Dynamics of opinions in social systems

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100 years of living science

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4. Presence of communities

Claudio Castellano, Santo Fortunato, Vittorio Loreto, *Statistical physics of social dynamics,* Eprint arXiv: 0710.3256
M. Buchanan, *The social atom* (CYAN 2007)

Statistical physics and social systems

Complex Patterns, collective phenomena observed in social systems originate from the interactions between the individuals.

Complexity does not necessarily originate from the complexity of each individual, but from the interactions between even “simplistic” models of individuals

- crowd dynamics

Pedestrian streams (lane formation in a street), panic

Figure 89 - The officers (bottom left) affect the crowd - leaving unused space.

- traffic

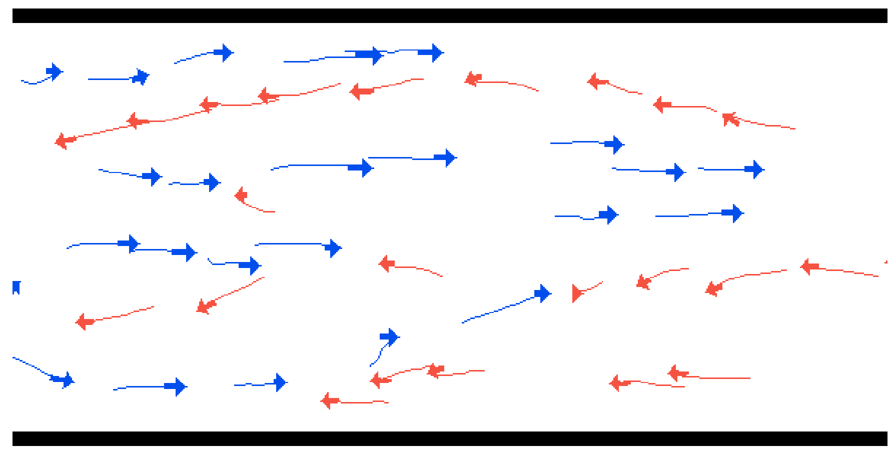
Traffic jam, Phantom Traffic Jams

- “ideas”

sudden shifts of opinions, fashions

Crow dynamics and traffic: direct relation with physics: position, velocity, local interactions

Figure 89 - The officers (bottom left) affect the crowd - leaving unused space.



Microscopic scale

Individual motion of the individuals

Atoms

→

Mescopic scale

Distribution of positions and velocities

Boltzmann equation

→

Macroscopic scale

Local densities

Hydrodynamics

Tools from statistical physics (! systems are usually non-conservative)

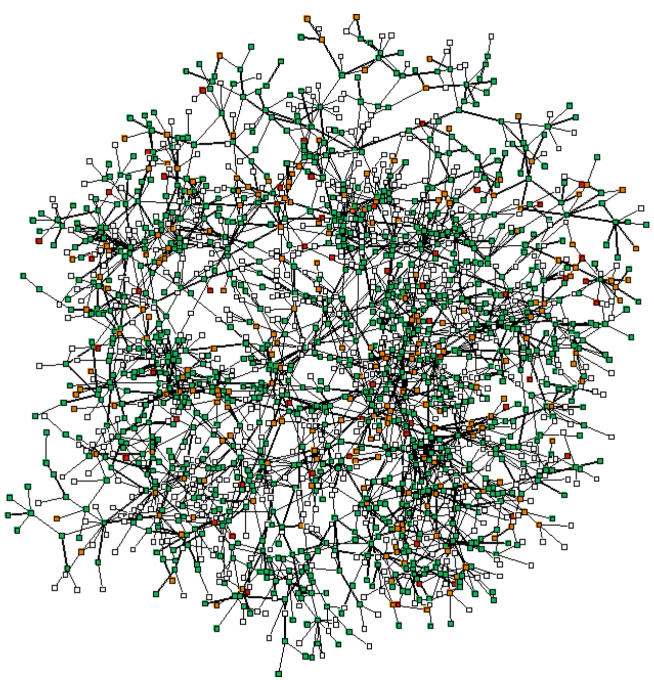
Opinion formation: analogy with spin systems

The opinion of an agent may take one among several values, the simplest case being the binary +-1, which can be viewed as the answer yes/no to a question. The opinion of each agent is then determined by the opinions of its friends on the network, in order to account for the peer pressure on this agent.

1) the dynamics takes place is not planar, but that it takes on a complex network

Small-world, high degree heterogeneity, communities, etc.

Social network plays a fundamental role as a medium for the spread of INFLUENCE among its members: Opinions, ideas, information, innovation…



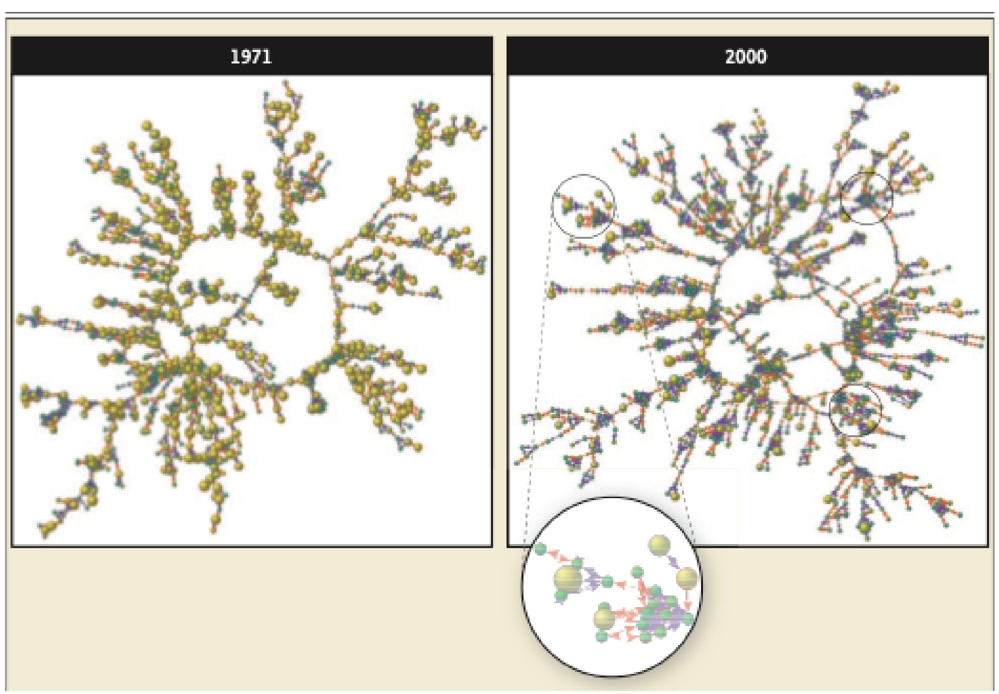
2000 users from a Belgian
Mobile Phone Network
1 day of communication
~ 1 million active users

