Dynamics of non-conservative voters

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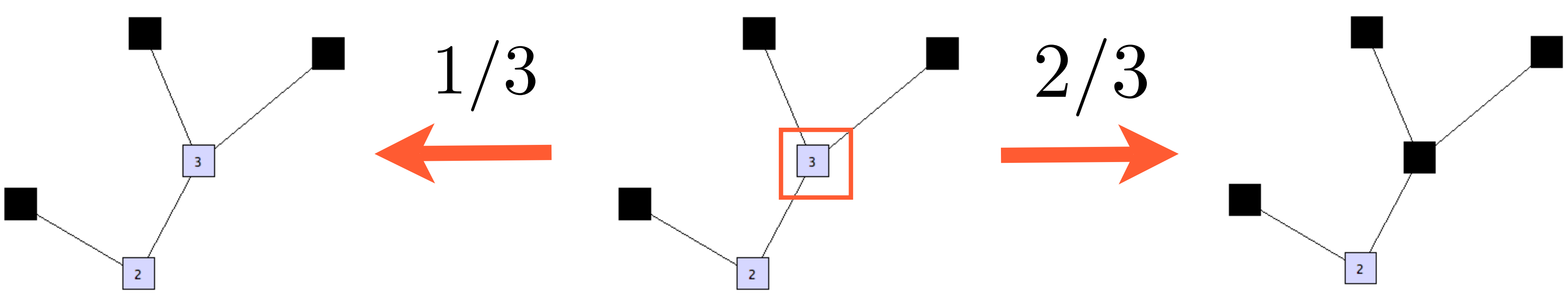
Claudio Castellano, Santo Fortunato, Vittorio Loreto, Statistical physics of social dynamics, Eprint arXiv: 0710.3256

Simplest interaction possible: imitation. People copy the behaviour of their friend, acquaintances, neighbours, etc.

Voter Model

N agents have an opinion: -1 or 1

The population evolves by:
(i) picking a random voter
(ii) the selected voter adopts the state of a randomly-chosen neighbor
(iii) repeating these steps *ad infinitum* or until a finite system necessarily reaches consensus.



Time evolution the density *x* of +1voters in the MF

$$\partial_t x = -x(1-x) + (1-x)x = 0$$

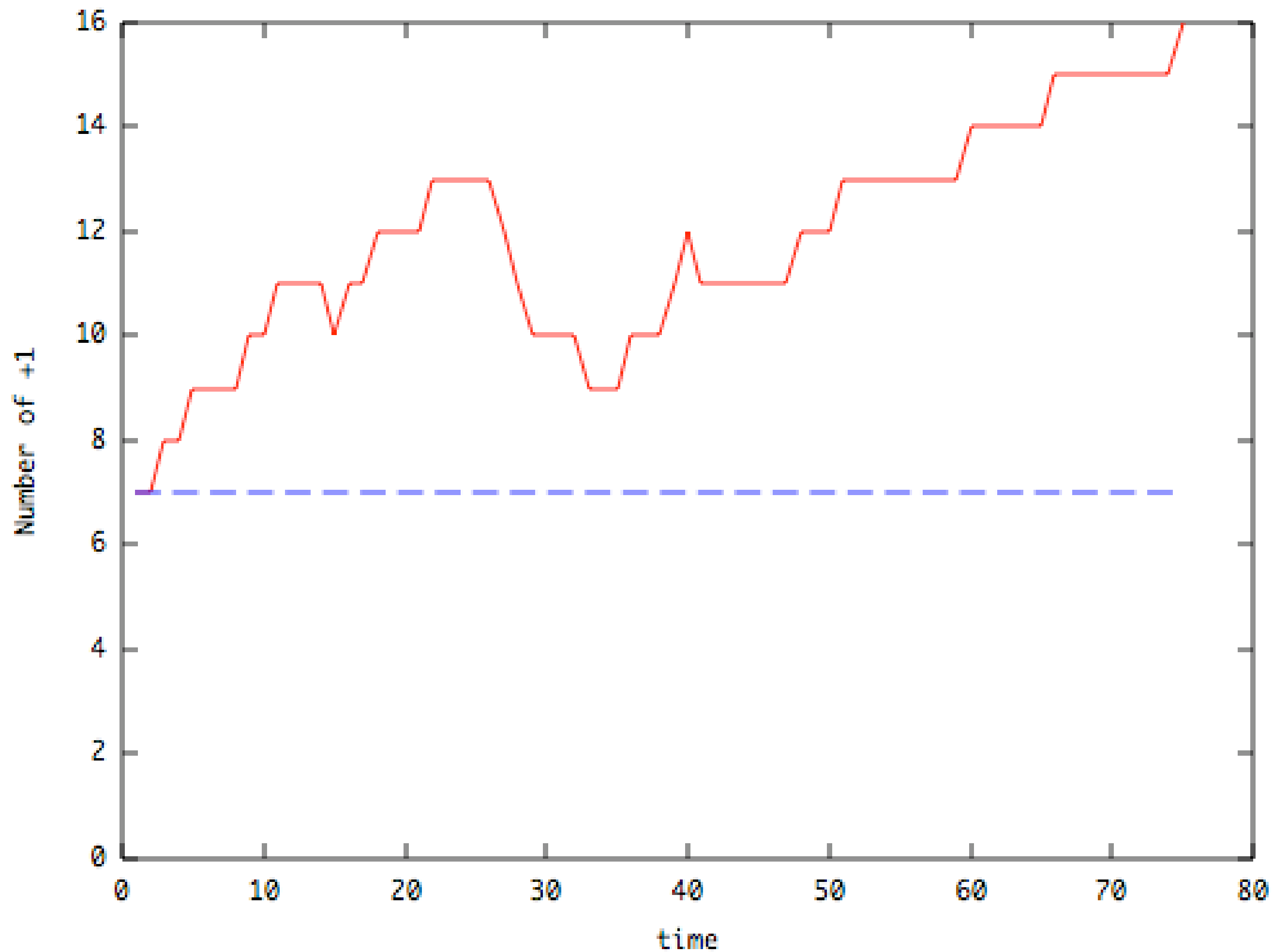
- $-x$: The selected node is +1
- $(1-x)$: The neighbour node is -1
- $(1-x)$: The selected node is -1
- x: The neighbour node is +1

$$m \equiv x - (1-x) = 2x - 1$$

=> **The average magnetization is conserved**

With this dynamics, a voter chooses a state with a probability equal to the fraction of neighbors in that state.

