

# Dynamics of non-conservative voters

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100 years of living science

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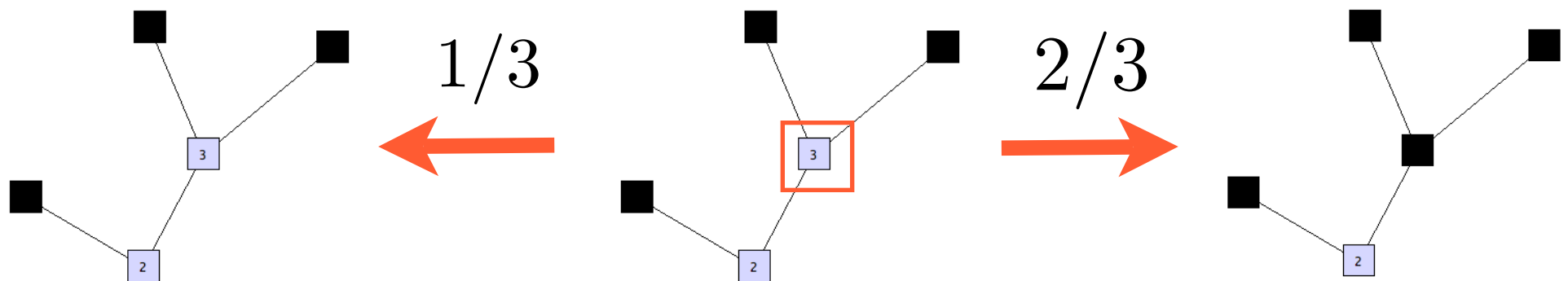
Simplest interaction possible: imitation. People copy the behaviour of their friend, acquaintances, neighbours, etc.

## Voter Model

$N$  agents have an opinion:  $-1$  or  $1$

The population evolves by:

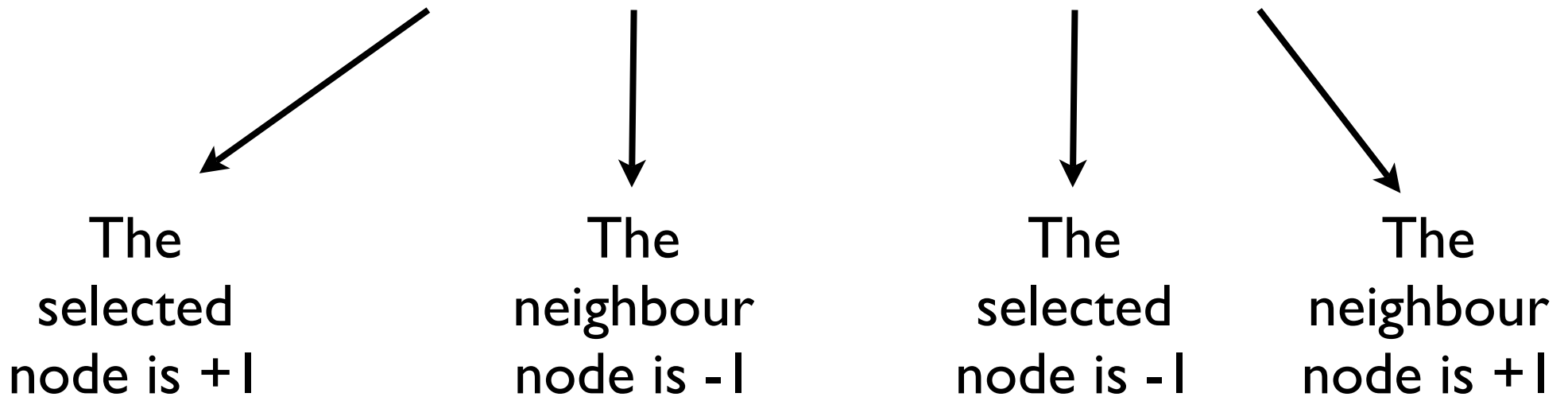
- (i) picking a random voter
- (ii) the selected voter adopts the state of a randomly-chosen neighbor
- (iii) repeating these steps *ad infinitum* or until a finite system necessarily reaches consensus.



With this dynamics, a voter chooses a state with a probability equal to the fraction of neighbors in that state.