2) Difficulty to perform experiments in order to validate the models: opinion AND social network

The Collective Dynamics of Smoking in a Large Social Network

12,067 people from 1971 to 2003 (Framingham Heart Study).
Who are your friends, colleagues?
Do you smoke?

The percentage of smokers decreased from 43% to 21%
Cascades inside communities
Segregation: the smokers lose their links the non-smokers



Online social networks offer promising perspectives

The Dynamics of Viral Marketing, Jure Leskovec , Lada A. Adamic ,Bernardo A. Huberman

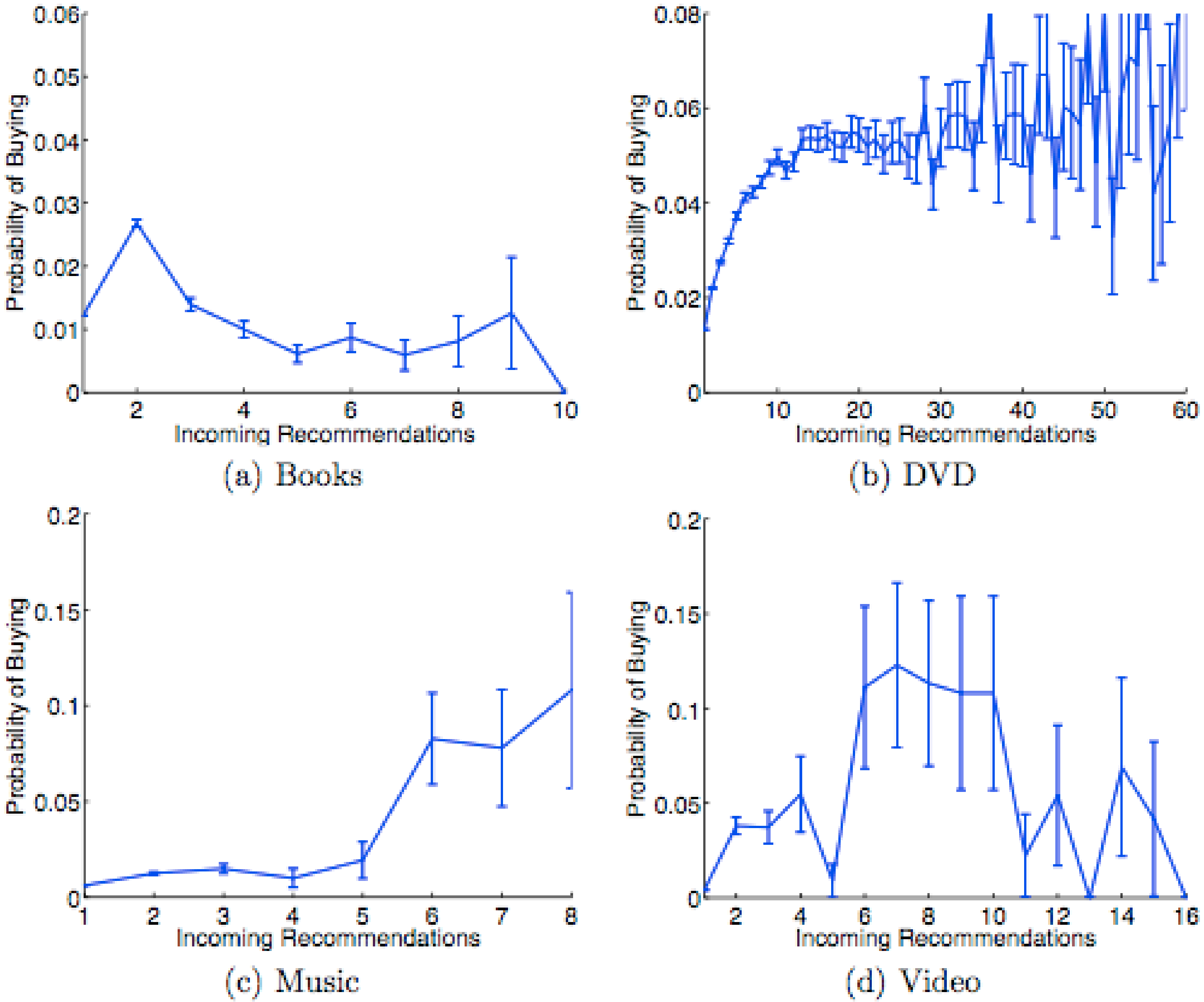


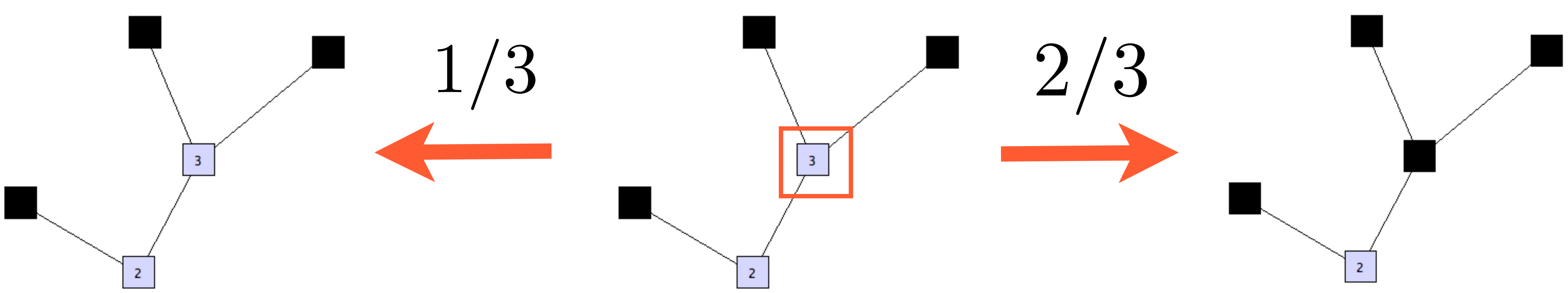
Figure 8: Probability of buying a book (DVD) given a number of incoming recommendations.

