Structure and evolution of social networks

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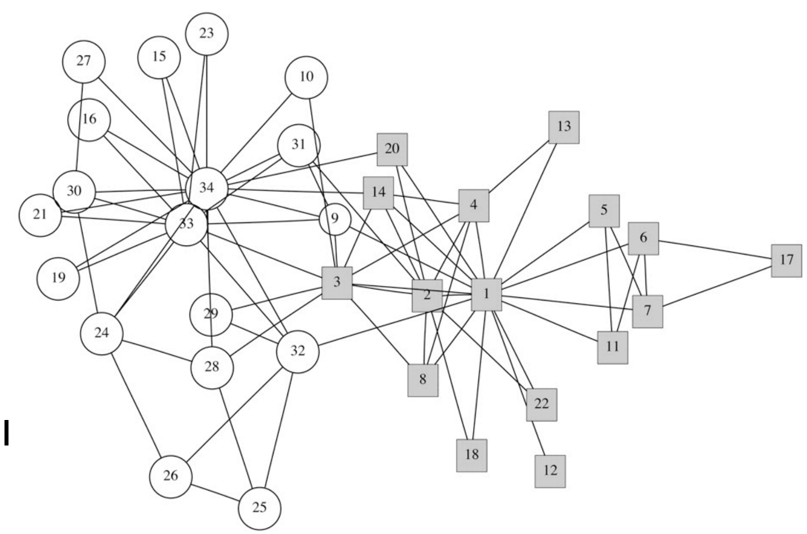
and J. Holyst

Examples of complex networks:

- Internet
- Citation networks
- Transport networks
- Protein interaction networks
- Food webs
- Social networks, e.g. collaboration networks, friendship networks...

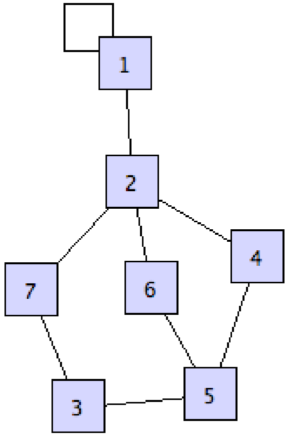


The friendship network from Zachary's karate club study

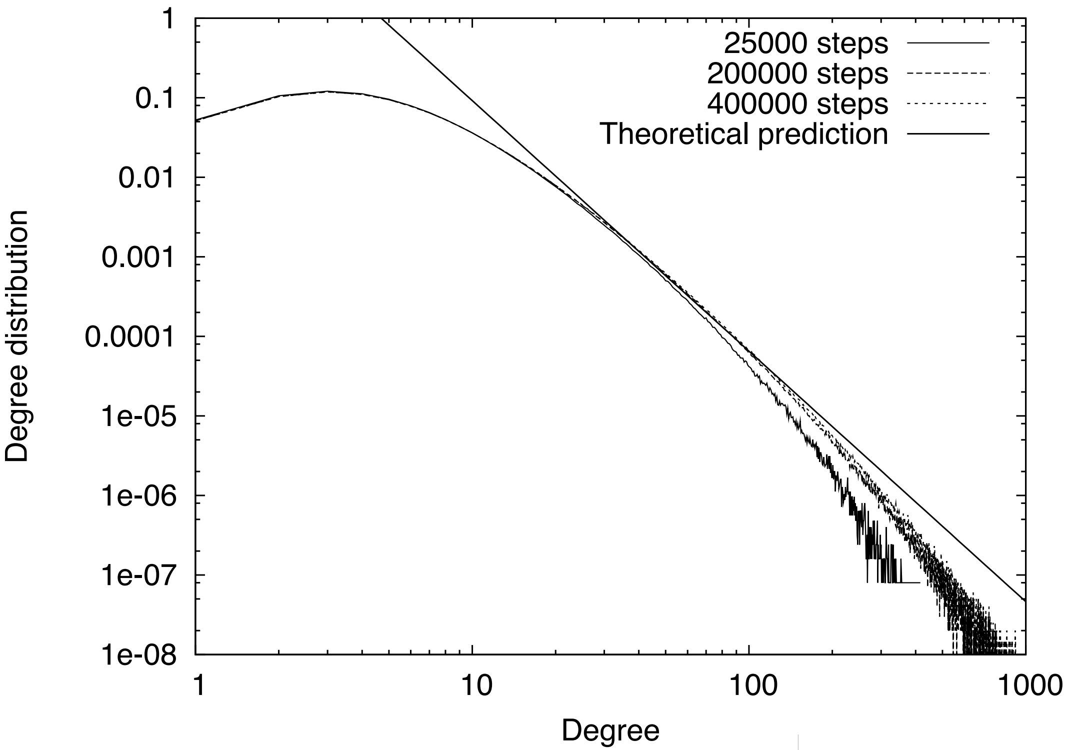
M. Girvan and M. E. J. Newman, PNAS | June 11, 2002 | vol. 99 7821
W.W. Zachary (1977) J. Anthropol. Res. 33, 452-473

Local properties

node degree: numbers of links arriving at one node

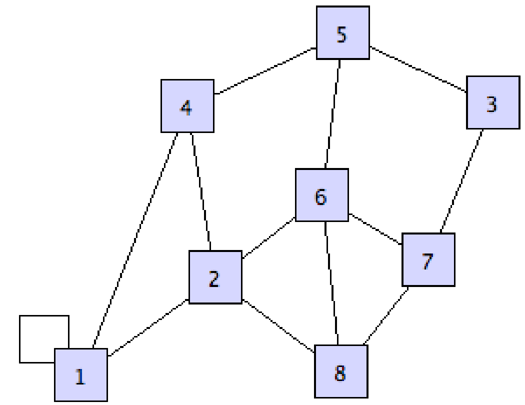


the degree distribution is known to behave like power-laws in many situations, usually with an exponential cut-off



Local properties

clustering coefficient: number of triangles



local correlations: assortativity...

High-degree vertices prefer to attach other high-degree vertices (social networks)
High-degree vertices prefer to attach low-degree vertices (technological networks)

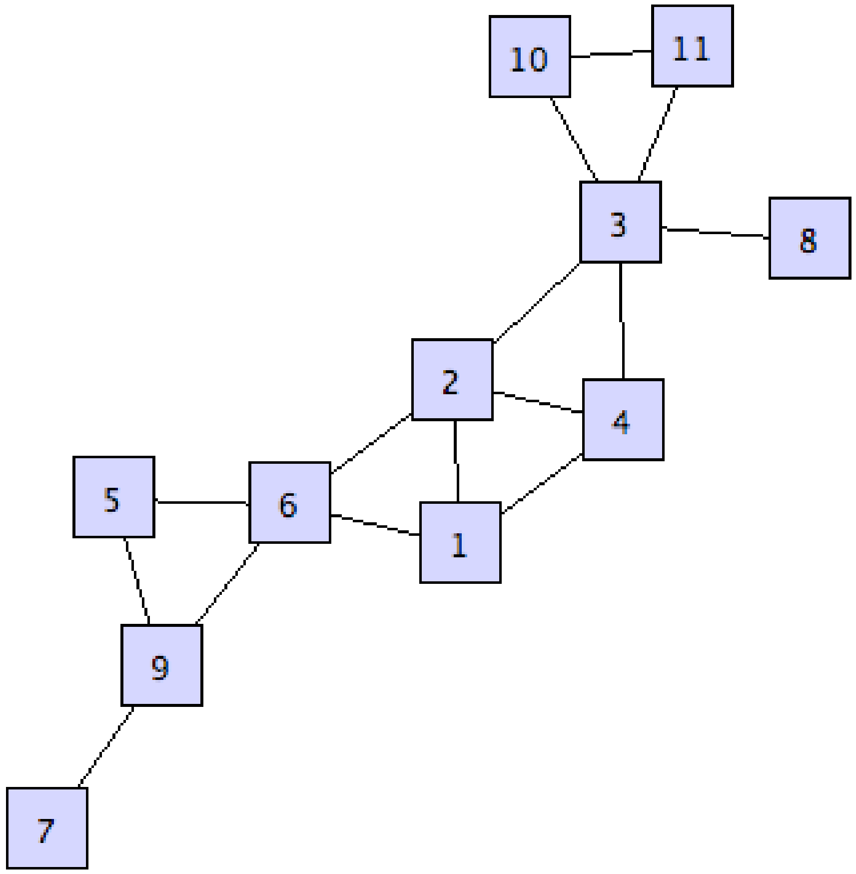
	network	n	r
real-world networks	physics coauthorship[a]	52 909	0.363
	biology coauthorship[a]	1 520 251	0.127
	mathematics coauthorship[b]	253 339	0.120
	film actor collaborations[c]	449 913	0.208
	company directors[d]	7 673	0.276
	Internet[e]	10 697	−0.189
	World-Wide Web[f]	269 504	−0.065
	protein interactions[g]	2 115	−0.156
	neural network[h]	307	−0.163
	food web[i]	92	−0.276
models	random graph[u]		0
	Callaway *et al.*[v]		$\delta/(1+2\delta)$
	Barabási and Albert[w]		0