Time evolution the density  $x$  of +1 voters in the MF

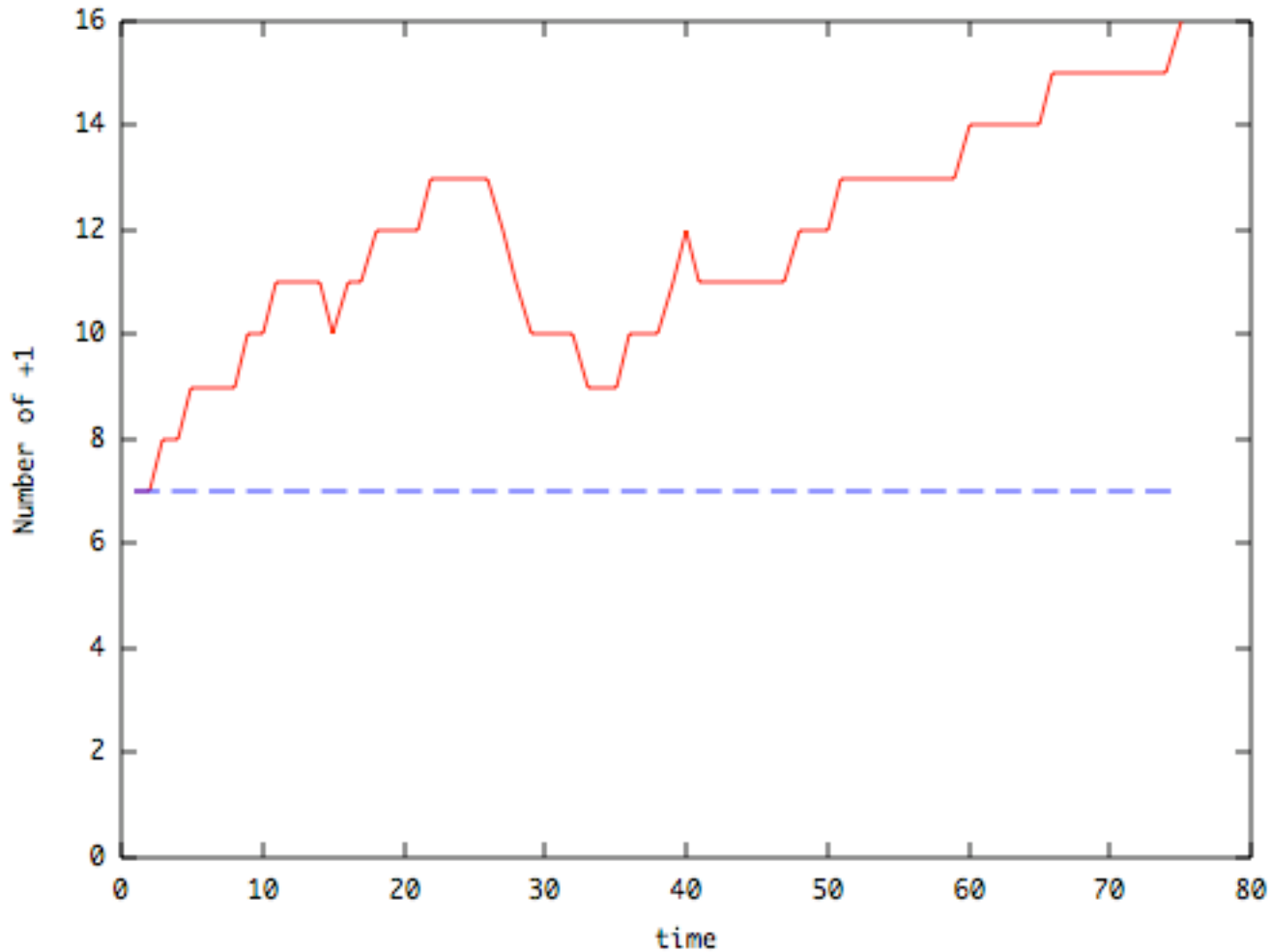
$$\partial_t x = -x(1-x) + (1-x)x = 0$$



$$m \equiv x - (1-x) = 2x - 1$$

=> The average magnetization is conserved

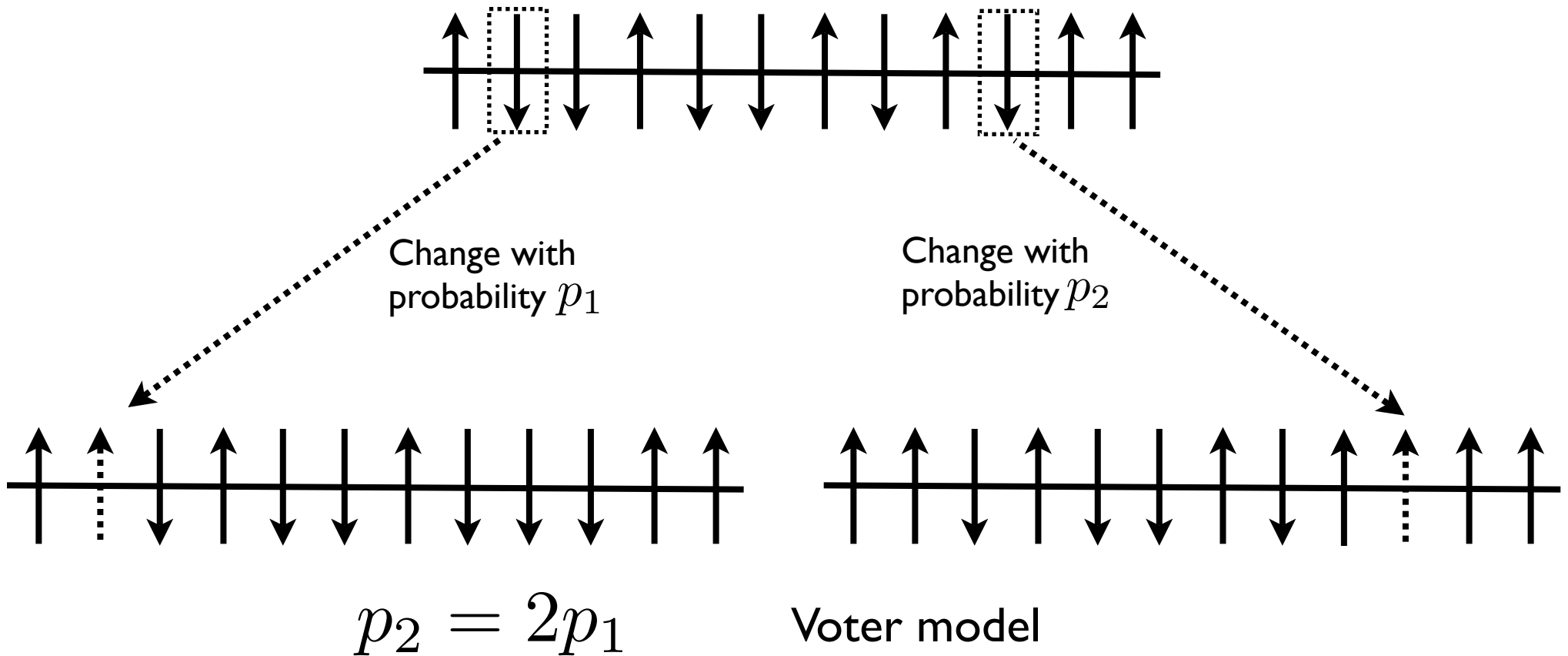
# The dynamics is stochastic



What is the probability to reach +1 consensus as a function of the initial condition?

What's the exit time to reach consensus?

# Non-conservative Voter models



$$\gamma = p_2/p_1$$

! voters change opinion only when in contact with another opinion

*Dynamics of Vacillating Voters*, R. Lambiotte and S. Redner, *JSTAT*, (2007) L10001

*Dynamics of non-conservative Voters*, R. Lambiotte and S. Redner, *EPL* **82** (2008) 18007

$\gamma = 2$  one recovers the classical voter model

$\gamma > 2$  the combined effect of two neighbors is more than twice that of one neighbor. Equivalently, voters can be viewed as having a conviction for their opinion and strong peer pressure is needed to change their opinion.

$\gamma \rightarrow \infty$  voters only change opinion when are confronted by a unanimity of opposite-opinion voters

$\gamma < 2$  one disagreeing neighbor is more effective in triggering an opinion change than in the classical voter model.

$\gamma = 1$  one recovers the *vacillating* voter model where voters change opinion at a fixed rate if either 1 or 2 of their neighbors disagree with them.

$\gamma < 1$  *contrarian* regime where a voter is less likely to change opinion as the fraction of neighbors in disagreement increases.

$\gamma$  conviction parameter

Let us first consider first the mean-field limit (the spins of neighboring nodes are uncorrelated).

$$\frac{\partial m}{\partial t} = 2(\gamma - 2)(m - m^3)$$

$$m \equiv x - (1 - x) = 2x - 1$$

where  $m$  is the average magnetization (opinion)  
and  $x$  is the density of +1 voters