Voter Model

Simplest interaction possible: imitation. People copy the behaviour of their friend, acquaintances, neighbours, etc.

N agents have an opinion: -1 or 1

The population evolves by:
(i) picking a random voter
(ii) the selected voter adopts the state of a randomly-chosen neighbor
(iii) repeating these steps *ad infinitum* or until a finite system necessarily reaches consensus.



Time evolution the density *x* of +1voters in the MF (random network)

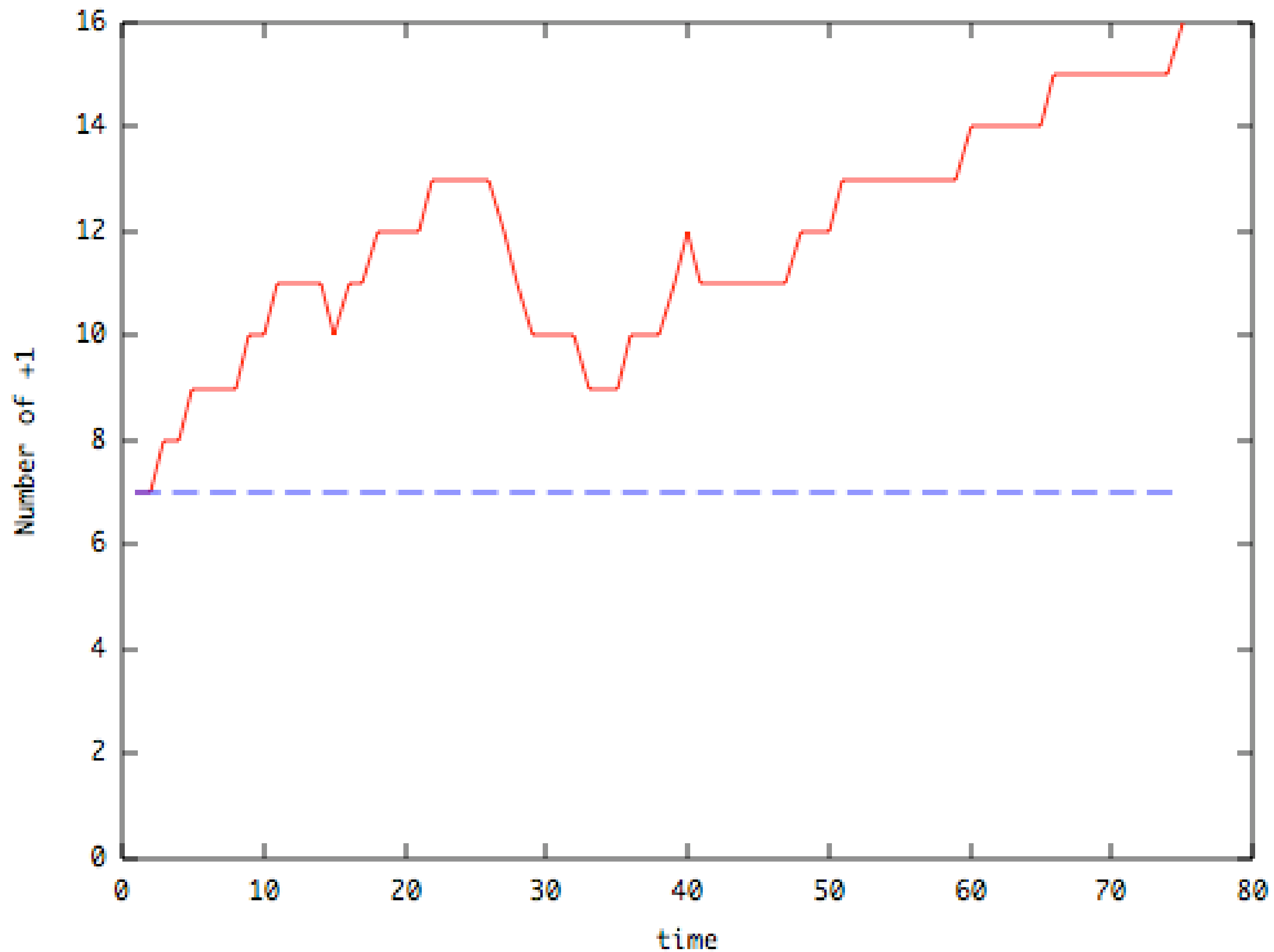
$$\partial_t x = -x(1-x) + (1-x)x = 0$$

- x: The selected node is +1
- $(1-x)$: The neighbour node is -1
- $(1-x)$: The selected node is -1
- x: The neighbour node is +1