Global properties

distance: length of the shortest path between nodes

small-world behaviour

$$d \sim \ln N$$



	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363	311, 313
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127	311, 313
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	−0.029	45
	sexual contacts	undirected	2 810				3.2				265, 266
information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	−0.067	14, 34
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7				74
	citation network	directed	783 339	6 716 198	8.57		3.0/–				351
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157	244
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	−0.189	86, 148
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	−0.003	416
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	−0.033	366
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	−0.016	318
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	−0.119	395
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	−0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	−0.366	6, 354
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	−0.240	214
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	−0.156	212
	marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	−0.263	204
	freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	−0.326	272
	neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	−0.226	416, 421

BA model: C=0, r=0, l=log(log(N))

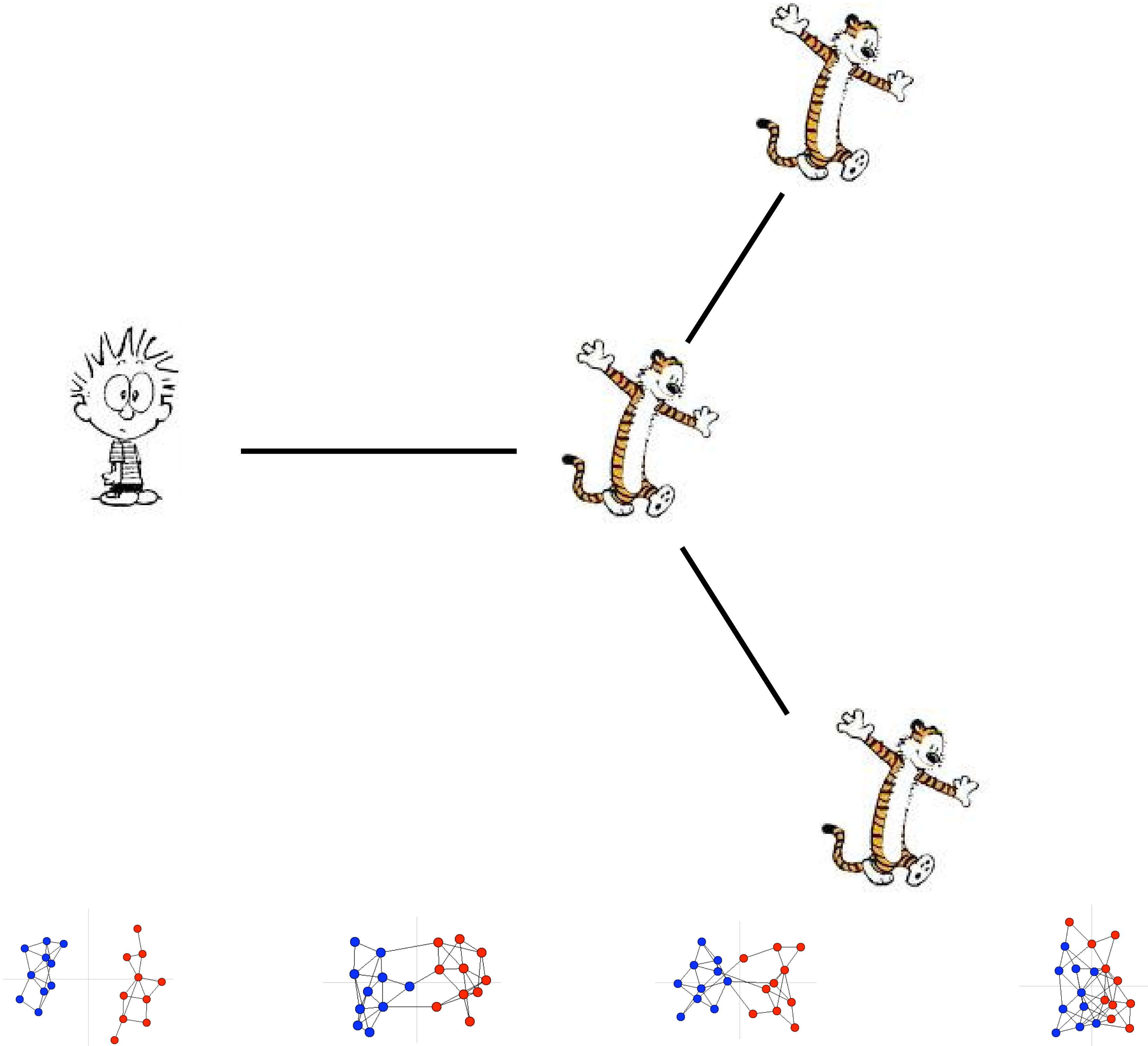
Questions

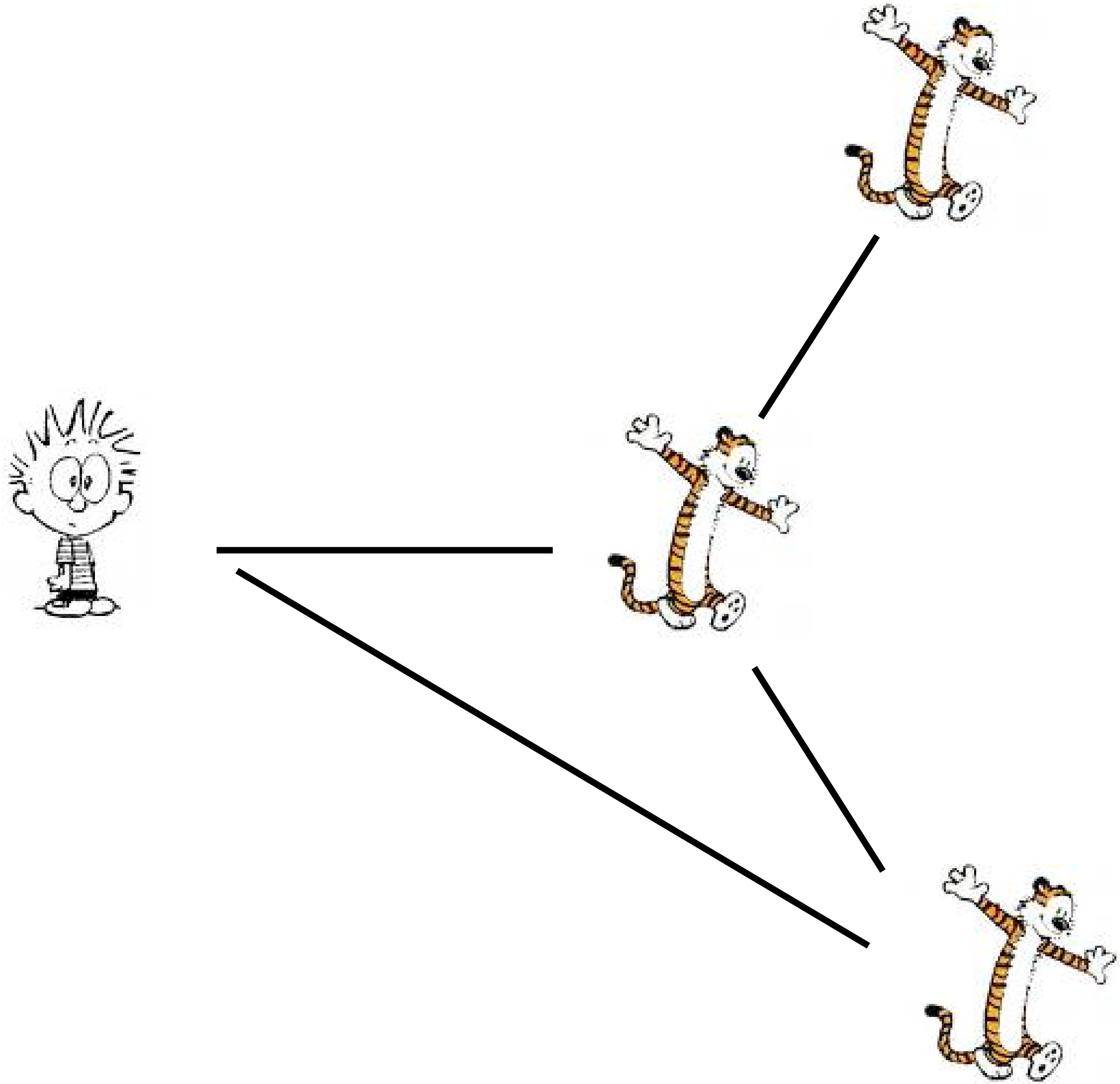
- How do these networks evolve and structure themselves?

  Copying mechanisms, redirecting mechanisms, ageing...

- How do the properties of the nodes evolve on such complex topologies?