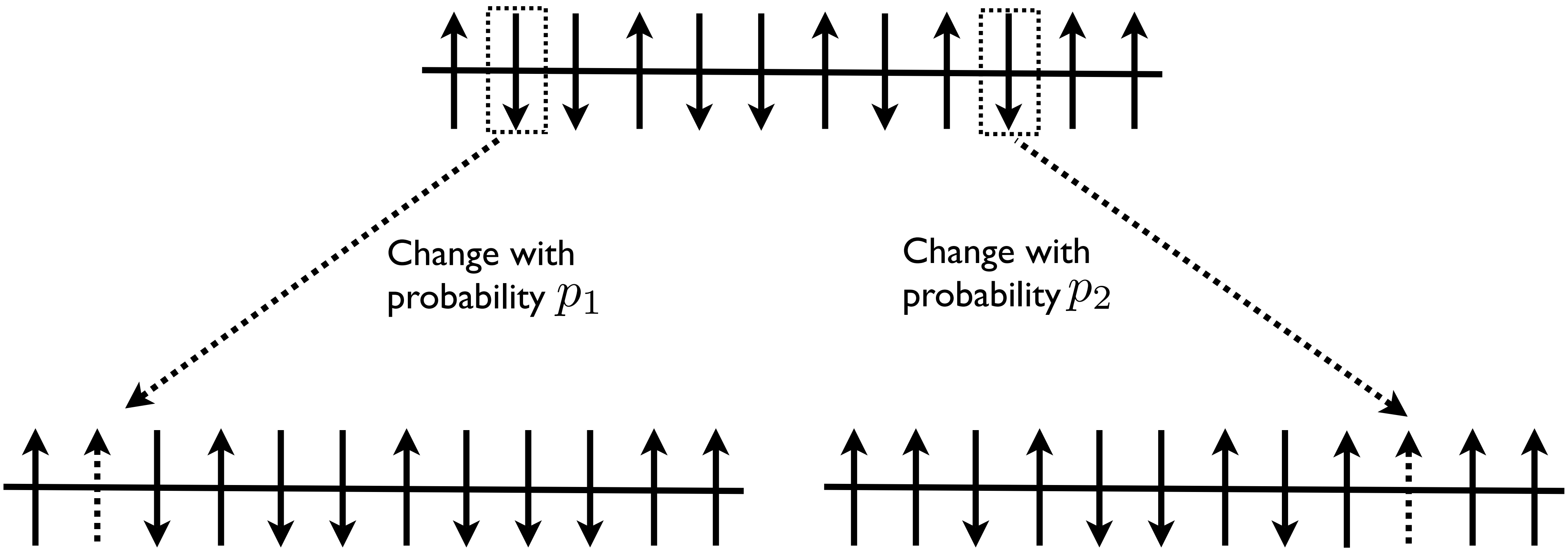
The dynamics is stochastic



What is the probability to reach +1 consensus as a function of the initial condition?

What's the exit time to reach consensus?

Non-conservative Voter models



$$p_2 = 2p_1$$

Voter model

$$\gamma = p_2/p_1$$

! voters change opinion only when in contact with another opinion

Dynamics of Vacillating Voters, R. Lambiotte and S. Redner, *JSTAT*, L10001 (2007)
Dynamics of non-conservative Voters, R. Lambiotte and S. Redner, *arXiv:0712.0364*

$\gamma = 2$ one recovers the classical voter model

$\gamma > 2$ the combined effect of two neighbors is more than twice that of one neighbor. Equivalently, voters can be viewed as having a conviction for their opinion and strong peer pressure is needed to change their opinion.

$\gamma \to \infty$ voters only change opinion when are confronted by a unanimity of opposite-opinion voters

$\gamma < 2$ one disagreeing neighbor is more effective in triggering an opinion change than in the classical voter model.

$\gamma = 1$ one recovers the *vacillating* voter model where voters change opinion at a fixed rate if either 1 or 2 of their neighbors disagree with them.

$\gamma < 1$ *contrarian* regime where a voter is less likely to change opinion as the fraction of neighbors in disagreement increases.

γ *conviction parameter*

Let us first consider first the mean-field limit (the spins of neighboring nodes are uncorrelated).