$$m \equiv x - (1-x) = 2x - 1$$

=> The average magnetization is conserved

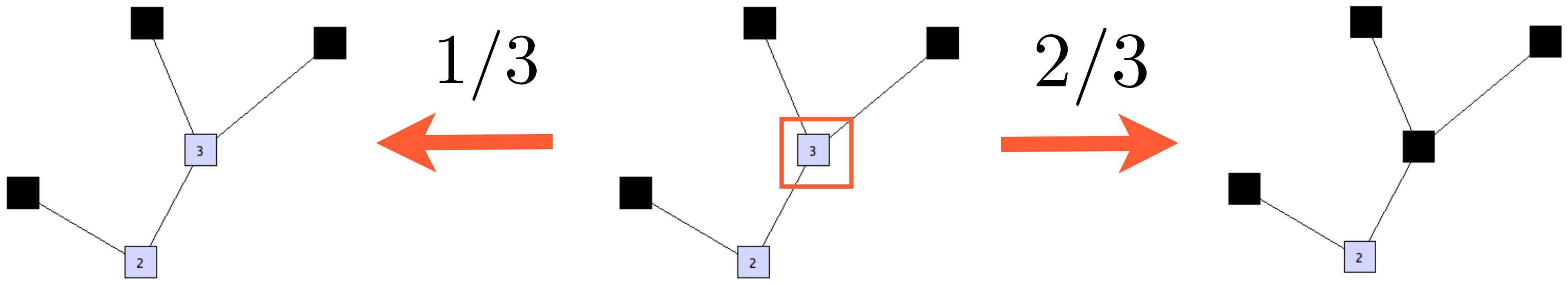
The dynamics is stochastic



What is the probability to reach +1 consensus as a function of the initial condition?

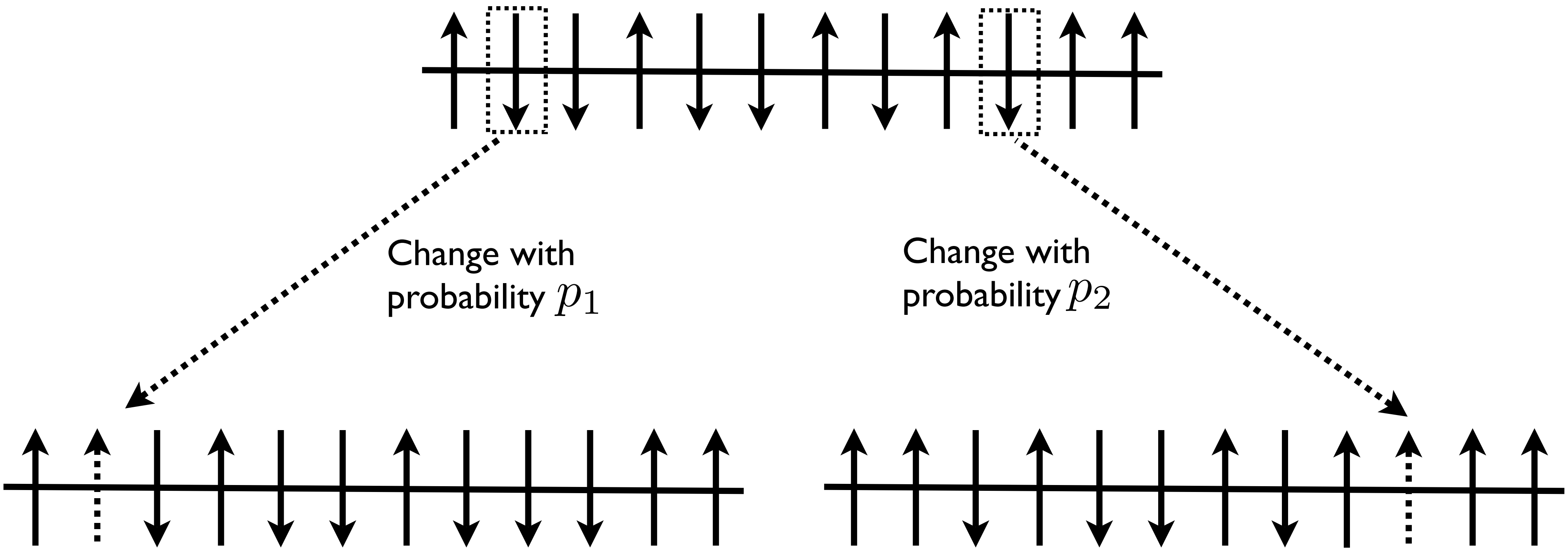
What's the exit time to reach consensus?

Voter Model



With this dynamics, the probability to switch state is the fraction of people who disagree with you.

Non-conservative Voter models



$p_2 = 2p_1$ Voter model

$$\gamma = p_2/p_1$$

! voters change opinion only when in contact with another opinion

Dynamics of Vacillating Voters, R. Lambiotte and S. Redner, *JSTAT*,(2007) L10001
Dynamics of non-conservative Voters, R. Lambiotte and S. Redner, *EPL* **82** (2008) 18007

$\gamma = 2$ one recovers the classical voter model

$\gamma > 2$ the combined effect of two neighbors is more than twice that of one neighbor. Equivalently, voters can be viewed as having a conviction for their opinion and strong peer pressure is needed to change their opinion.

$\gamma \to \infty$ voters only change opinion when are confronted by a unanimity of opposite-opinion voters

$\gamma < 2$ one disagreeing neighbor is more effective in triggering an opinion change than in the classical voter model.