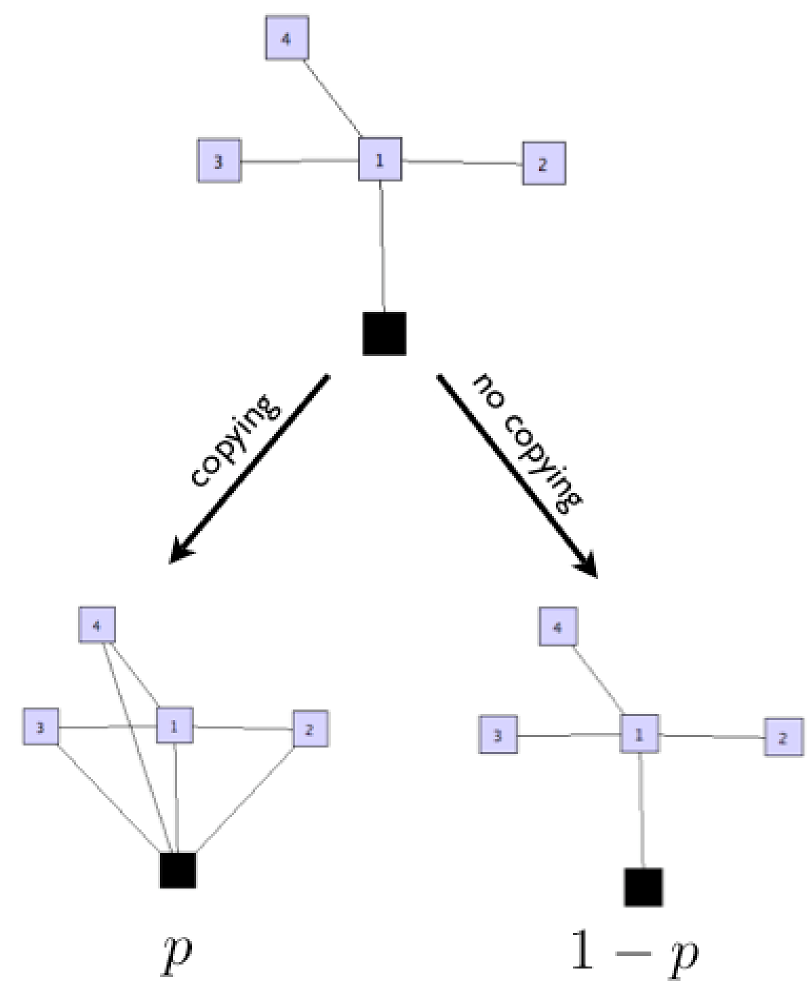
  Majority and Unanimity models, role of communities...



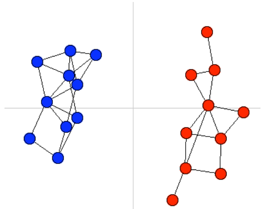


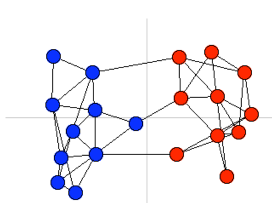
“All-or-none” model

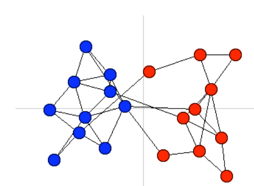
At each time step, a new node randomly connects to a target node. With probability *p*, all the neighbours of the target node are linked; otherwise, none is linked.

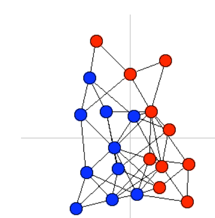


With P. Krapivsky

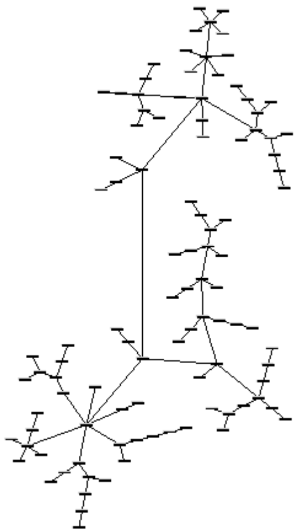


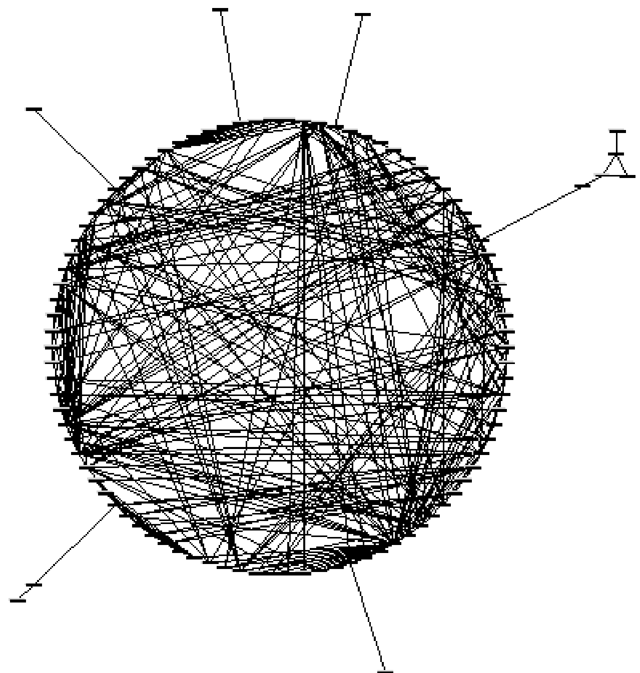


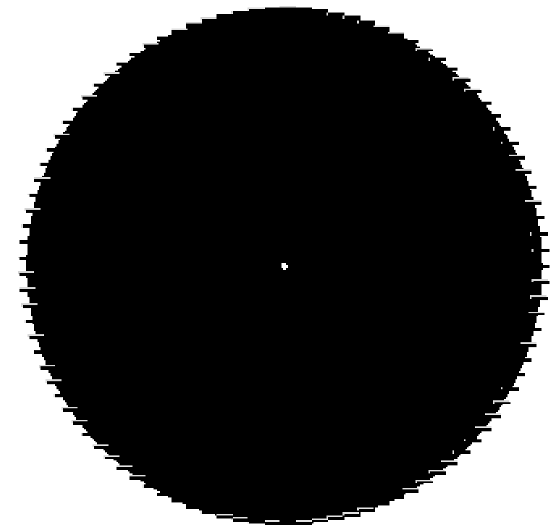




By construction, starting from *1* root node, the system grows (*N+1* nodes after N steps) and behaves like:



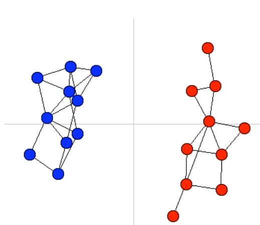


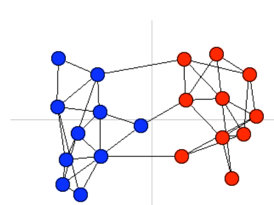


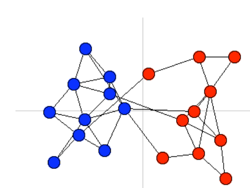
Purely random network when *p=0*

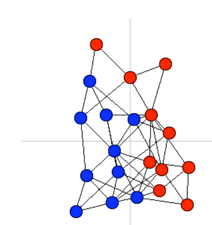
What are its properties when $p \in]0,1[$?

Fully connected network when p=1







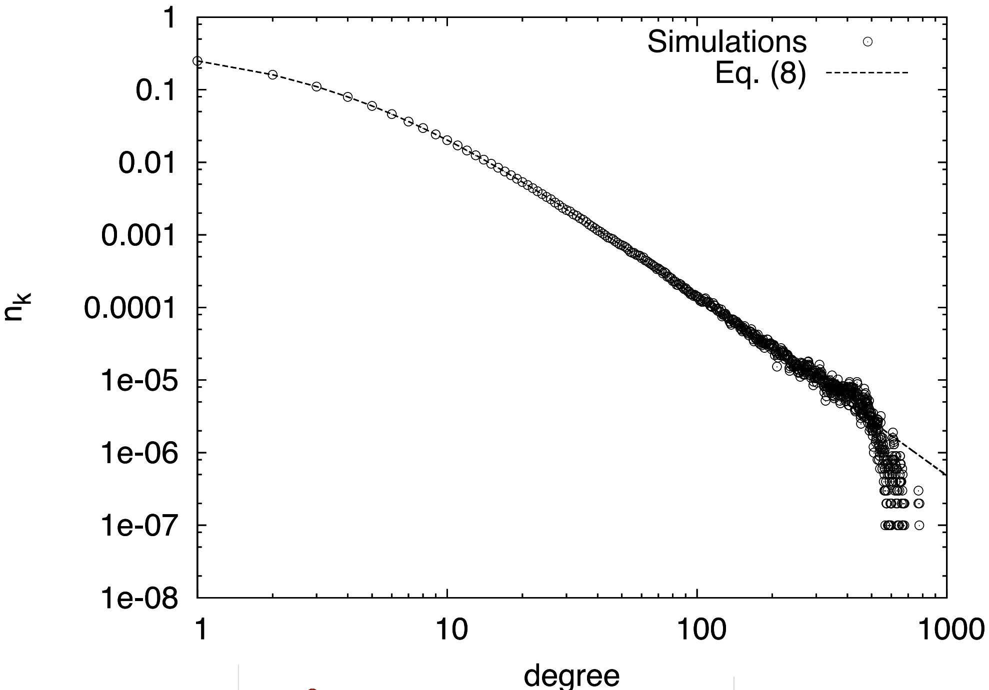


The oversimplified model reproduces all the characteristics of social models

The degree distribution satisfies the non-trivial set of equations:

$$\frac{dN_k}{dN} = \frac{N_{k-1} - N_k}{N} + p\,\frac{(k-1)N_{k-1} - kN_k}{N} + m_k$$

$$m_k = (1-p)\delta_{k,1} + p\frac{N_{k-1}}{N}$$



$$n_k = Y\frac{\Gamma(k+1+1/p)}{\Gamma(k+1+2/p)}$$

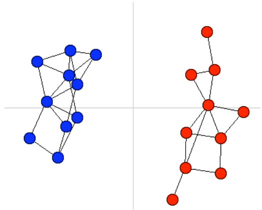
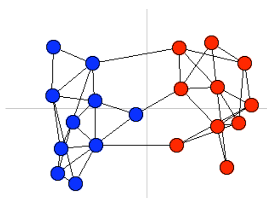
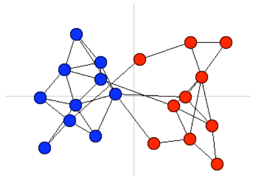
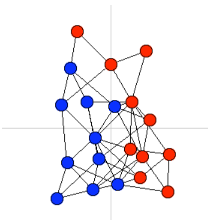
The oversimplified model reproduces all the characteristics of social models

Distance distribution in the "*all-or-none*" version of the model

$$P(d,N+1) = \left[1 + \frac{2p}{N}\right] P(d,N) + \frac{2(1-p)}{N} P(d-1,N)$$

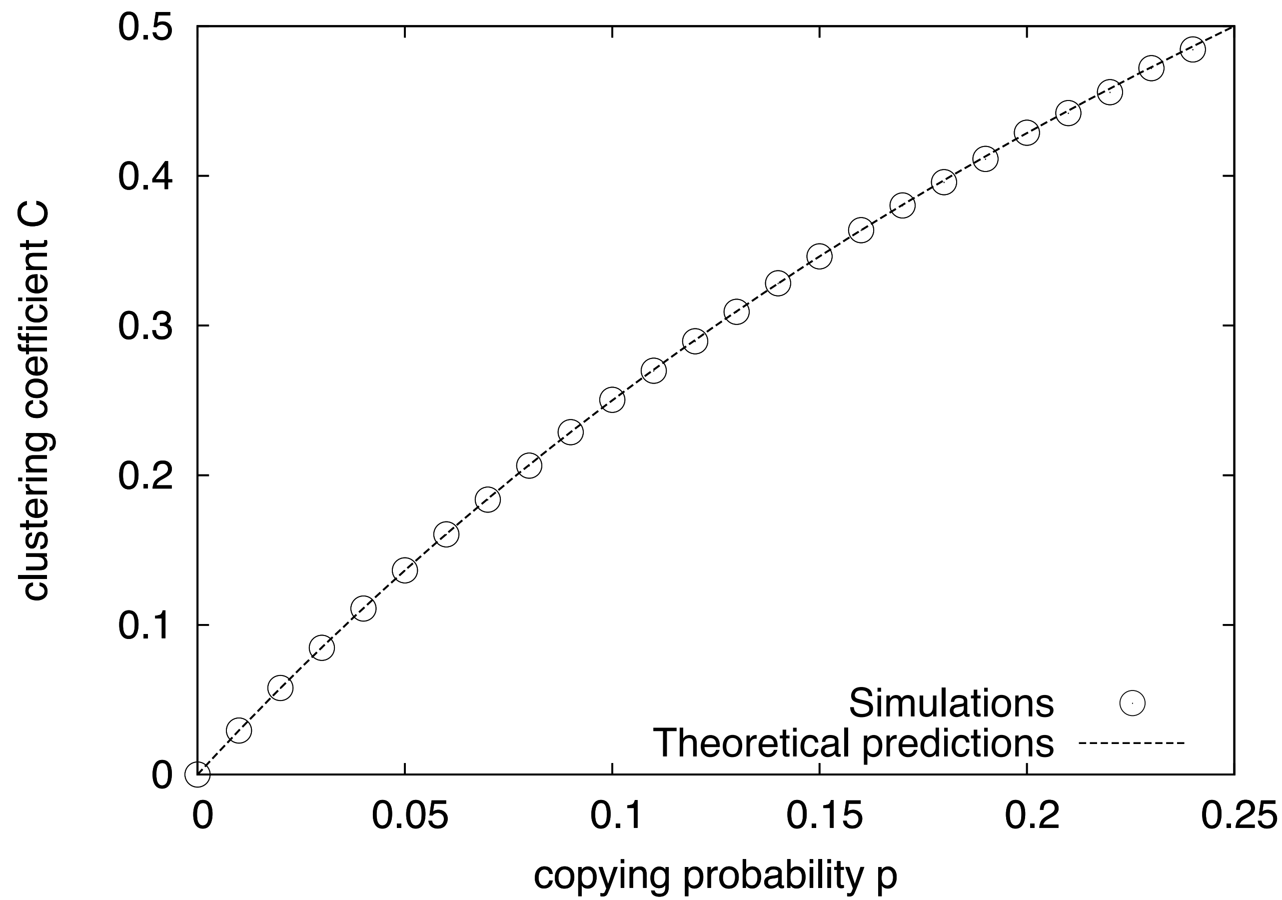
$$d \sim 2(1-p)\ln N$$

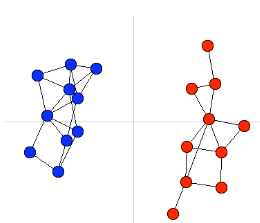
while the fluctuations are gaussian and asymptotically vanish.

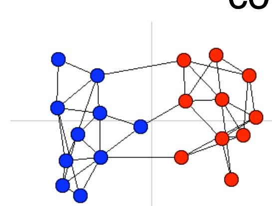





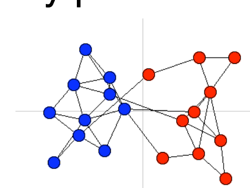
The oversimplified model reproduces all the characteristics of social models

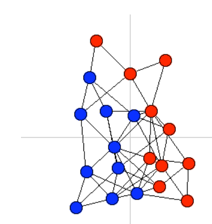
Non-vanishing clustering coefficient $C = 3p/(1+2p)$