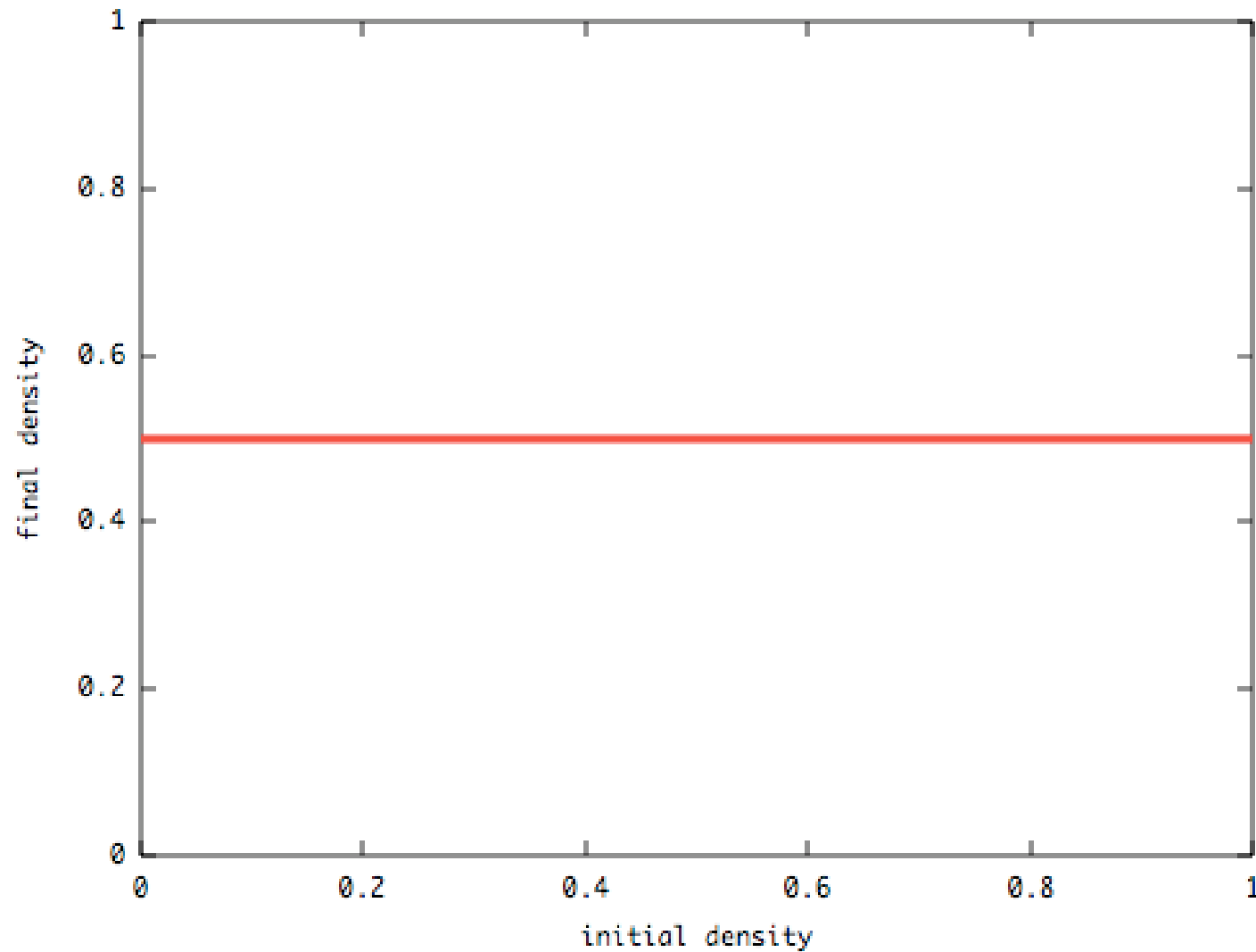
Qualitative change at  $\gamma = 2$

$$\gamma > 2 \quad \text{Population is driven toward consensus}$$
$$m = [-1, 1] \quad x = [0, 1]$$

$$\gamma < 2 \quad \text{Population is driven toward zero-magnetisation}$$
$$m = 0 \quad x = 1/2$$