$$\frac{\partial m}{\partial t} = 2(\gamma - 2)(m - m^3)$$

$$m \equiv x - (1 - x) = 2x - 1$$

where m is the average magnetization (opinion) and x is the density of +1voters

Qualitative change at $\gamma = 2$

$\gamma > 2$ Population is driven toward consensus

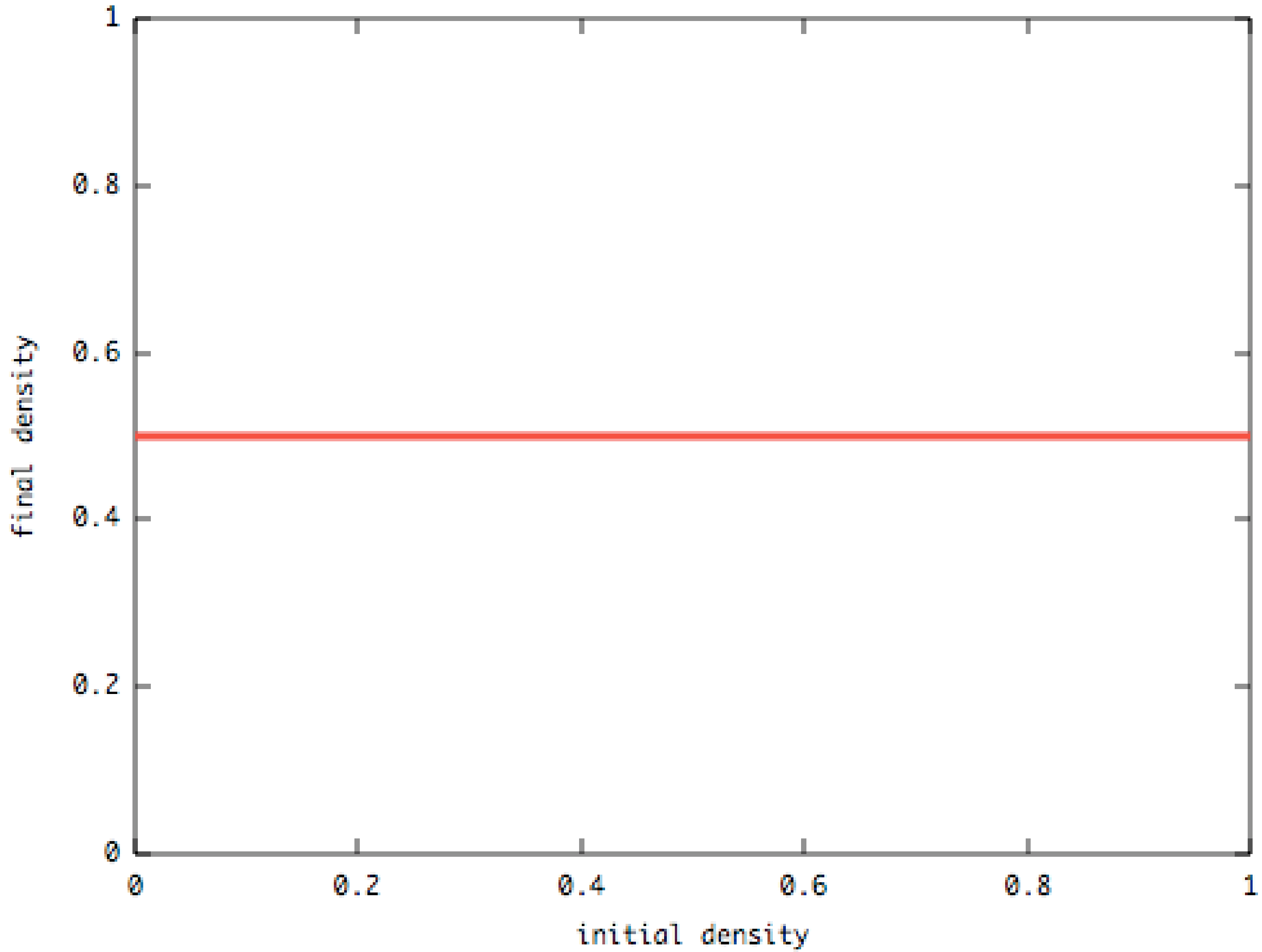
$m = [-1, 1] \quad x = [0, 1]$

$\gamma < 2$ Population is driven toward zero-magnetisation

$m = 0 \quad x = 1/2$

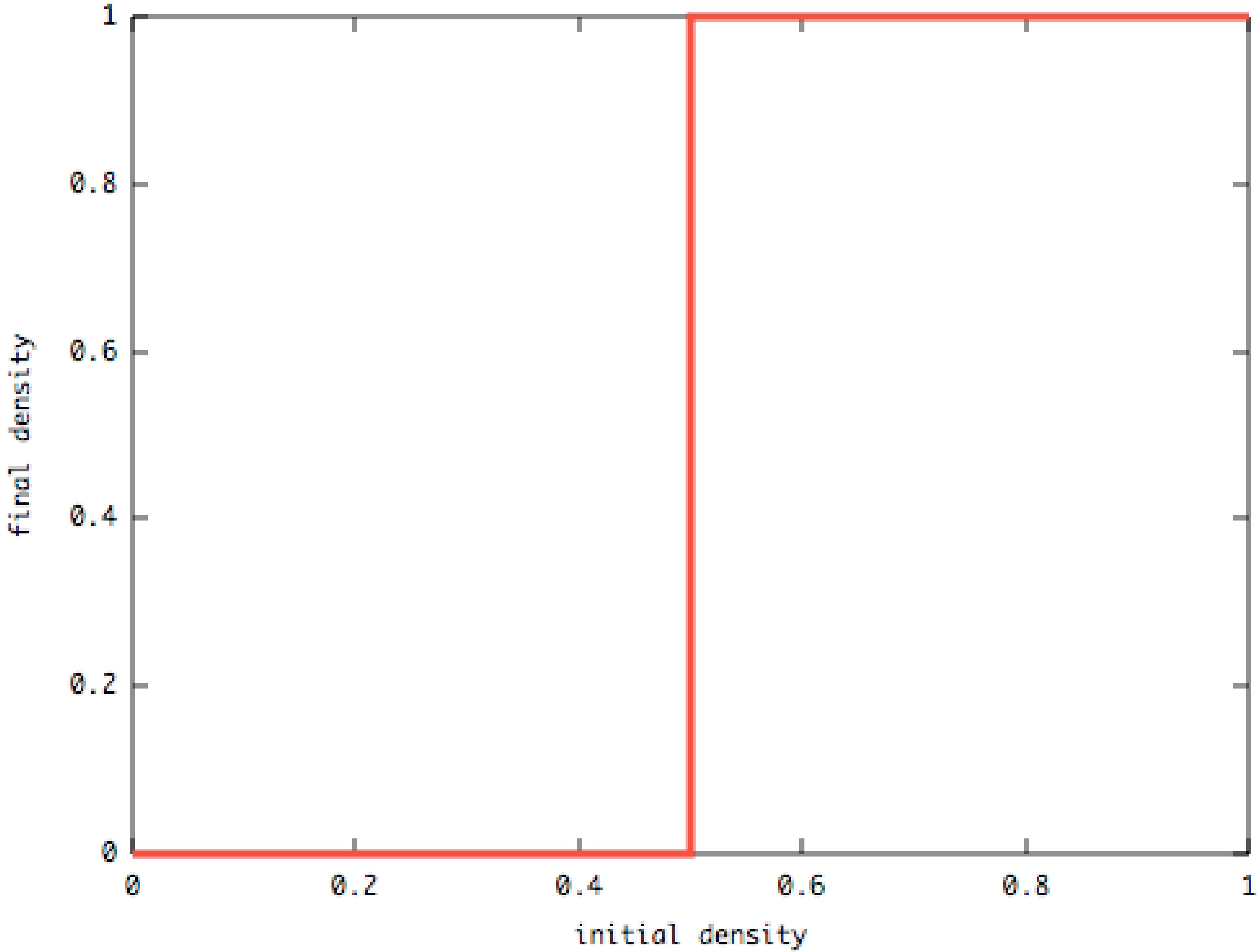
Dependence on the initial conditions

$$\gamma < 2$$



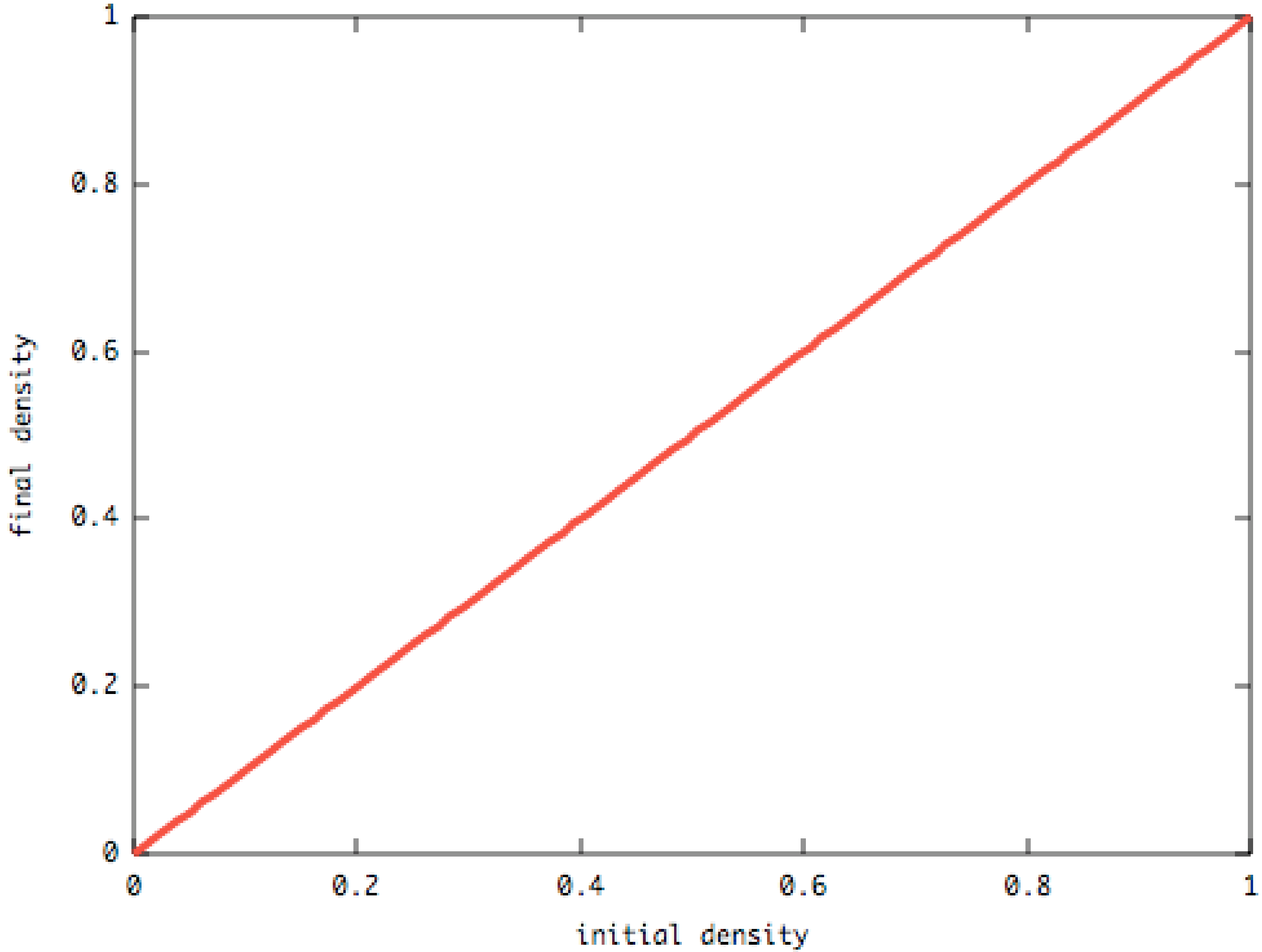