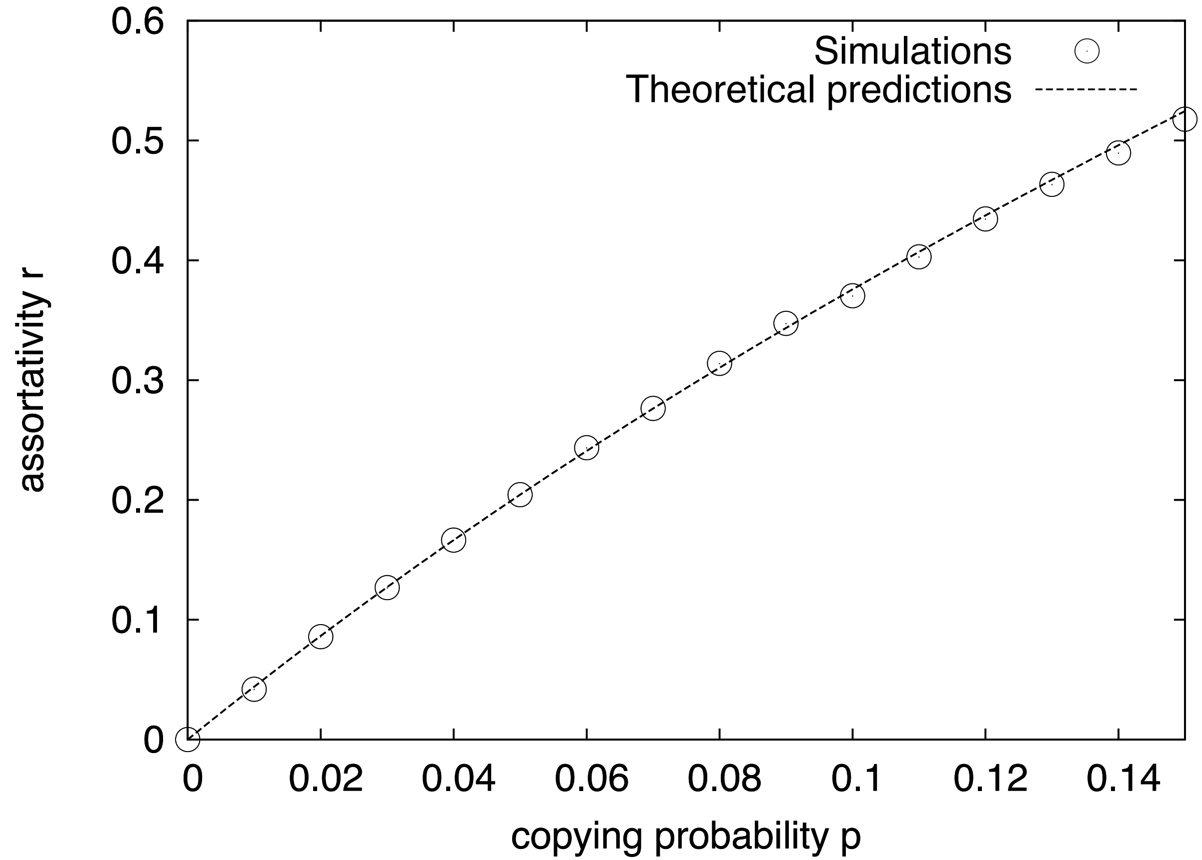


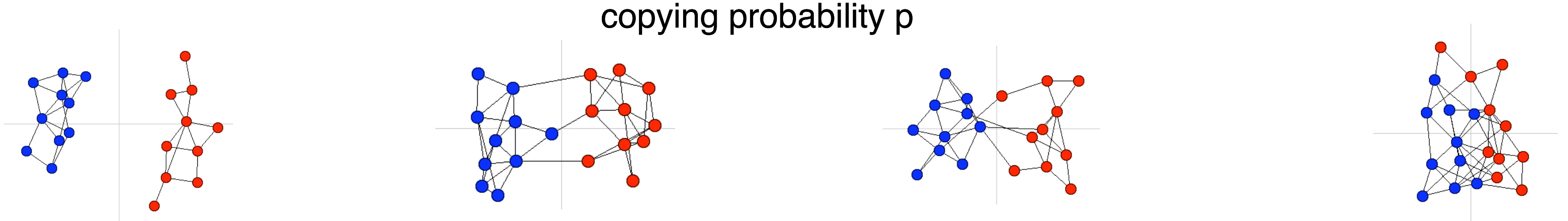


The oversimplified model reproduces all the characteristics of social models

Positive assortativity

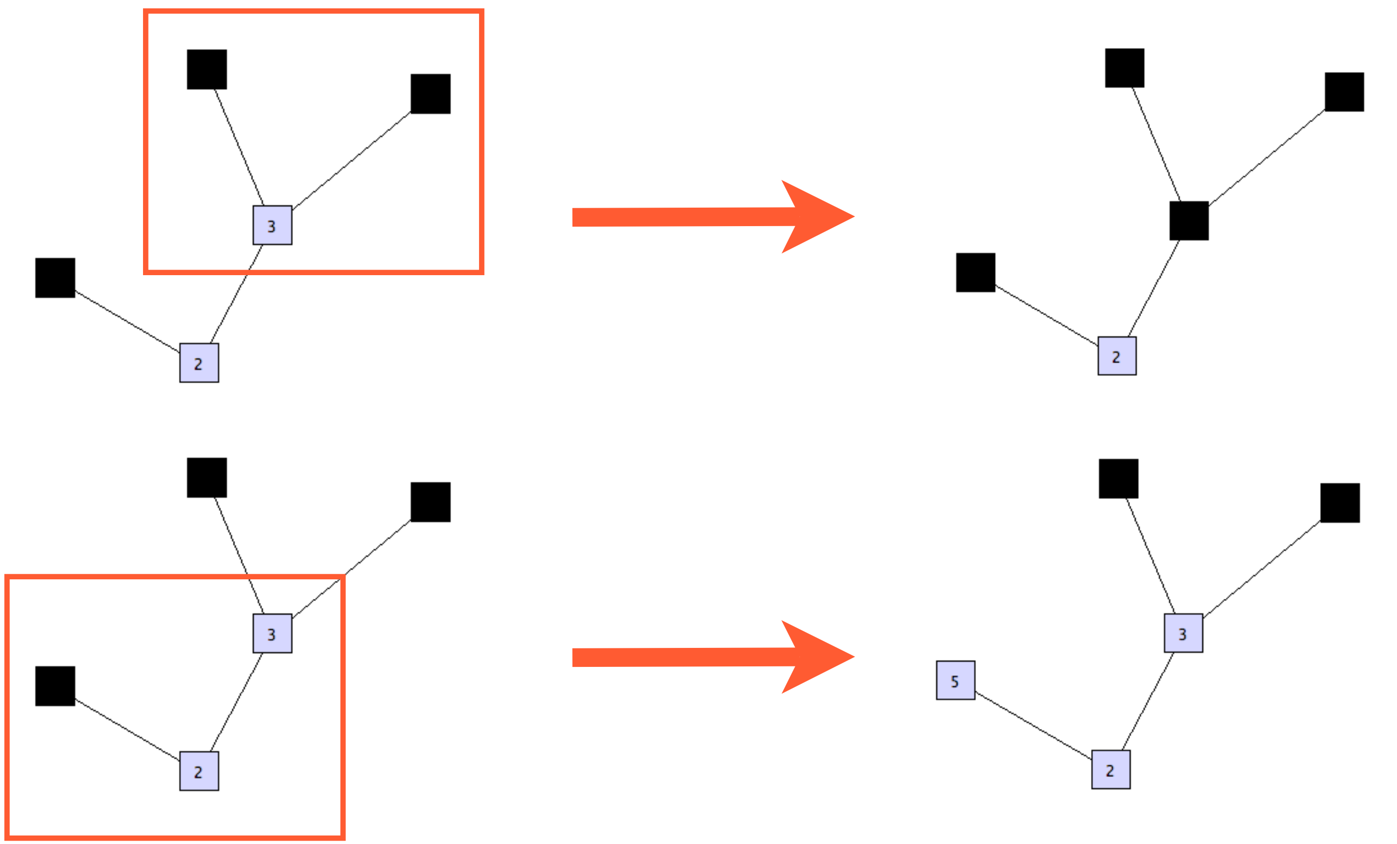
$$r = \frac{p(9 - 14p + 11p^2)}{2 + p - 5p^2 + 2p^3}$$





Majority model: role of communities

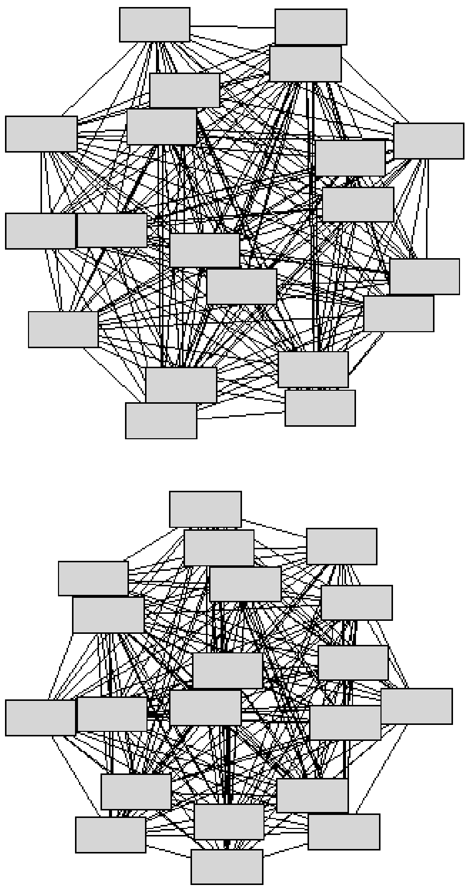
In a network composed of N persons, 3 people meet at each time step and adapt the opinion of the majority. ! No randomness, except in the selection of the three nodes.