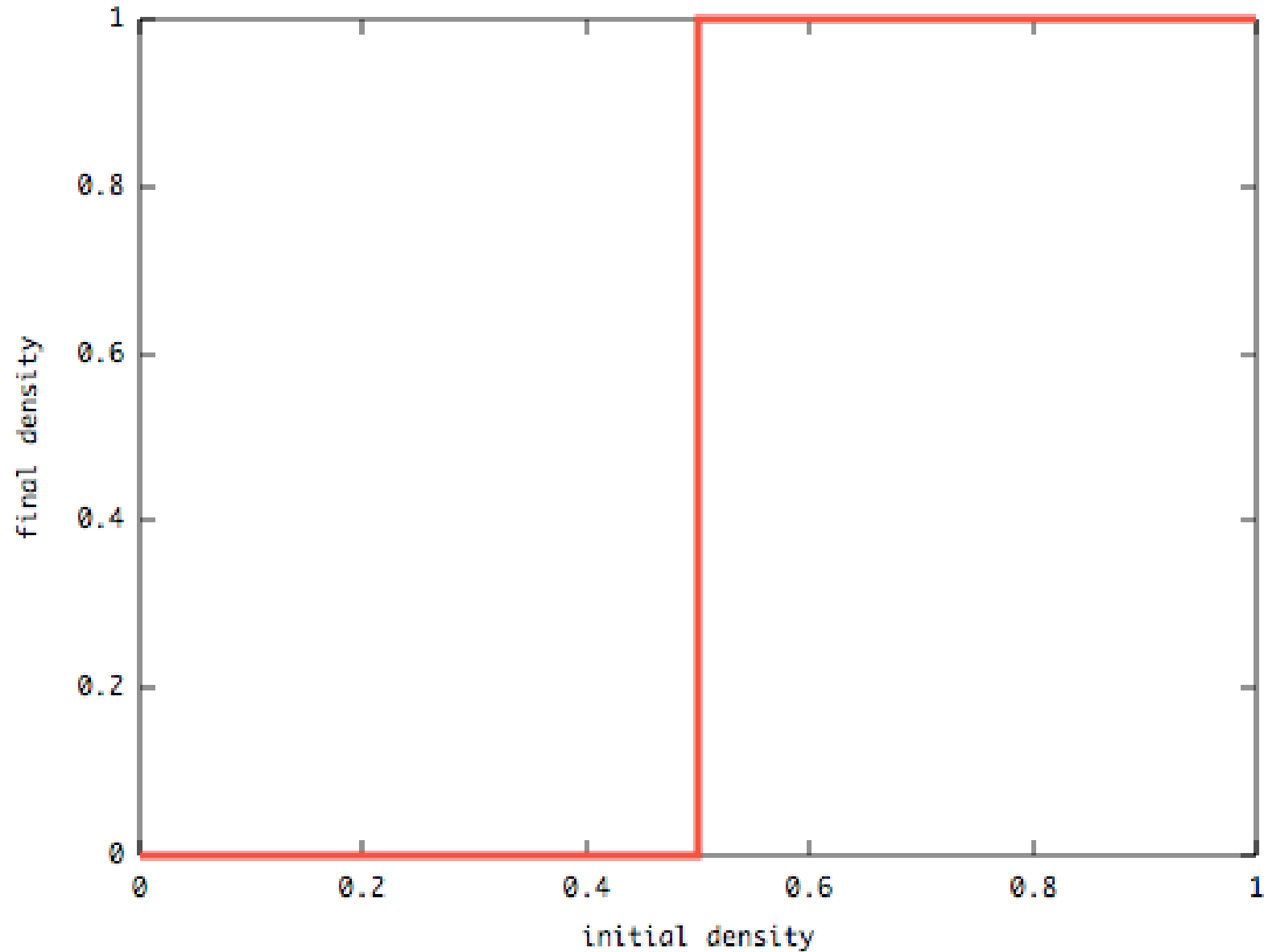
# Dependence on the initial conditions

$$\gamma < 2$$



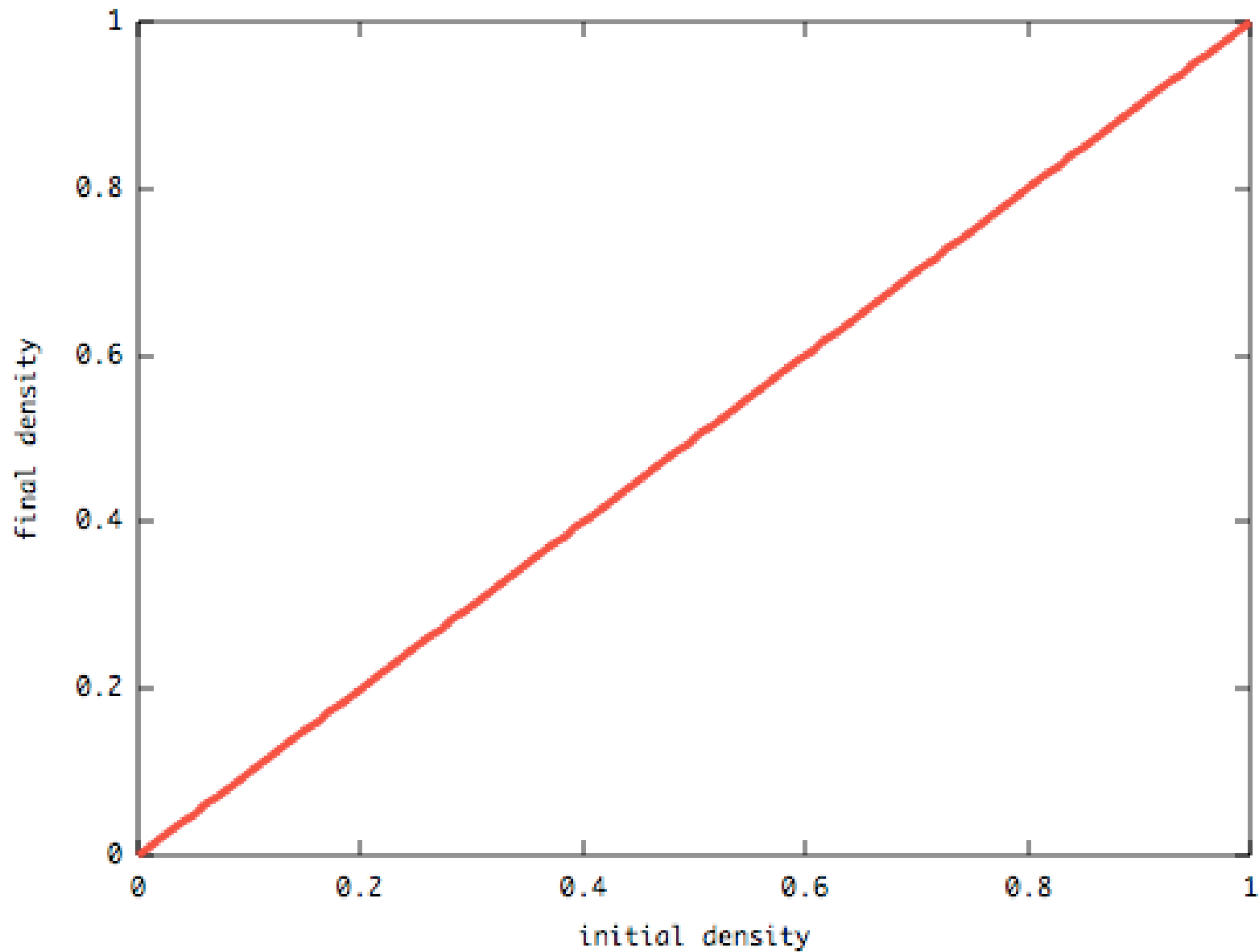
# Dependence on the initial conditions

$$\gamma > 2$$

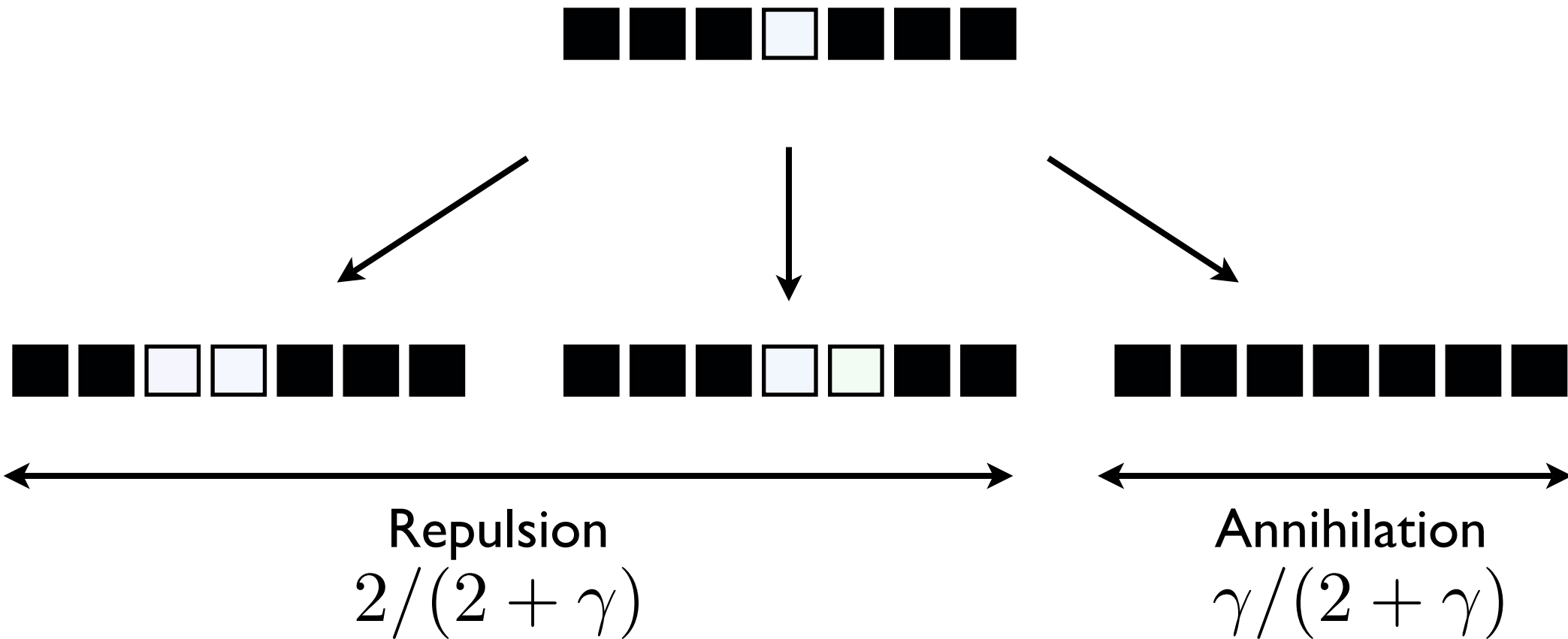


# Dependence on the initial conditions

$$\gamma = 2$$

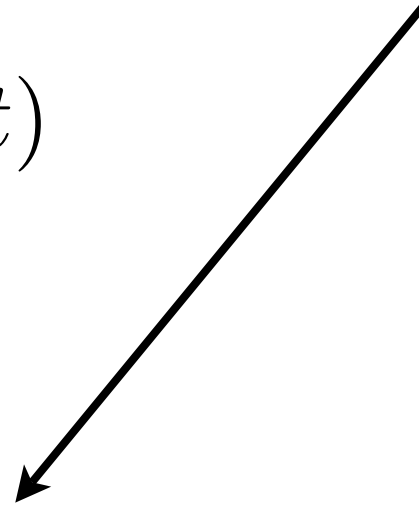


In one dimension, the system coarsens and the interface between domains performs a symmetric random walk, except when domain walls are adjacent



$$\frac{\partial s_j}{\partial t} = 2\gamma(s_{j+1} + s_{j-1}) - 2(\gamma + 2)s_j - 2(\gamma - 2)\langle\sigma_{j-1}\sigma_j\sigma_{j+1}\rangle$$

$$s_j \equiv \langle\sigma_j\rangle = \sum_{\{\sigma\}} \sigma_j P(\{\sigma\}; t)$$