Dependence on the initial conditions

$$\gamma > 2$$

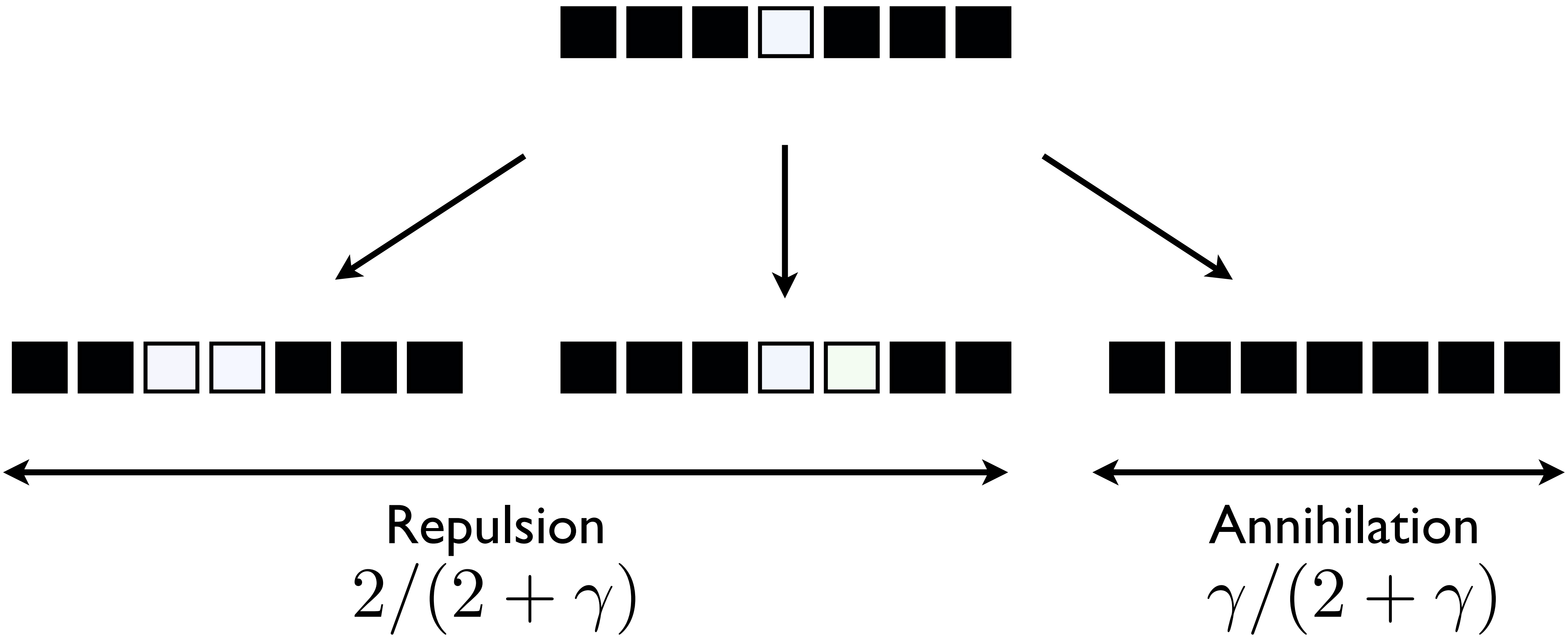


Dependence on the initial conditions

$\gamma = 2$



In one dimension, the system coarsens and the interface between domains performs a symmetric random walk, except when domain walls are adjacent



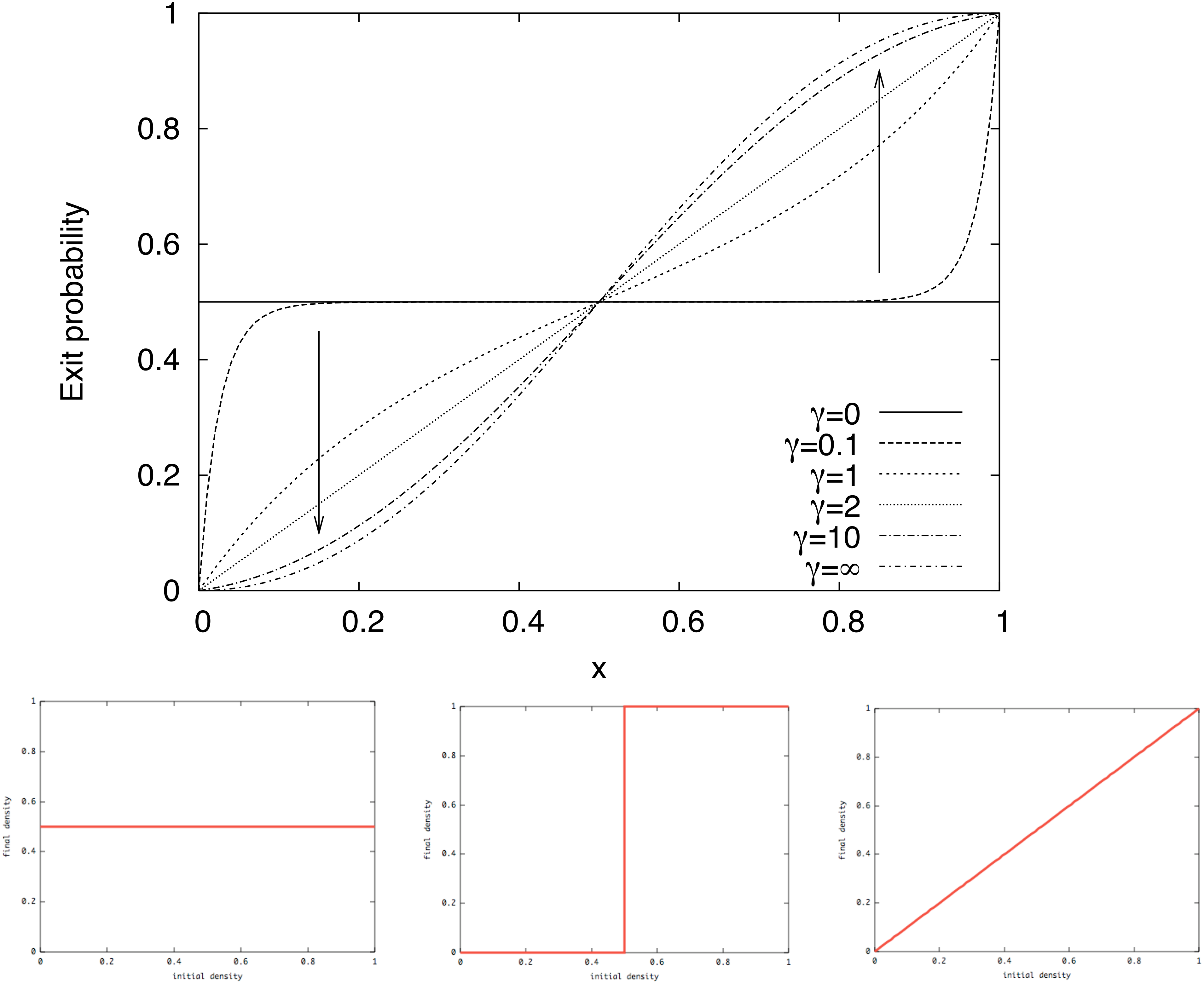
$$\frac{\partial s_j}{\partial t} = 2\gamma(s_{j+1} + s_{j-1}) - 2(\gamma + 2)s_j - 2(\gamma - 2)\langle \sigma_{j-1}\sigma_j\sigma_{j+1}\rangle$$

$$s_j \equiv \langle \sigma_j \rangle = \sum_{\{\sigma\}} \sigma_j P(\{\sigma\}; t)$$