$\gamma = 1$ one recovers the *vacillating* voter model where voters change opinion at a fixed rate if either 1 or 2 of their neighbors disagree with them.

$\gamma < 1$ *contrarian* regime where a voter is less likely to change opinion as the fraction of neighbors in disagreement increases.

γ *conviction parameter*

Unanimity Rule on networks, R. Lambiotte, S. Thurner and R. Hanel, Phys. Rev. E, **76** (2007) 046101

Let us first consider first the mean-field limit (the spins of neighboring nodes are uncorrelated ~ random network).

$$\frac{\partial m}{\partial t} = 2(\gamma - 2)(m - m^3)$$

$$m \equiv x - (1 - x) = 2x - 1$$

where m is the average magnetization (opinion) and x is the density of +1voters

Qualitative change at $\gamma = 2$

$\gamma > 2$ Population is driven toward consensus

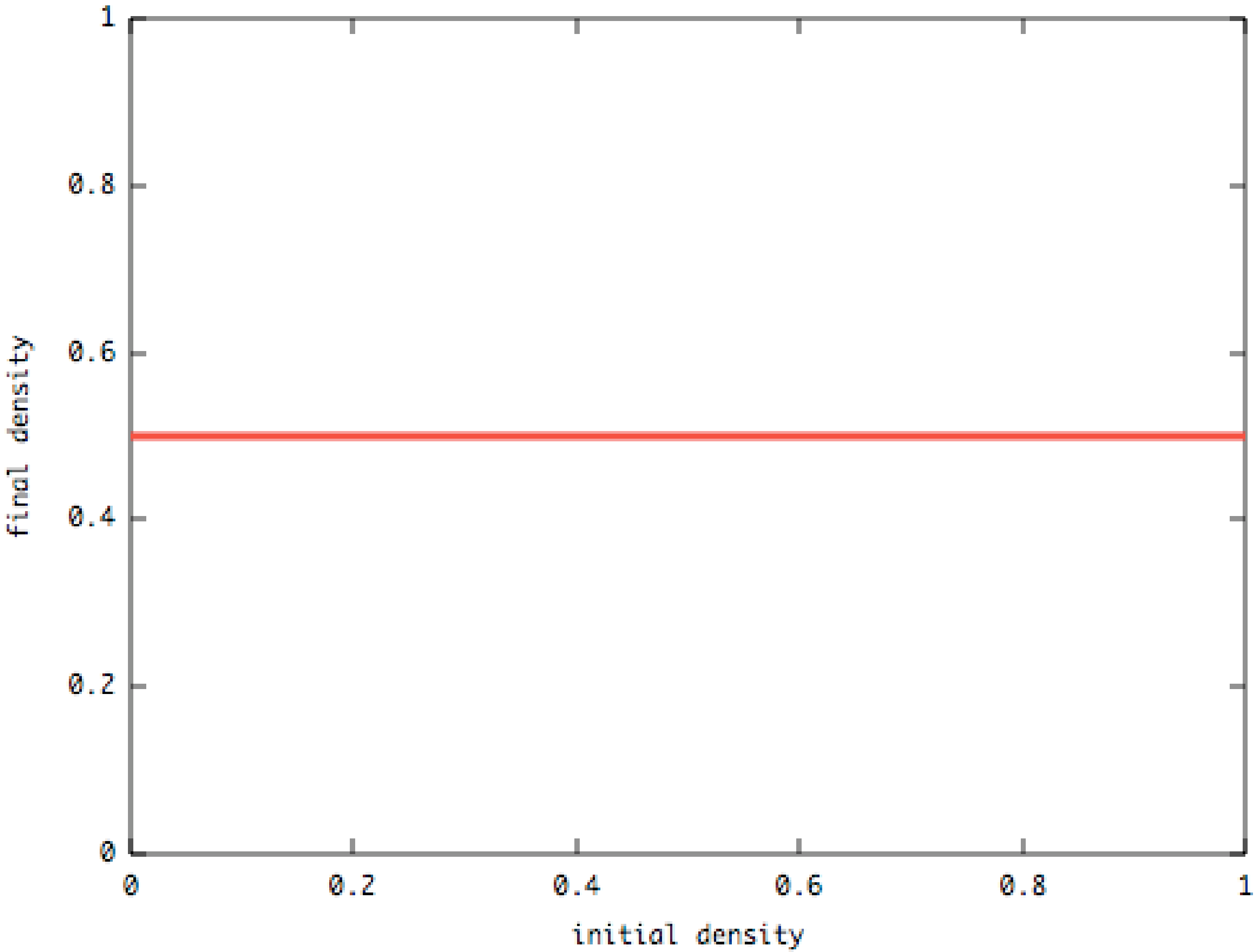
$m = [-1, 1] \quad x = [0, 1]$

$\gamma < 2$ Population is driven toward zero-magnetisation

$m = 0 \quad x = 1/2$

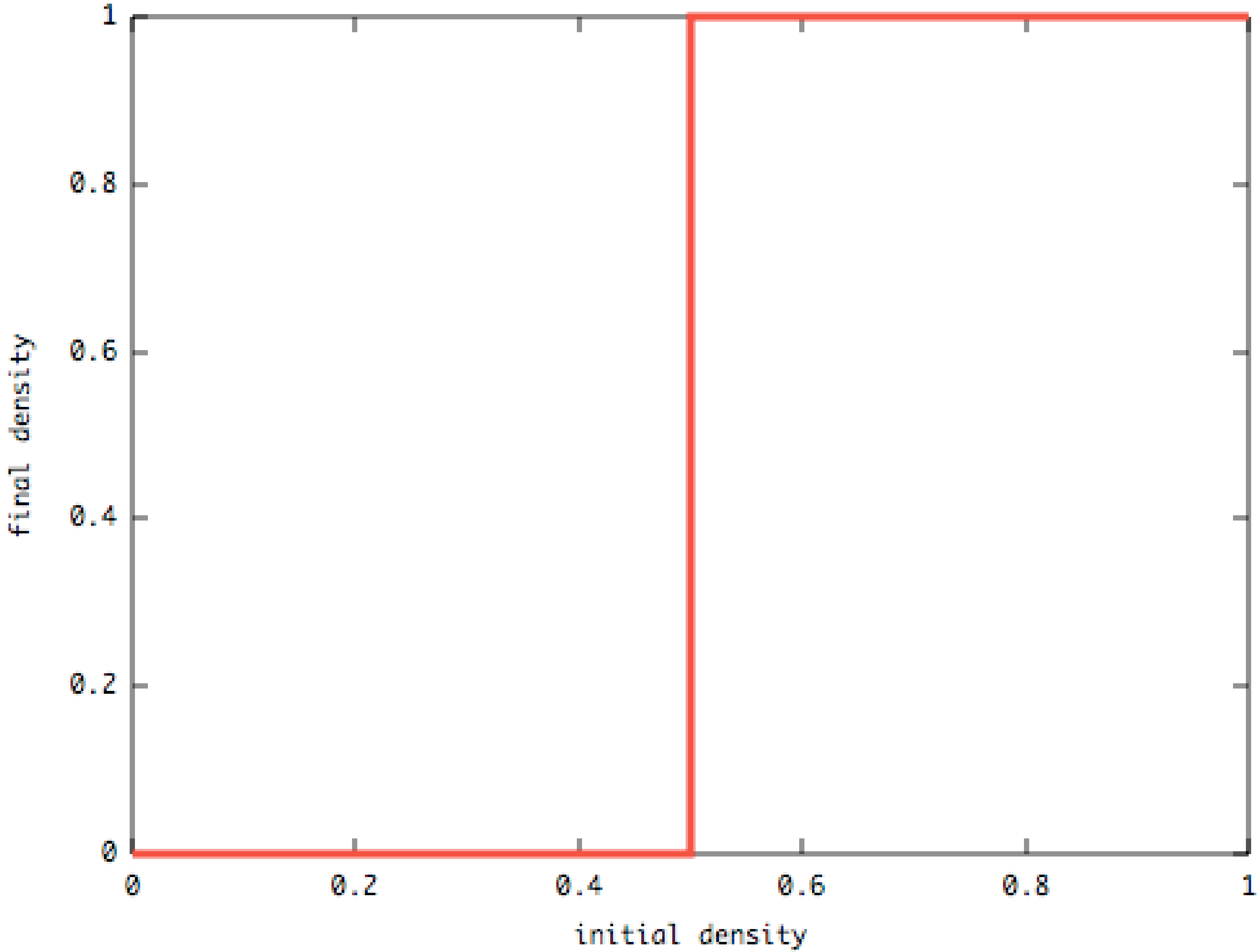
Dependence on the initial conditions

$$\gamma < 2$$