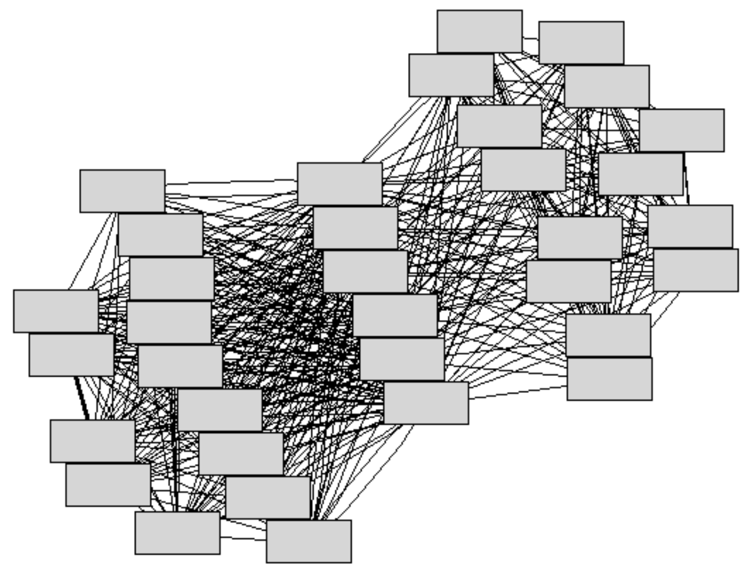


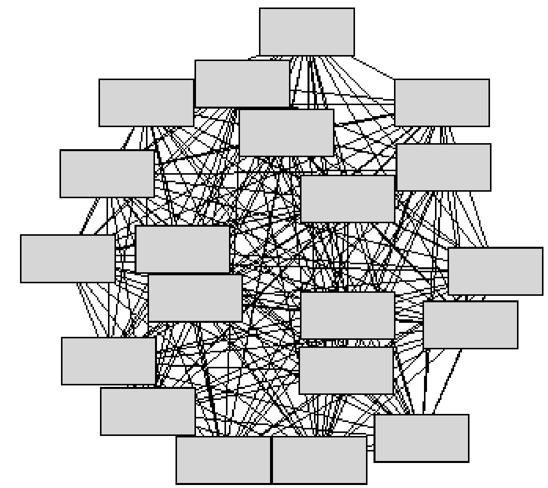
With J. Holyst

Role of communities

$$\nu = N_0/N$$

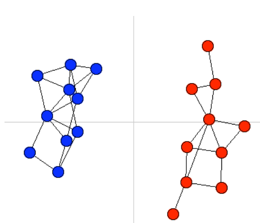


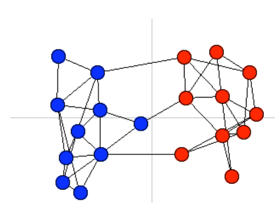


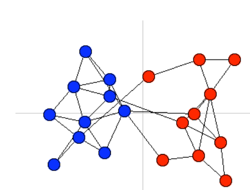


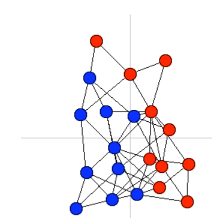
$\nu = 0$ $\nu = 0.3$ $\nu = 1$

$N = 20$





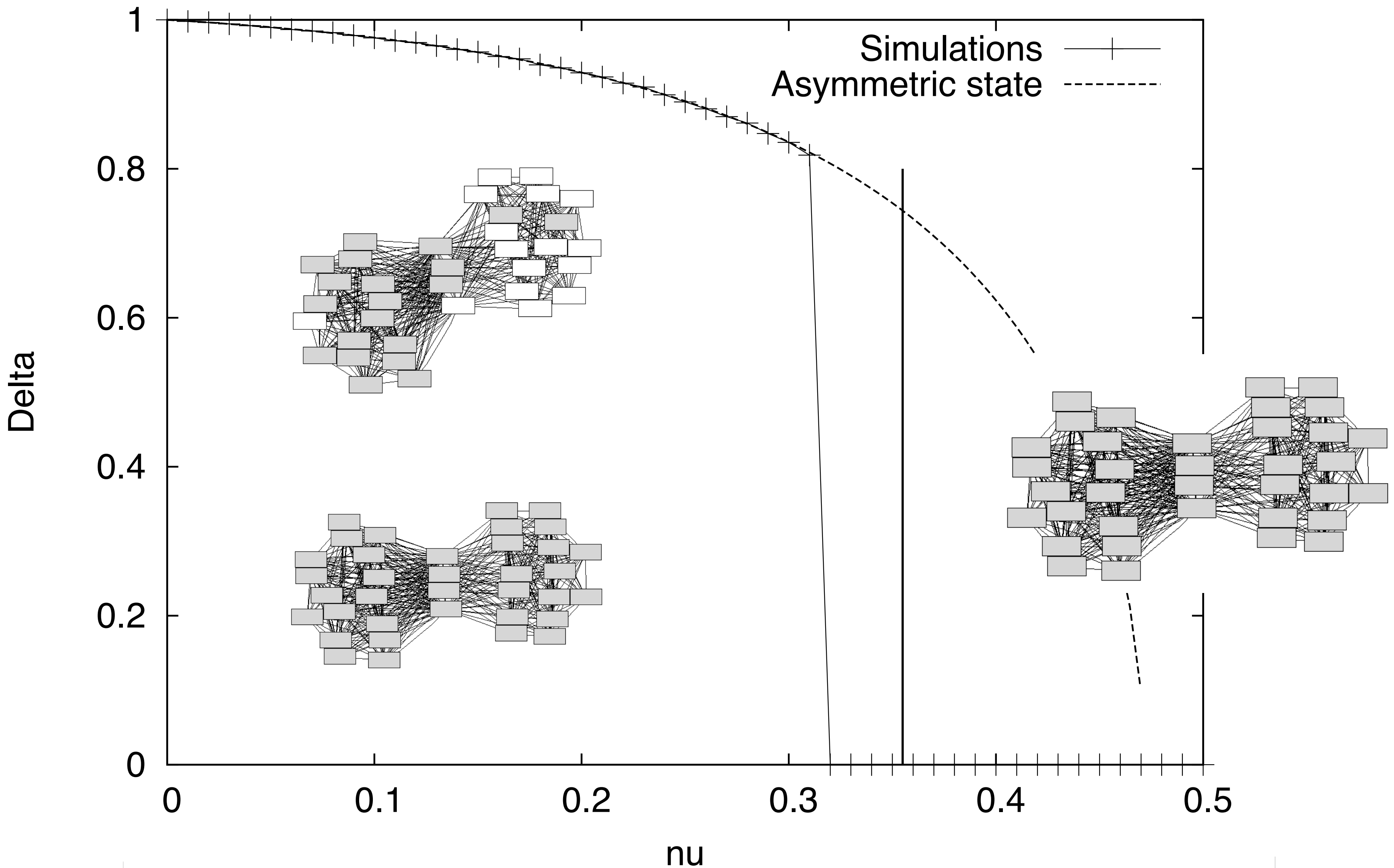




A simple model but lengthy calculations...

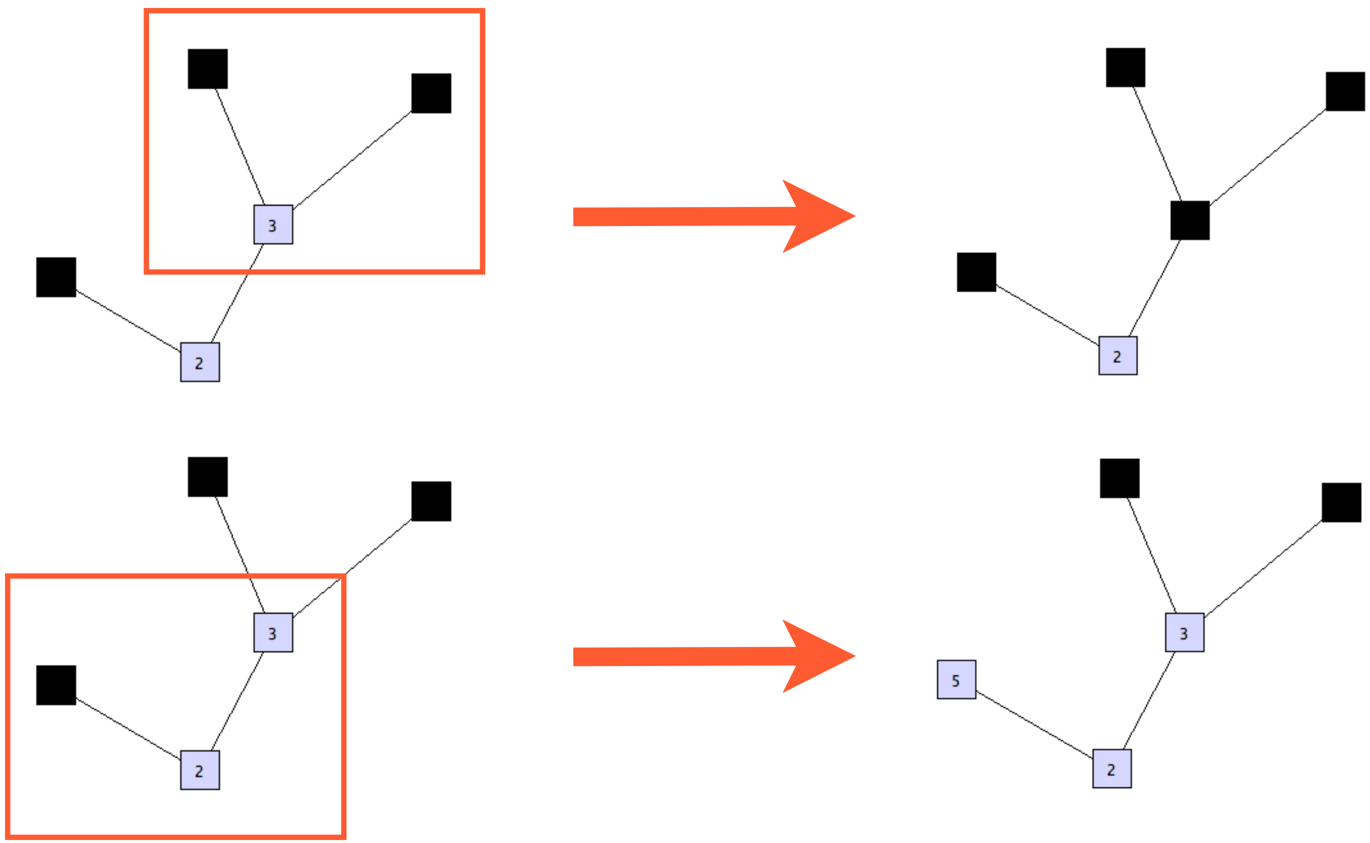
$$
\begin{aligned}
A_{0;t+1} - A_{0;t} &= p_{300}(a_0^2 b_0 - a_0 b_0^2) + \frac{2}{3} p_{210}(a_0 a_1 b_0 - a_0 b_0 b_1) + \frac{1}{3} p_{120}(a_1^2 b_0 - a_0 b_1^2) + \frac{2}{3} p_{201}(a_0 a_2 b_0 - a_0 b_0 b_2) \\
&+ \frac{1}{3} p_{102}(a_2^2 b_0 - a_0 b_2^2) + \frac{1}{3} p_{111}(a_1 a_2 b_0 - a_0 b_1 b_2) \\
A_{1;t+1} - A_{1;t} &= p_{030}(a_1^2 b_1 - a_1 b_1^2) + \frac{2}{3} p_{120}(a_0 a_1 b_1 - a_1 b_0 b_1) + \frac{1}{3} p_{210}(a_0^2 b_1 - a_1 b_0^2) + \frac{1}{3} p_{111}(a_0 a_2 b_1 - a_1 b_0 b_2) \\
A_{2;t+1} - A_{2;t} &= p_{003}(a_2^2 b_2 - a_2 b_2^2) + \frac{2}{3} p_{102}(a_0 a_2 b_2 - a_2 b_0 b_2) + \frac{1}{3} p_{201}(a_0^2 b_2 - a_2 b_0^2) + \frac{1}{3} p_{111}(a_0 a_1 b_2 - a_2 b_0 b_1)
\end{aligned}
$$