Coupling with higher order correlations
Need for a decoupling scheme

Non-trivial initial state dependence:

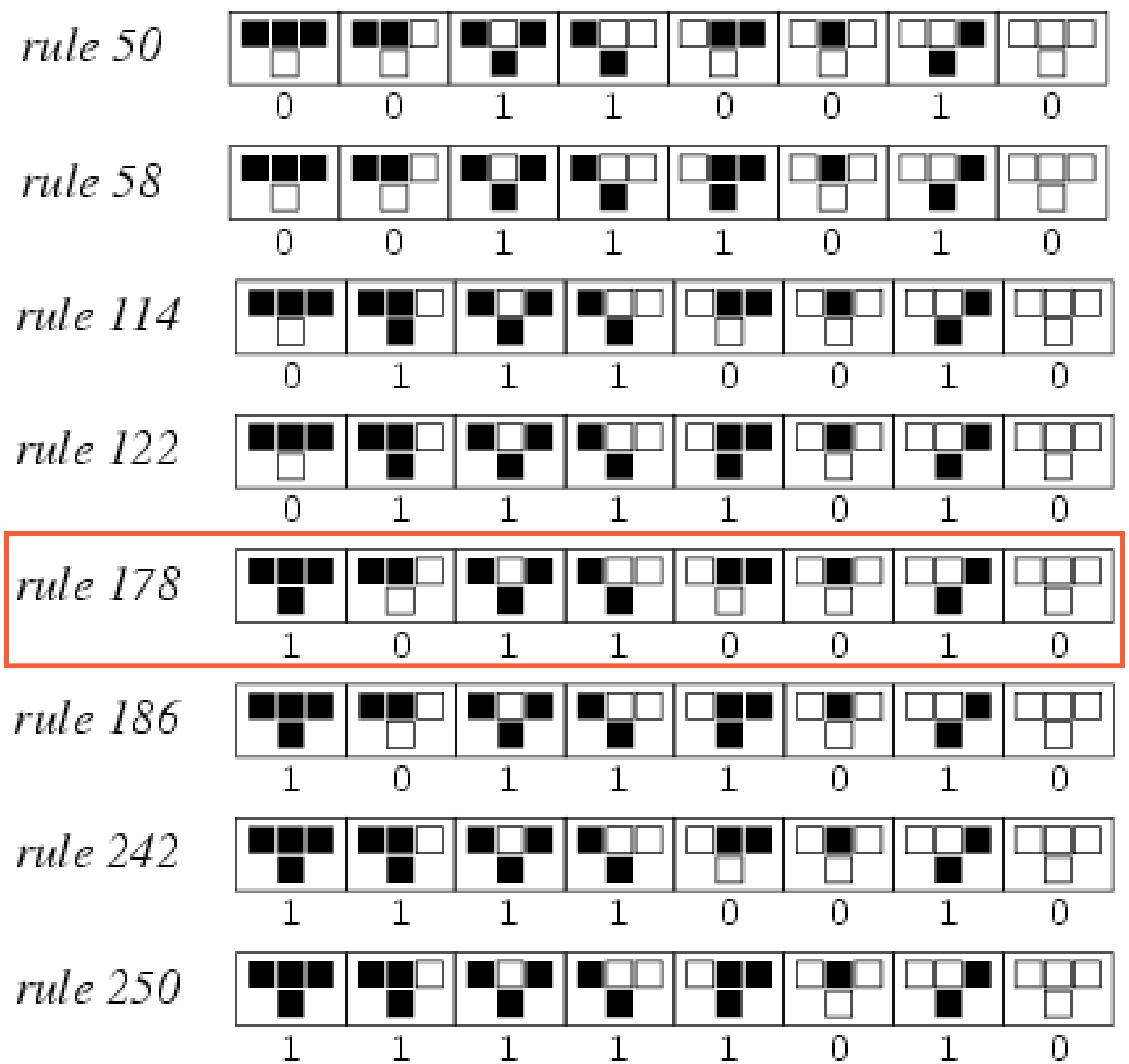
$$m(\infty) = m(0)\, e^{(2-\gamma)/[2\gamma\,(m(0)^2 - 1)]}$$



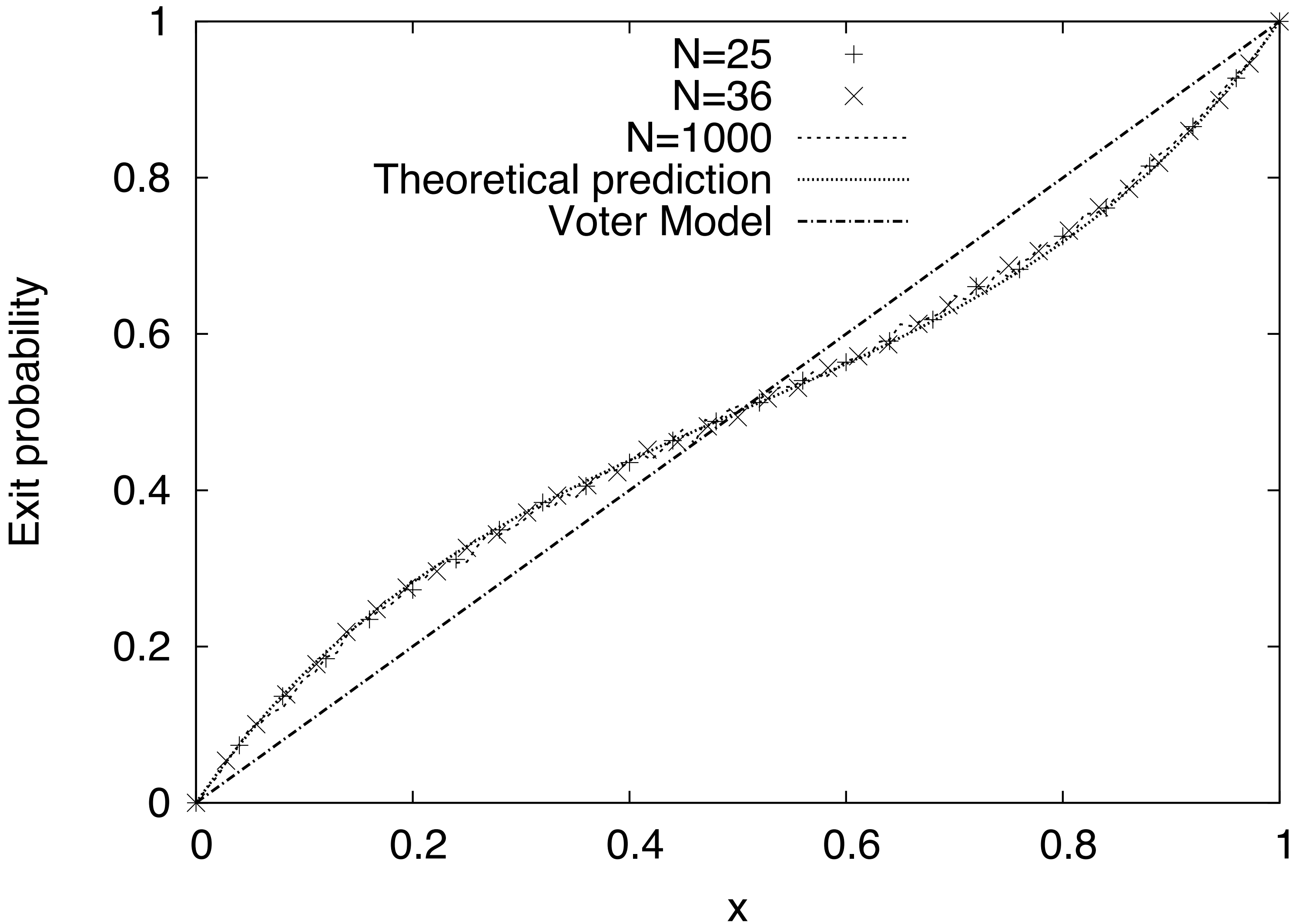
Exit probability
x
γ=0
γ=0.1
γ=1
γ=2
γ=10
γ=∞
final density
initial density