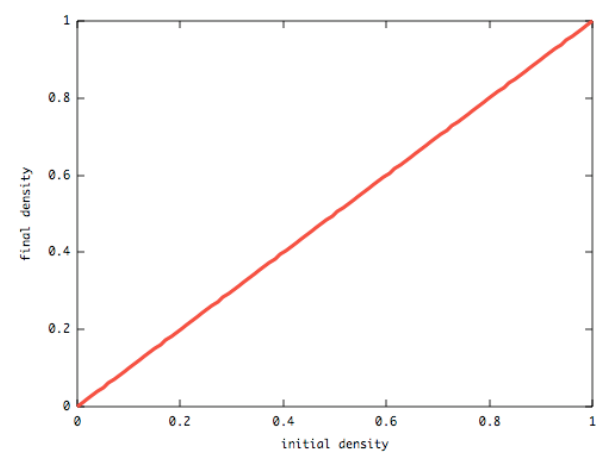
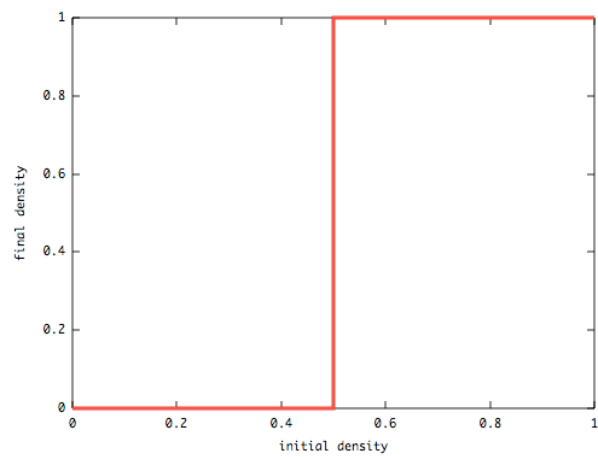
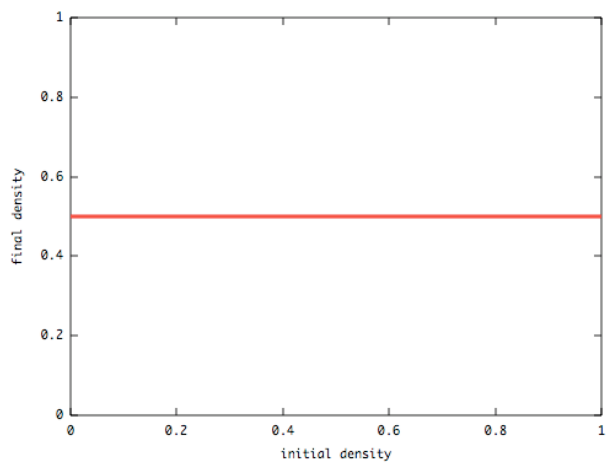
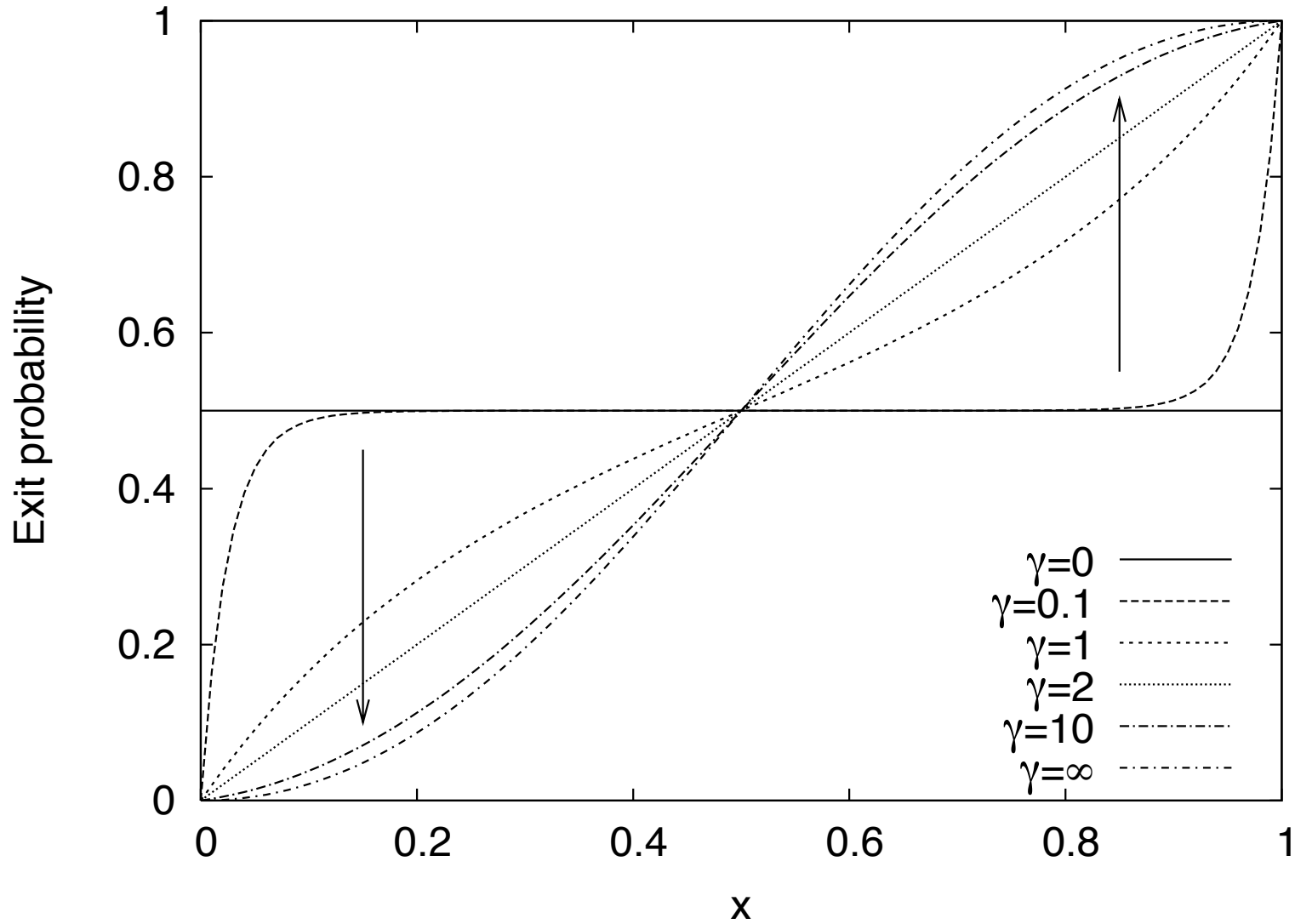