Bifurcation



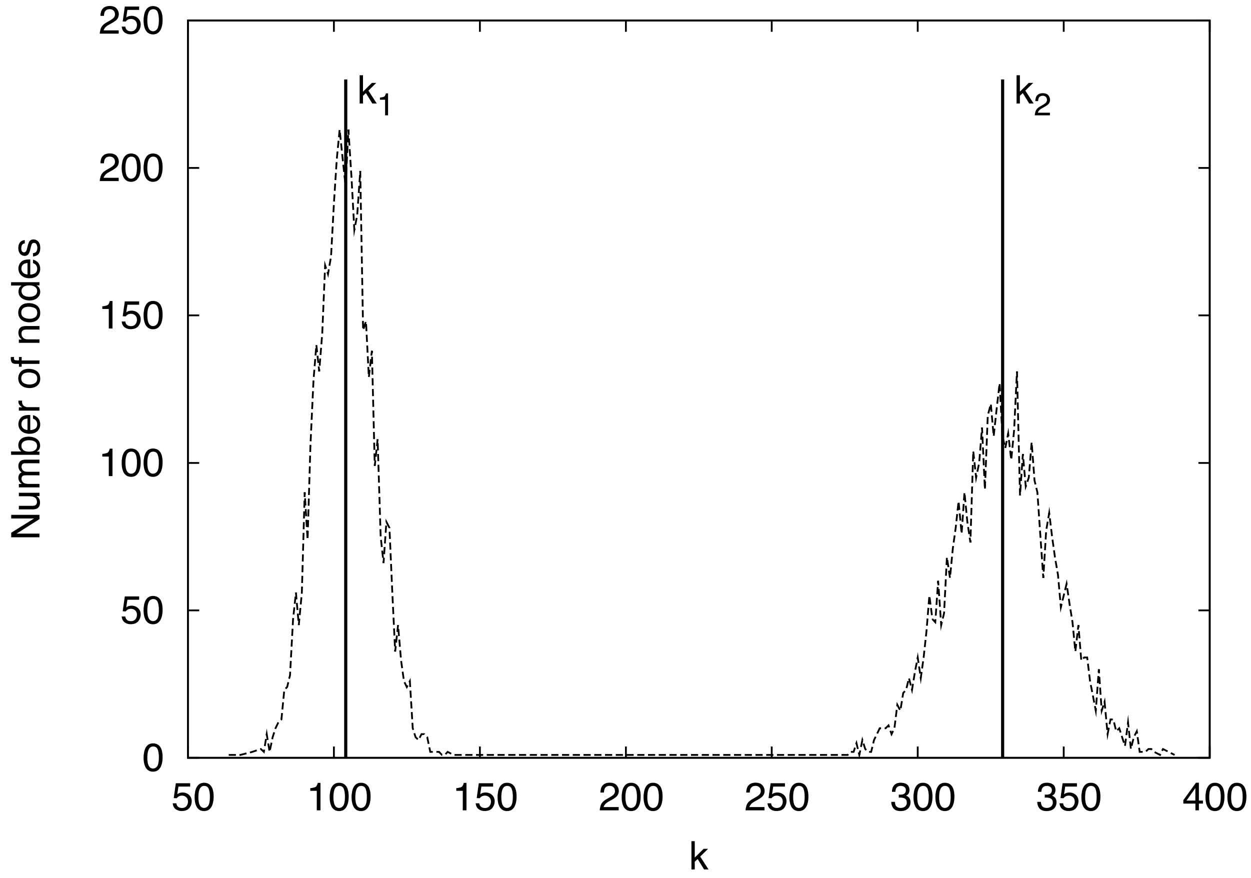
Majority model: role of the degree distribution

At each time step, one node is selected. Two processes:
i) With probability q, random change
ii) Else, 2 people around him reach local majority