$\gamma = 1$

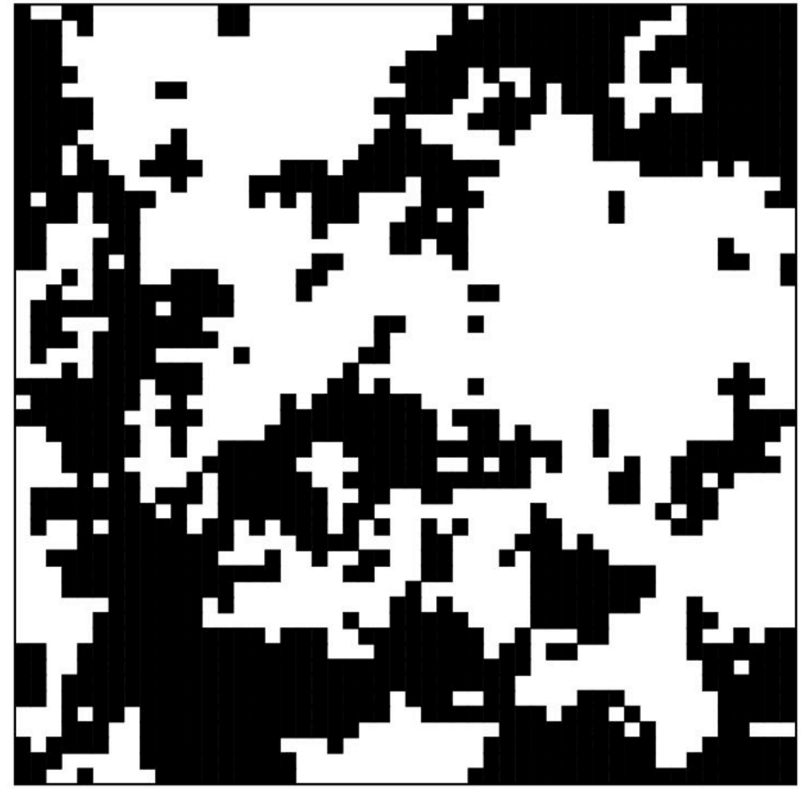
The *vacillating* voter model where voters change opinion at a fixed rate if either 1 or 2 of their neighbors disagree with them, equivalent to rule 178 of the one-dimensional cellular automaton (see Wolfram).



The probability to reach the final state of +1 consensus has a non-trivial initial state dependence.



Vacillating Voter Model



Voter Model

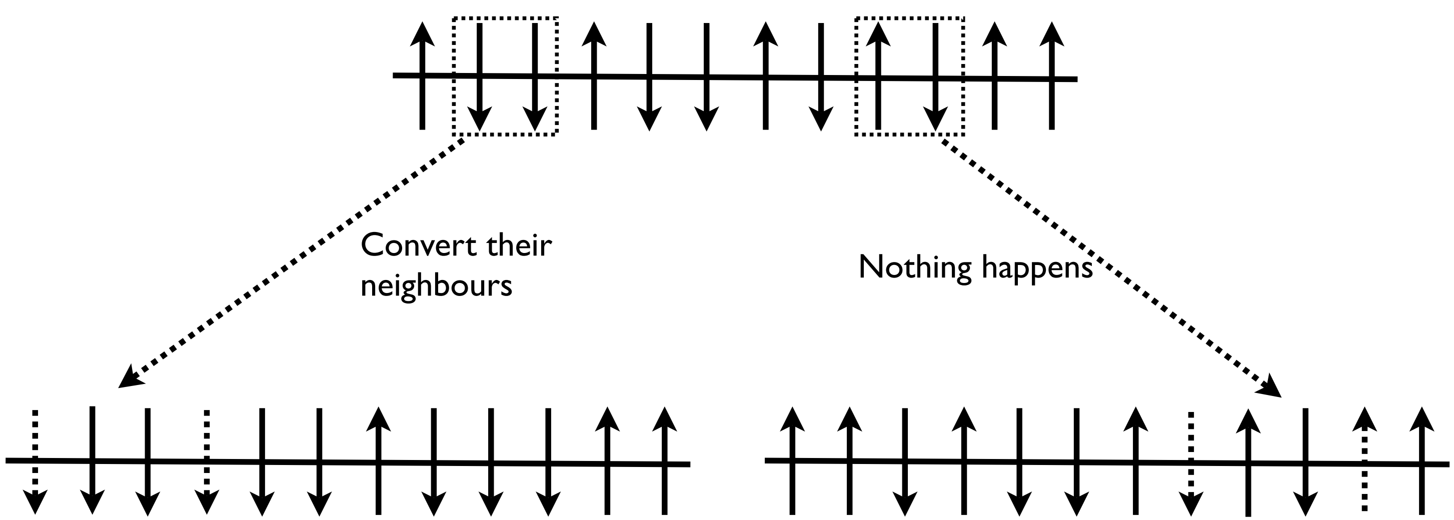
$$C_1 \equiv \langle \sigma_{i,j} \sigma_{i,j+1} \rangle \rightarrow 0.31$$