Coupling with higher order correlations

Need for a decoupling scheme

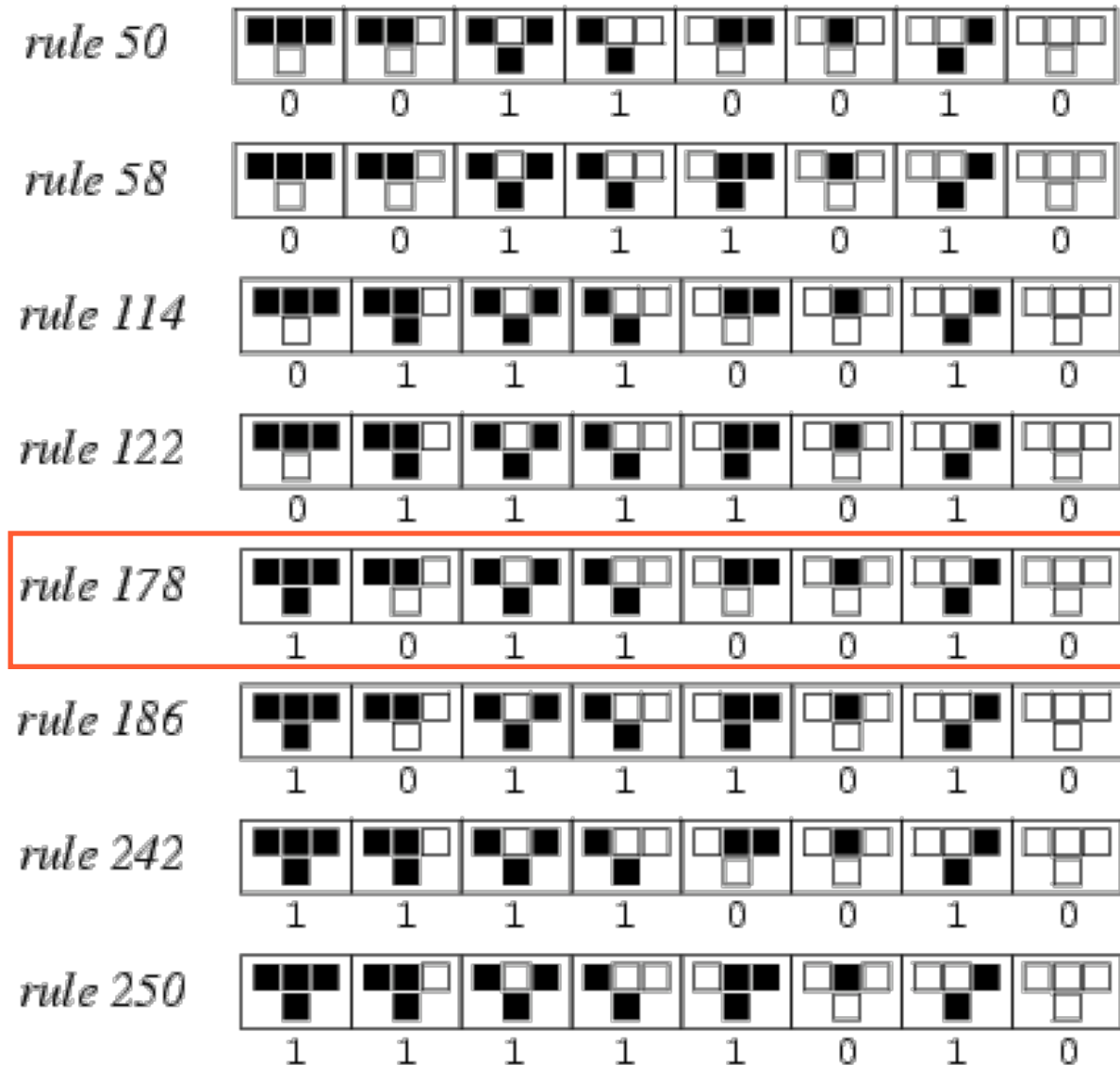
Non-trivial initial state dependence:

$$m(\infty) = m(0) e^{(2-\gamma)/[2\gamma(m(0)^2-1)]}$$

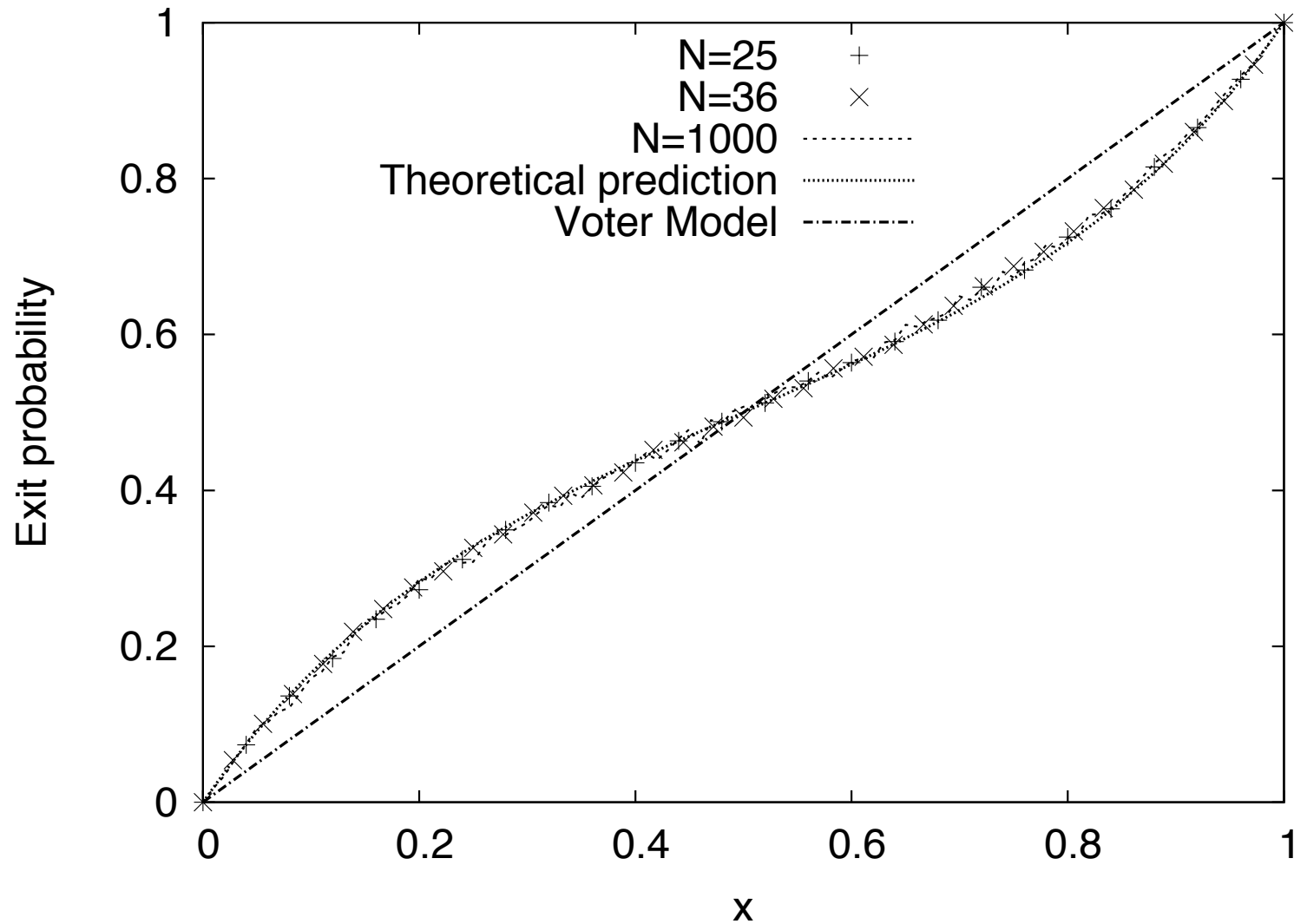


$$\gamma = 1$$

The *vacillating* voter model where voters change opinion at a fixed rate if either 1 or 2 of their neighbors disagree with them, equivalent to rule 178 of the one-dimensional cellular automaton (see Wolfram).