Dichotomous networks

There are two kinds of nodes, each kind *i* being characterised by a degree k_i



Node dispersity: $$\alpha = k_2/k_1$$

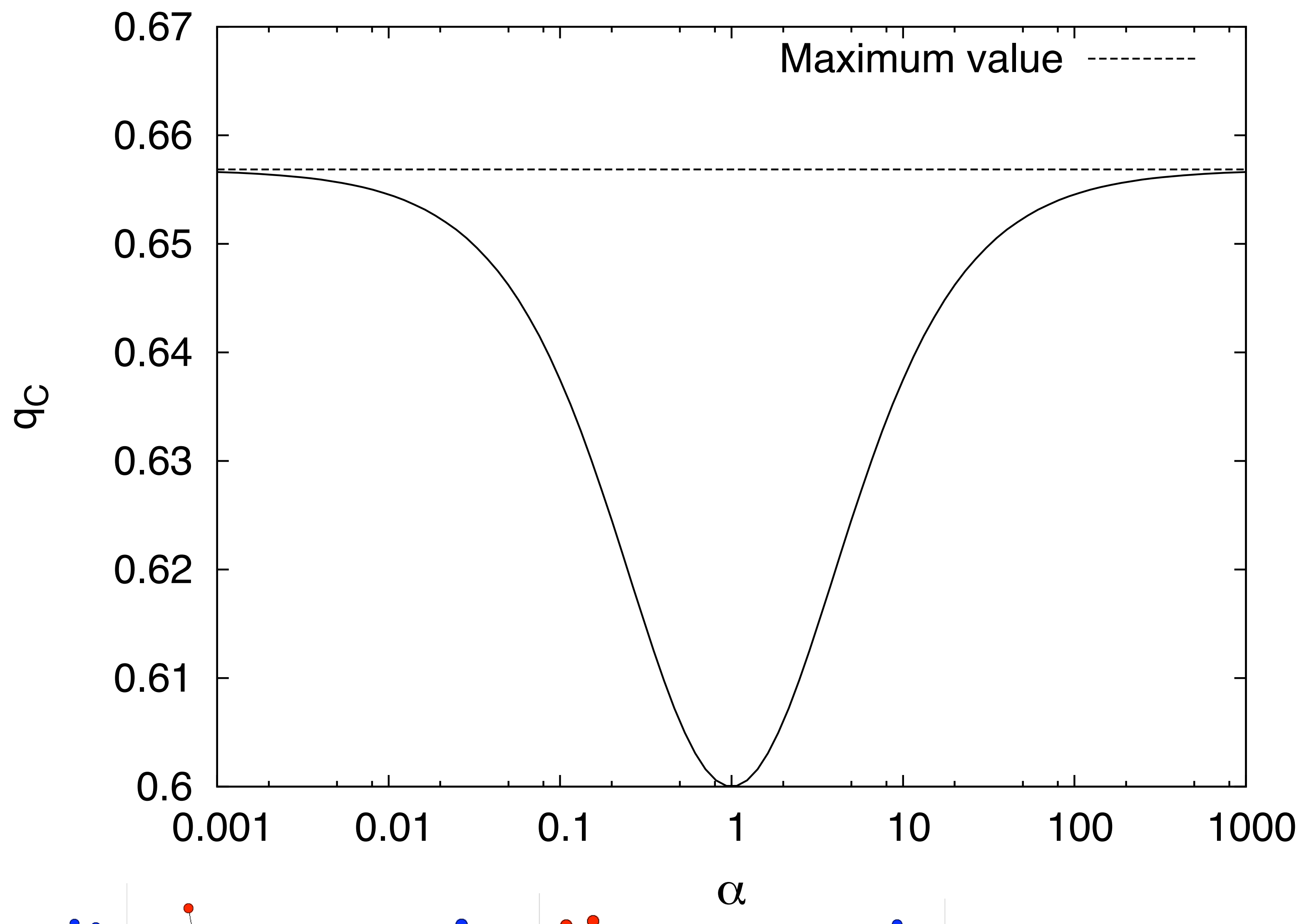
Order-disorder transition

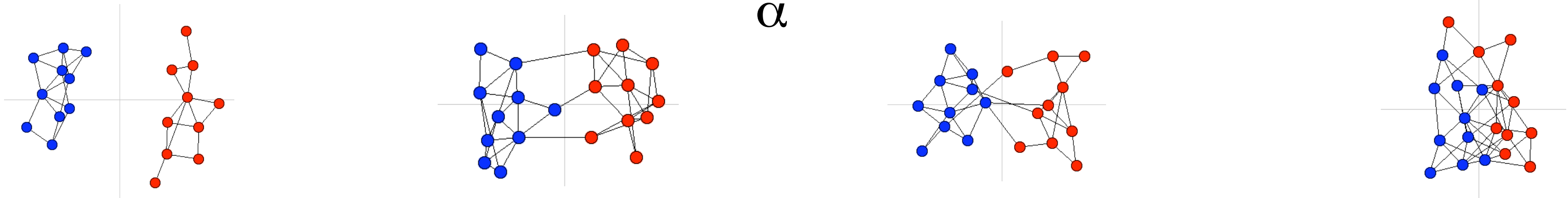
from which one finds that the symmetric solution $a = 1/2$ ceases to be stable when $q < 3/5$, and that the system reaches the following asymmetric solutions in that case

$$a_- = \frac{1}{2} - \sqrt{\frac{3-5q}{12(1-q)}}, \quad a_+ = \frac{1}{2} + \sqrt{\frac{3-5q}{12(1-q)}}. \qquad (5)$$

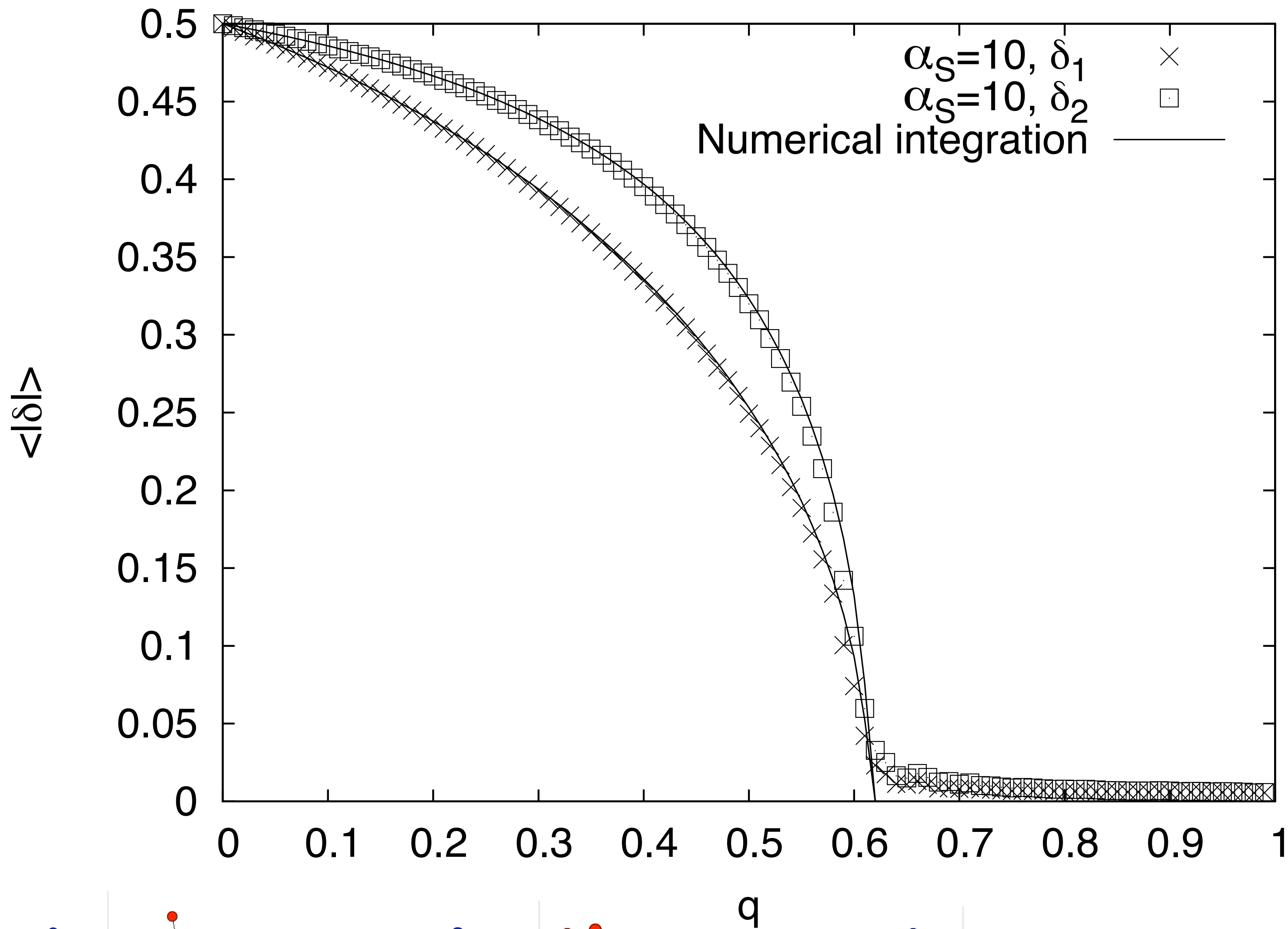
$$\begin{aligned}
A_{1;t+1} - A_{1;t} &= \frac{q}{4} - \frac{qa_1}{2} + \frac{(1-q)}{2(1+\alpha)^2}[3(a_1^2 b_1 - a_1 b_1^2) \\
&+ 2(1+2\alpha)(a_2 a_1 b_1 - a_1 b_2 b_1) \\
&+ (2\alpha + \alpha^2)(a_2^2 b_1 - a_1 b_2^2)] \\
A_{2;t+1} - A_{2;t} &= \frac{q}{4} - \frac{qa_2}{2} + \frac{(1-q)}{2(1+\alpha)^2}[3\alpha^2(a_2^2 b_2 - a_2 b_2^2) \\
&+ 2(2\alpha + \alpha^2)(a_1 a_2 b_2 - a_2 b_1 b_2) \\
&+ (1+2\alpha)(a_1^2 b_2 - a_2 b_1^2)], \qquad (11)
\end{aligned}$$

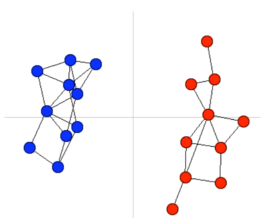
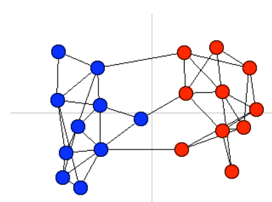
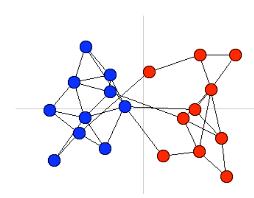
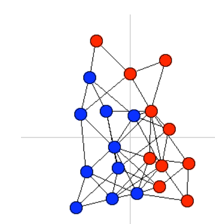
Degree diversity displaces the location of the transition





Degree diversity implies a *non-equipartition* of the opinion between species



Summary

- Simple model for social networks that reproduces qualitatively many empirical features
- Information networks: ageing and redirection may lead to qualitative changes in the structure
- Role of communities: co-existing communities
- Role of the degree: non-equipartition of *opinion*
- Unanimity model: "critical mass"

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R. Lambiotte and M. Ausloos, "Growing network with *j*-redirection"**,** *Europhys. Lett.* **77** (2007) 58002
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