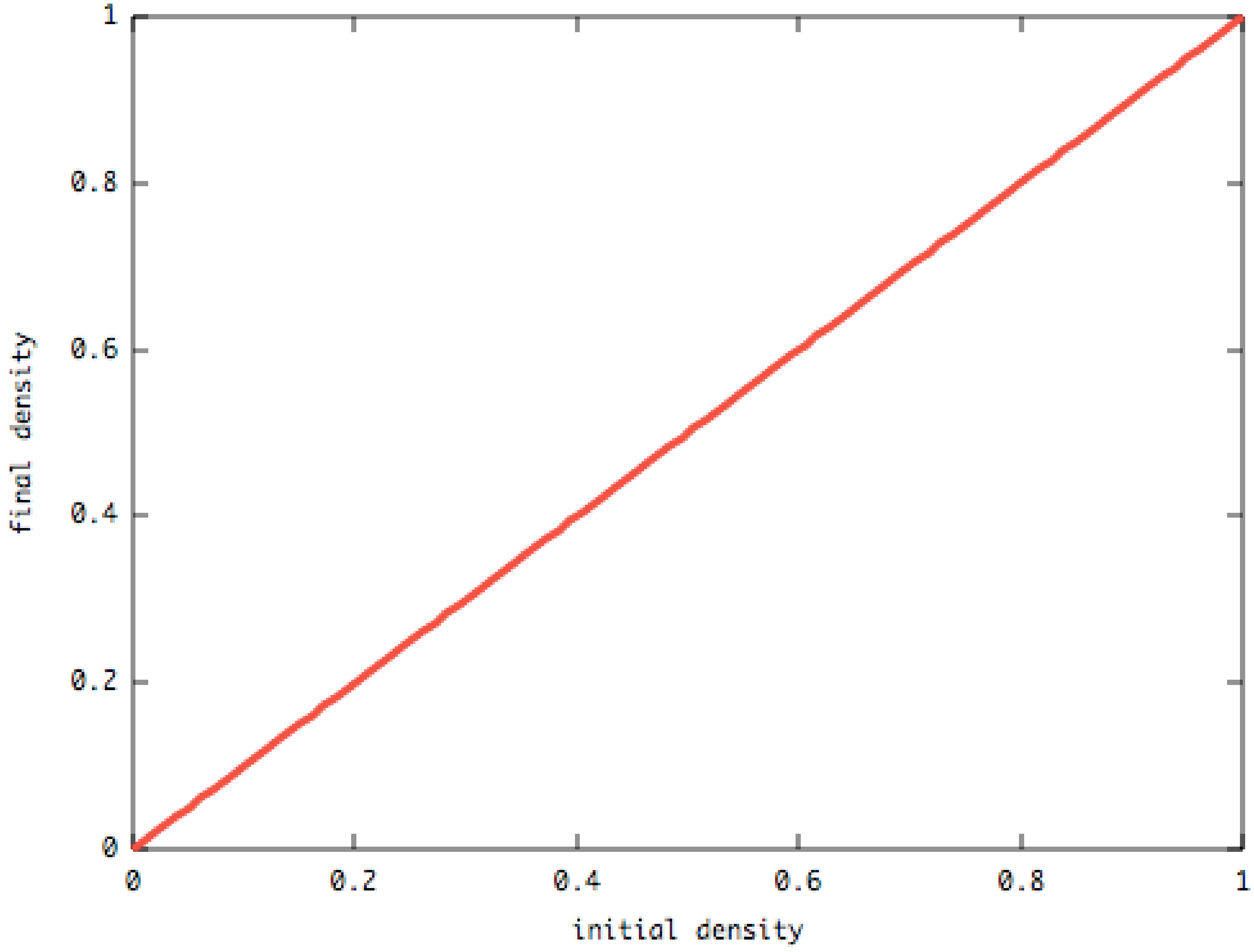
Dependence on the initial conditions

$$\gamma > 2$$

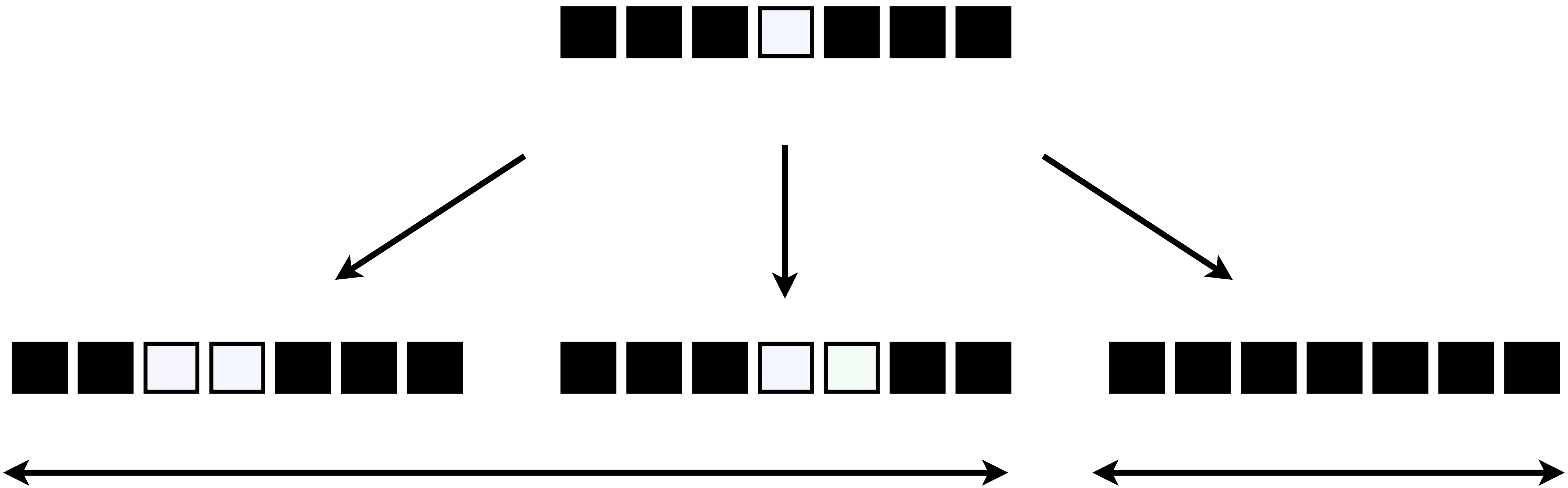


Dependence on the initial conditions

$\gamma = 2$



In one dimension, the system coarsens and the interface between domains performs a symmetric random walk, except when domain walls are adjacent



Repulsion

$2/(2+\gamma)$

Annihilation

$\gamma/(2+\gamma)$

$$\frac{\partial s_j}{\partial t} = 2\gamma(s_{j+1} + s_{j-1}) - 2(\gamma+2)s_j - \boxed{2(\gamma-2)\langle\sigma_{j-1}\sigma_j\sigma_{j+1}\rangle}$$