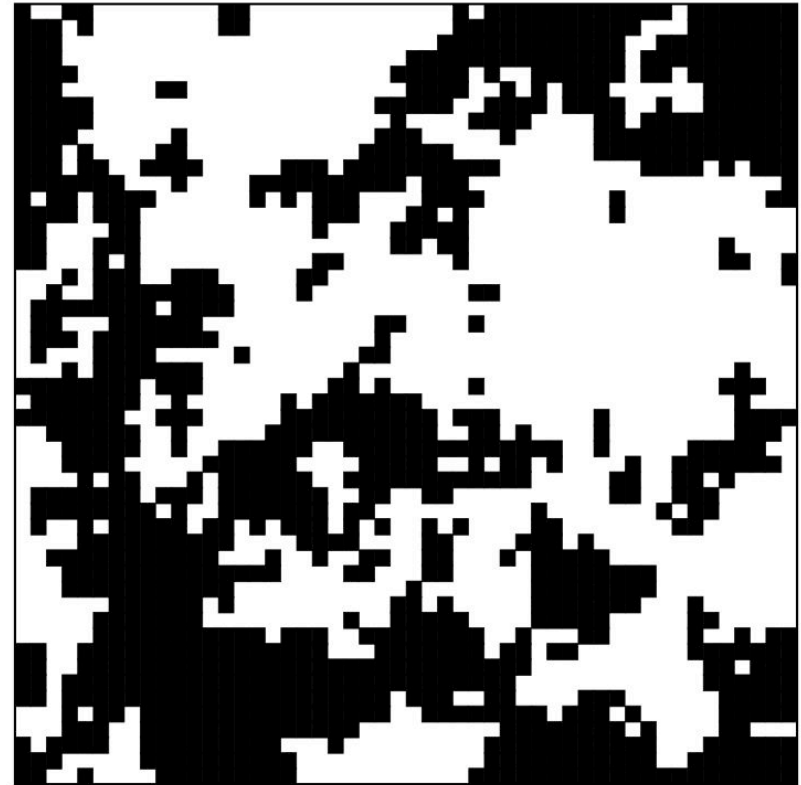


The probability to reach the final state of +I consensus has a non-trivial initial state dependence.





Vacillating Voter Model



Voter Model

$$C_1 \equiv \langle \sigma_{i,j} \sigma_{i,j+1} \rangle \rightarrow 0.31$$



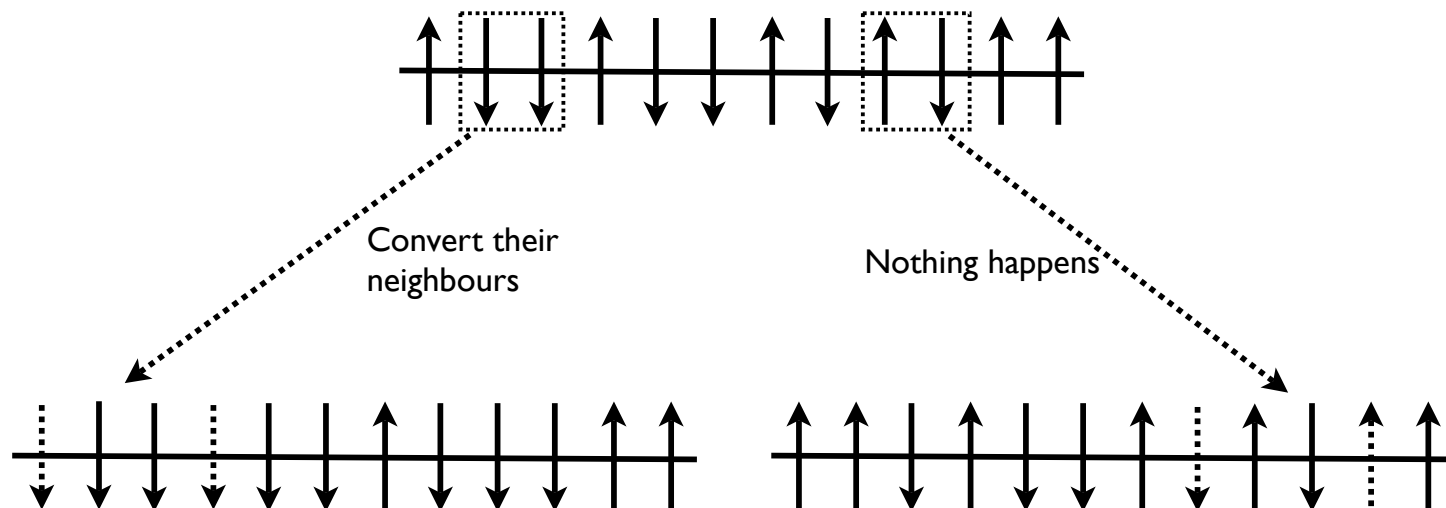
domains of opposite opinions coexist

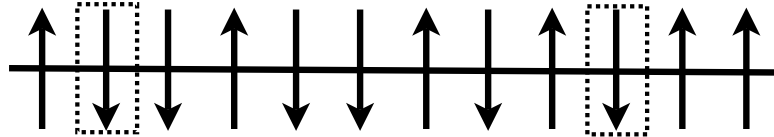
The approach developed above can also be applied to another simple opinion dynamics model in one dimension, namely, the Sznajd model.

Social validation: agents are only influenced by groups (e.g. pairs) of aligned voters and not by single individuals.

The Sznajd is defined by the following evolution rule:

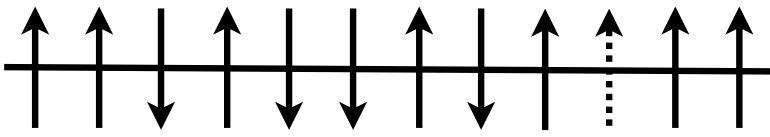
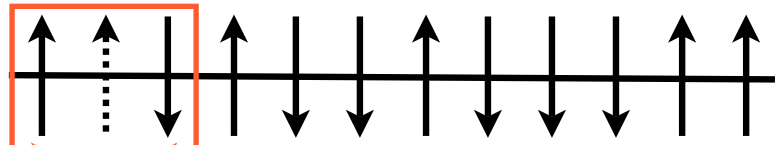
- (i) pick a pair of neighboring voters;
- (ii) if these voters have the same opinion, convert the opinion of the neighbors on either side of the initial pair;
- (iii) repeat these steps *ad infinitum* or until a finite system necessarily reaches consensus.





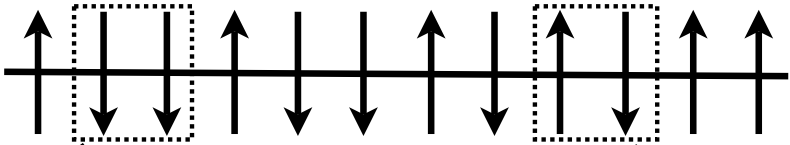
Change with probability  $p_1$

Change with probability  $p_2$



Inflow

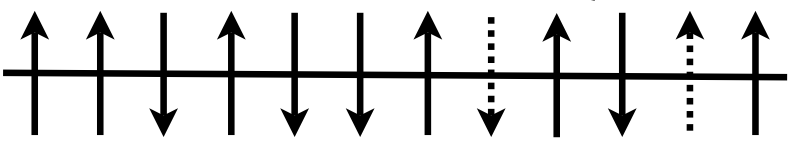
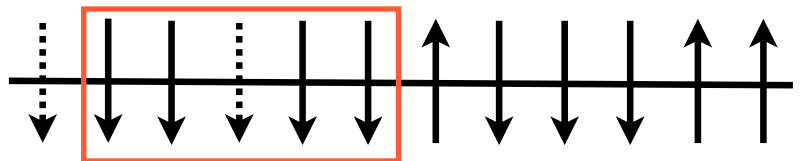
First neighbours



