multiplex networks (dynamics of motifs)




A vast majority of changes in the network are due to the creation of new positive and negative links, and not due to the switching of existing links from plus to minus or vice versa.

This result is in marked contrast with many dynamical models of structural balance which assume that a given social network is fully connected from the start and that only the signs of the relationships are the relevant dynamical parameters, which evolve to reduce stress in the system.

Our observation underpins that network sparsity and growth are fundamental properties and they need to be incorporated in any reasonable model of dynamics of positive and antagonistic forces in social systems.

T. Antal, P. L. Krapivsky, and S. Redner, Phys Rev E 72, 036121 (2005).

# Take home message

Know your classics...

**“IMPLICATIONS FOR FUTURE RESEARCH :**
**- Need for Studies of Multiplexity .....**
**- Need for Dynamic Data ....**
**- Need for Study of Co-evolution ....**
**”**

M. McPherson, L. Smith-Lovin and J.M. Cook (2001) Annu. Rev. Sociol. 27, 415.

**Methodological and modelling of multiplex networks**

**Large datasets**

# Take home message (2)

Need for new algorithms

New theoretical questions

# Take home message (3)

Help to answer old questions...