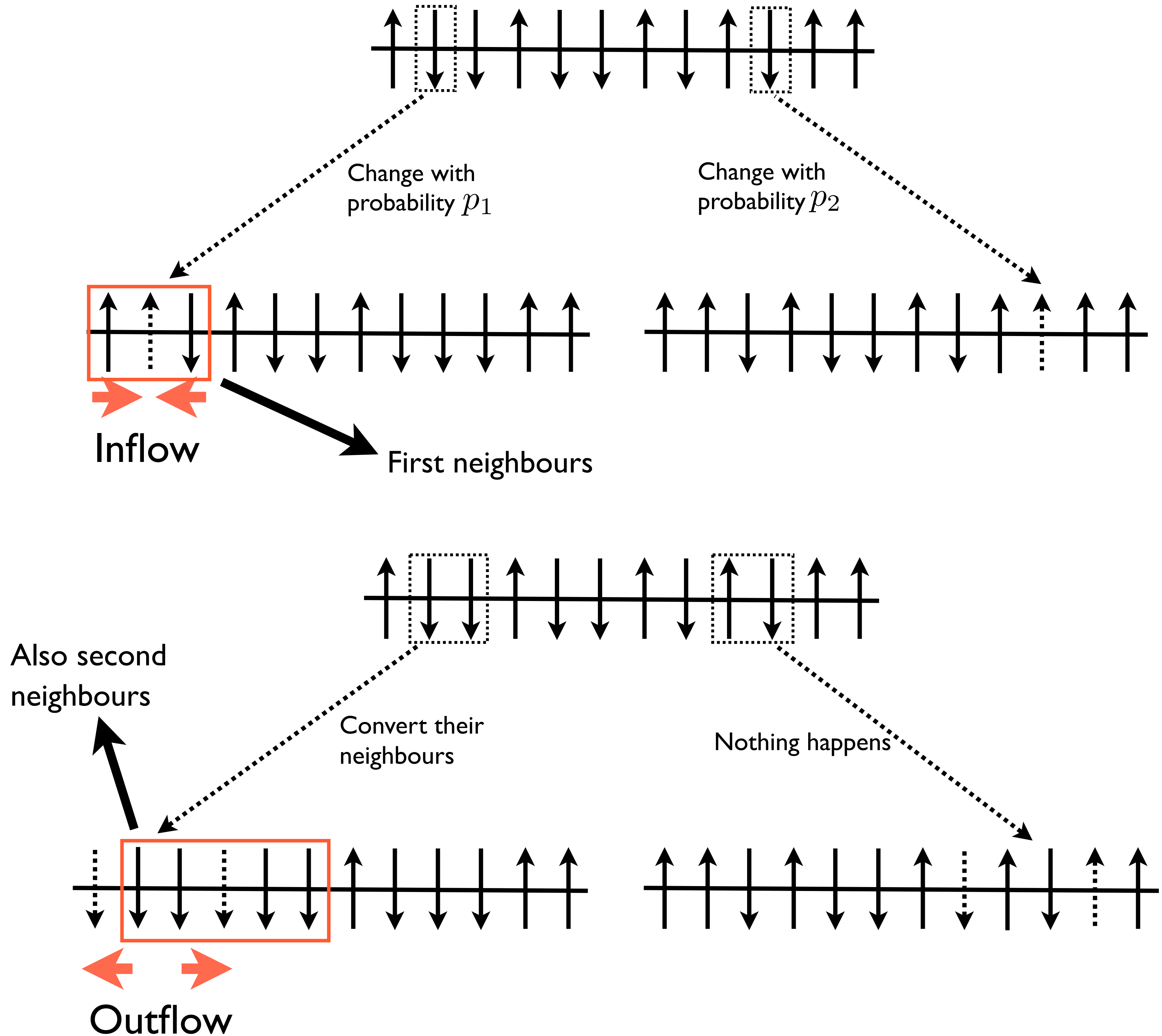
→ domains of opposite opinions coexist

The approach developed above can also be applied to another simple opinion dynamics model in one dimension, namely, the Sznajd model.

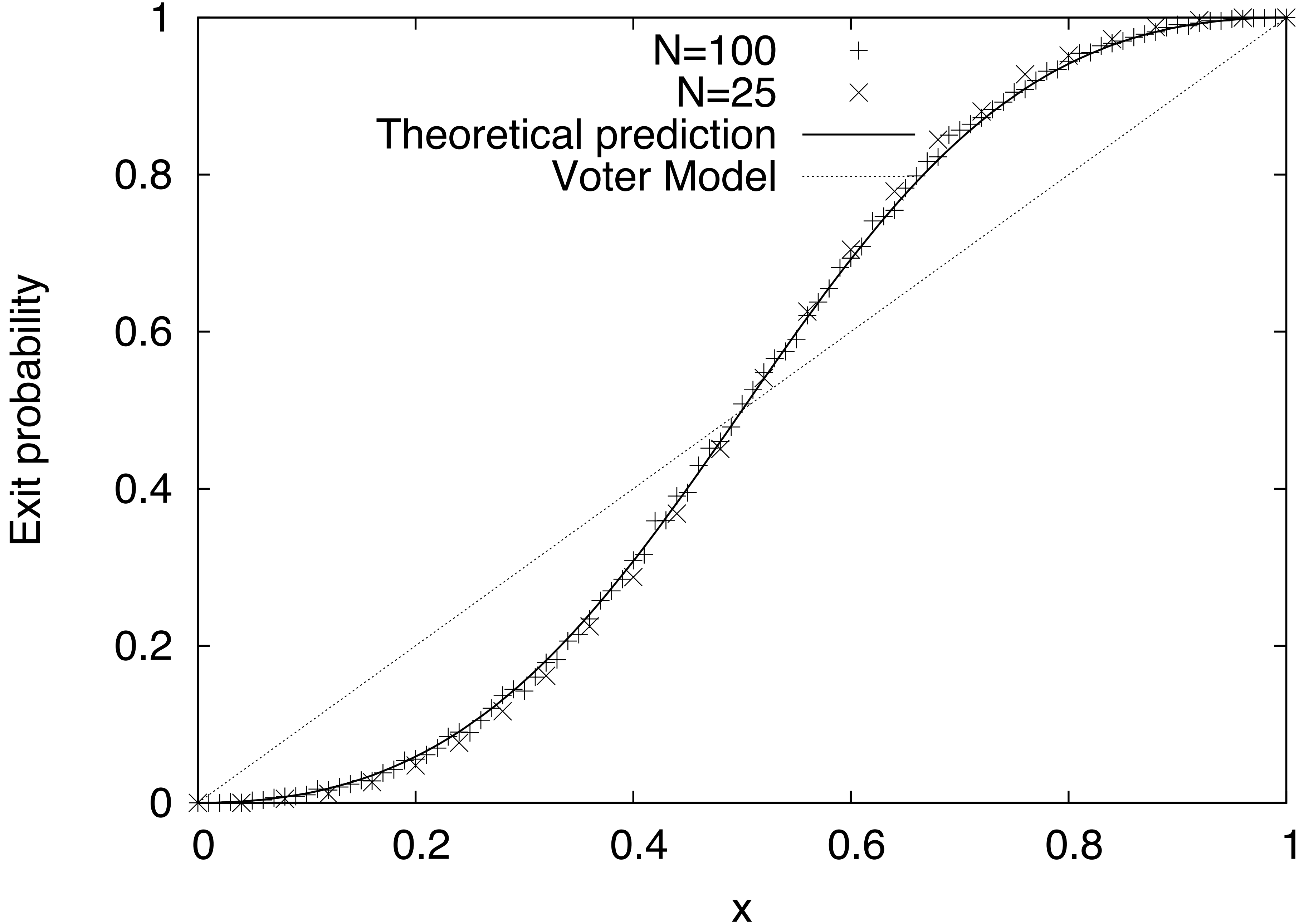
Social validation: agents are only influenced by groups (e.g. pairs) of aligned voters and not by single individuals.

The Sznajd is defined by the following evolution rule:
(i) pick a pair of neighboring voters;
(ii) if these voters have the same opinion, convert the opinion of the neighbors on either side of the initial pair;
(iii) repeat these steps *ad infinitum* or until a finite system necessarily reaches consensus.





Change with probability p_1
Change with probability p_2
Inflow
First neighbours
Also second neighbours
Convert their neighbours
Nothing happens
Outflow


N=100
N=25
Theoretical prediction
Voter Model
Exit probability
x
0
0.2
0.4
0.6
0.8
1

Role of communities: Coupled Random Networks

Distinct communities within networks are defined as subsets of nodes which are more densely linked when compared to the rest of the network.

The network is composed of N nodes divided into two types of nodes, 1 and 2. Different types of nodes have a probability p_{cross} to be linked, while nodes of the same type have a probability p_{in} to be linked.