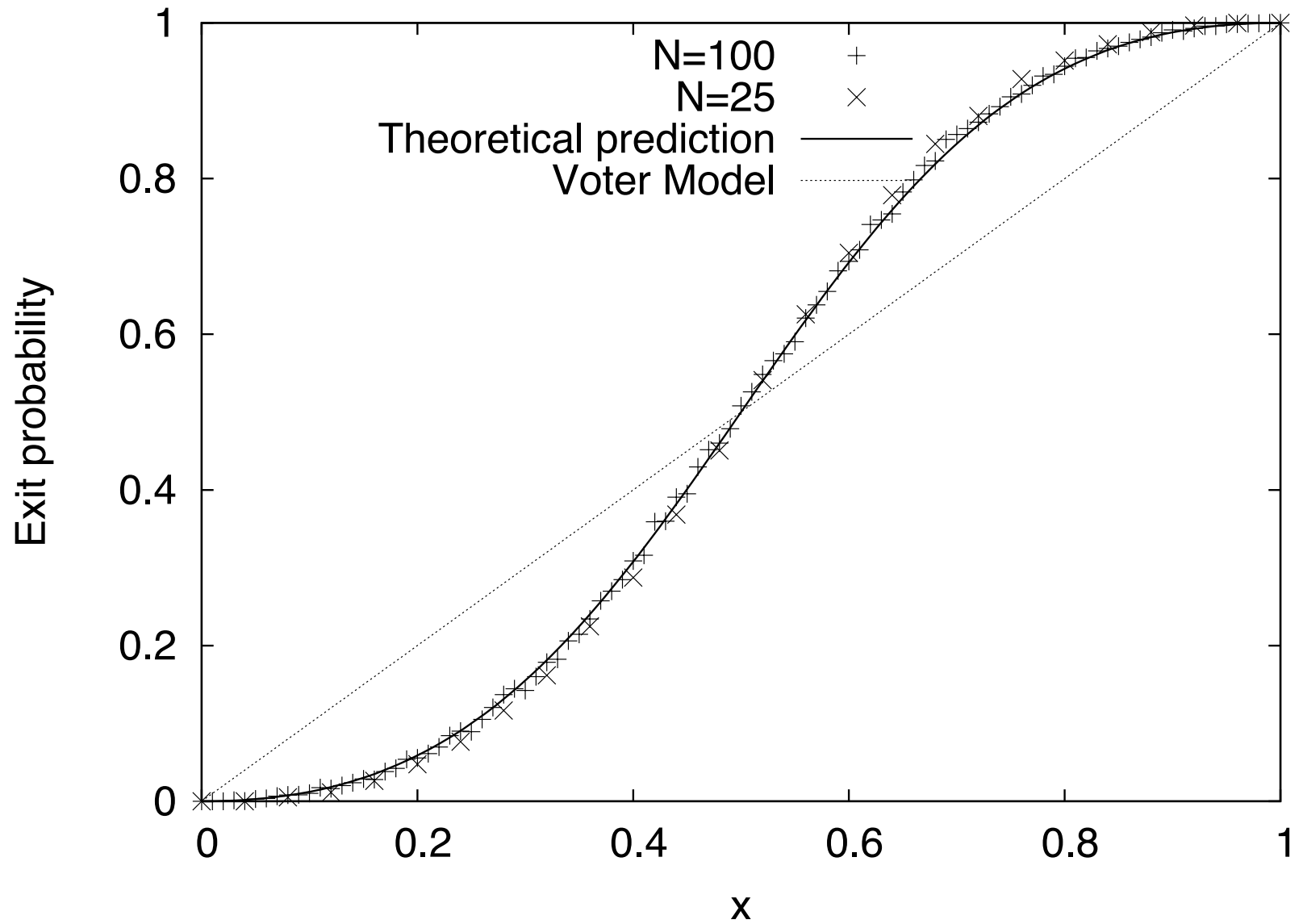
Convert their neighbours

Nothing happens

Also second neighbours



Outflow



*Dynamics of non-conservative Voters*, R. Lambiotte and S. Redner, *EPL* **82** (2008) 18007

*Some new results on one-dimensional outflow dynamics*, F. Slanina, K. Sznajd-Weron and P. Przybyła, *EPL* **82** (2008) 18006

# Role of communities: Coupled Random Networks

Distinct communities within networks are defined as subsets of nodes which are more densely linked when compared to the rest of the network.

The network is composed of  $N$  nodes divided into two types of nodes, 1 and 2. Different types of nodes have a probability  $p_{cross}$  to be linked, while nodes of the same type have a probability  $p_{in}$  to be linked.

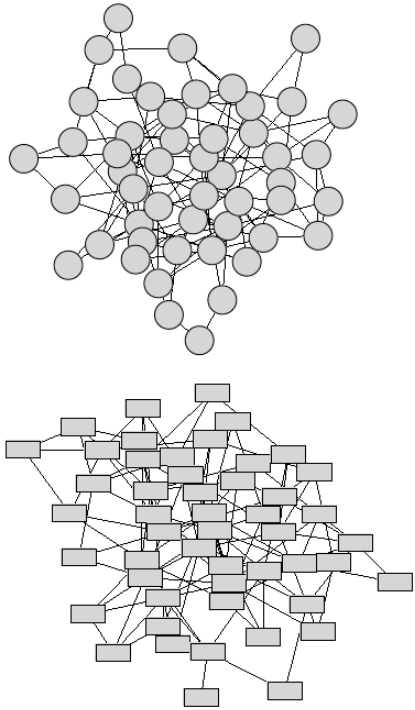
The inter-connectivity between the communities is tunable through the parameter

$$\nu = p_{cross}/p_{in}$$

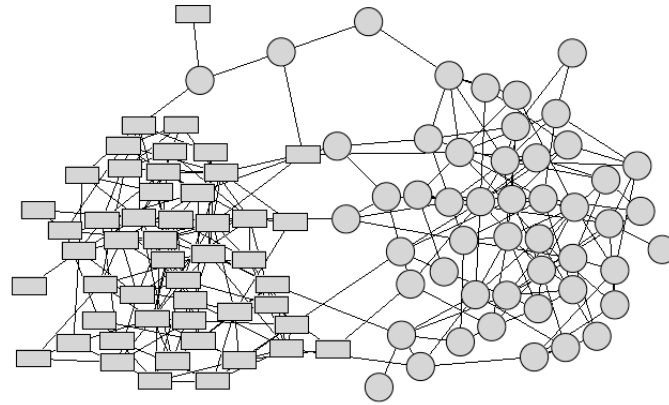
*Coexistence of opposite opinions in a network with communities*, R. Lambiotte and M. Ausloos, *JSTAT*, P08026 (2007)

*Majority Model on a network with communities*, R. Lambiotte, M. Ausloos and J. Holyst, *Phys. Rev. E*, **75** (2007) 030101(R)

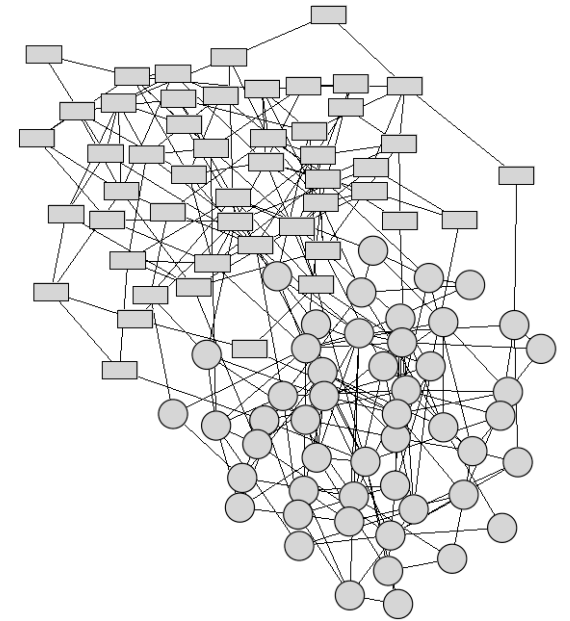
# Coupled Random Networks



$\nu = 0$



$\nu = 0.05$



$\nu = 0.1$