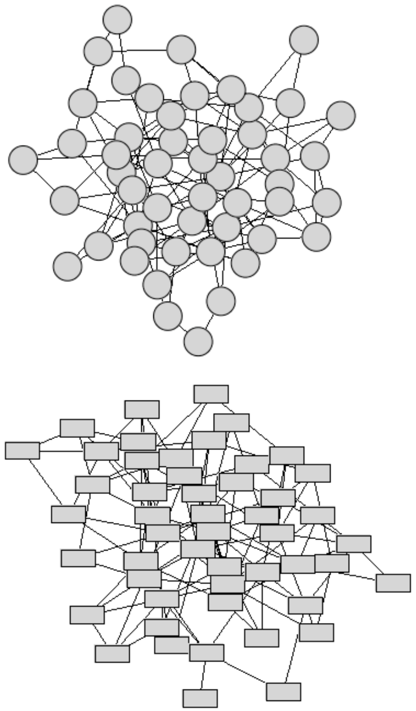
The inter-connectivity between the communities is tunable through the parameter

$$\nu = p_{cross}/p_{in}$$

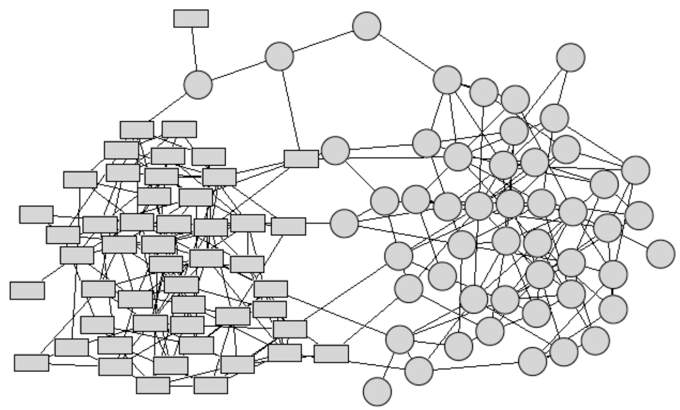
Coexistence of opposite opinions in a network with communities, R. Lambiotte and M. Ausloos, *JSTAT*, P08026 (2007)

Majority Model on a network with communities, R. Lambiotte, M. Ausloos and J. Holyst, *Phys. Rev. E*, 75 (2007) 030101(R)

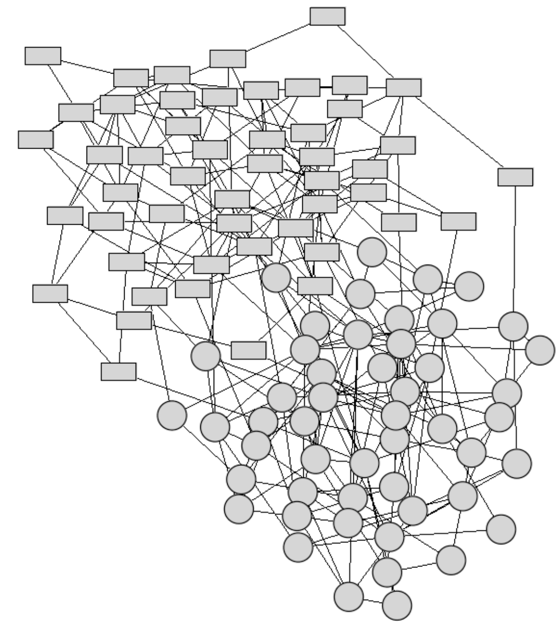
Coupled Random Networks



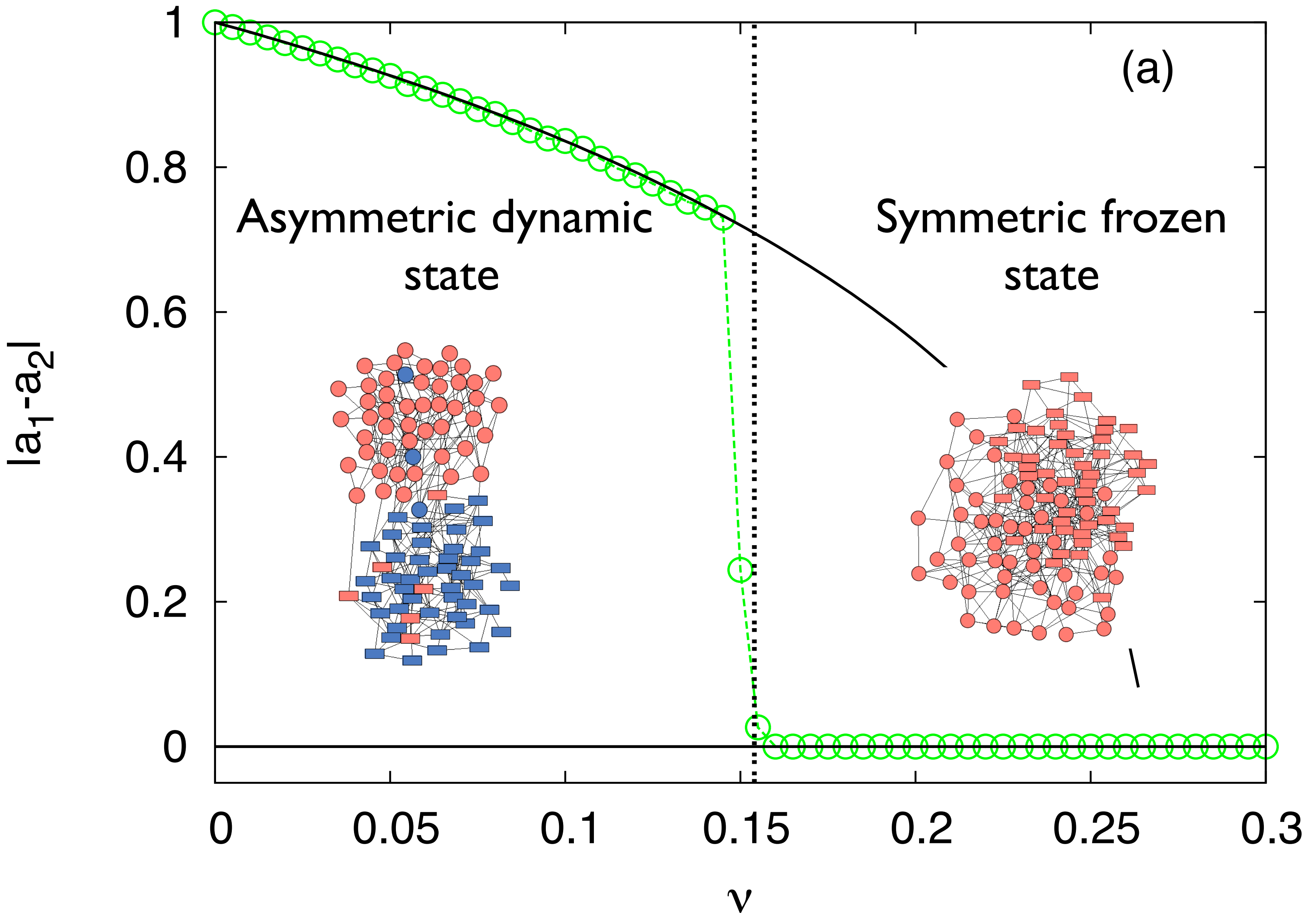
$\nu = 0$



$\nu = 0.05$



$\nu = 0.1$



(a)
Asymmetric dynamic state
Symmetric frozen state
|a1-a2|
ν
1
0.8
0.6
0.4
0.2
0
0
0.05
0.1
0.15
0.2
0.25
0.3