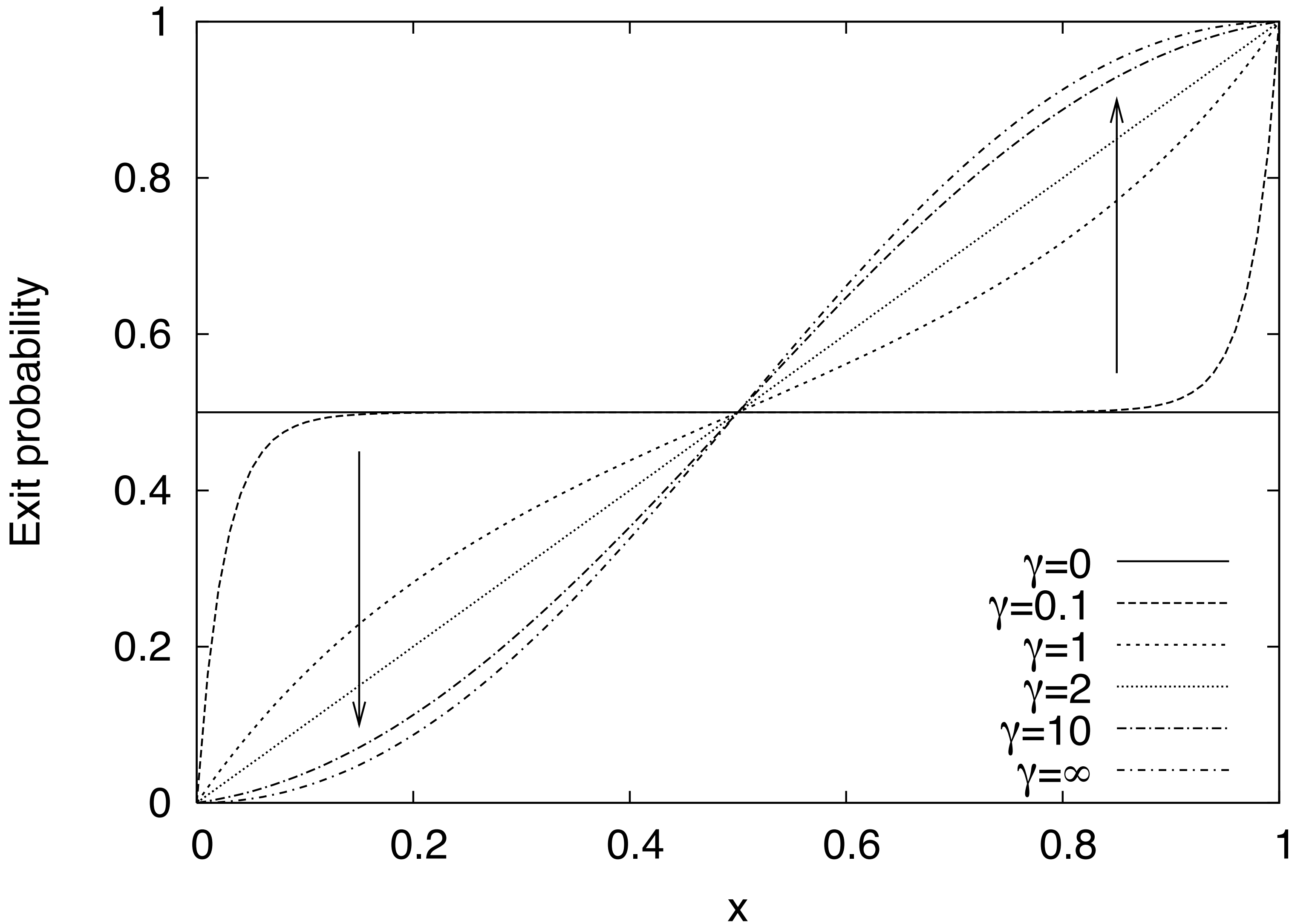
$$s_j \equiv \langle\sigma_j\rangle = \sum_{\{\sigma\}} \sigma_j P(\{\sigma\}; t)$$

Coupling with higher order correlations

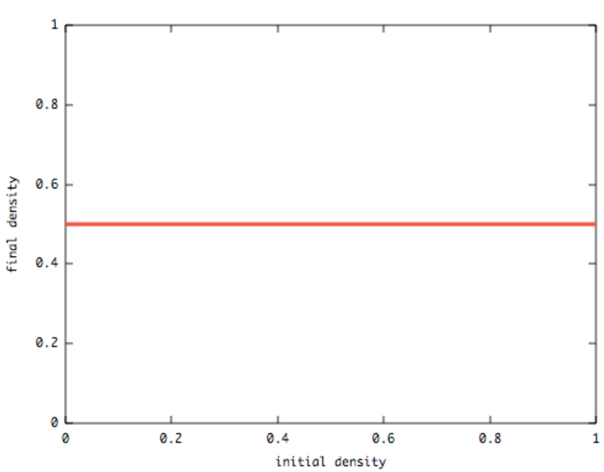
Need for a decoupling scheme

Non-trivial initial state dependence:

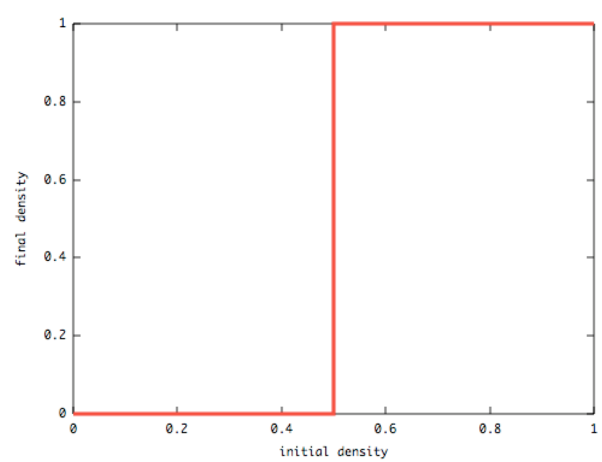
$$\boxed{m(\infty) = m(0)\, e^{(2-\gamma)/[2\gamma\,(m(0)^2-1)]}}$$



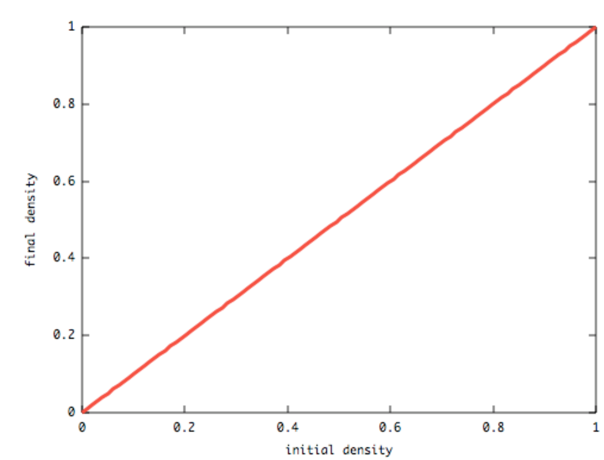
Exit probability
x
γ=0
γ=0.1
γ=1
γ=2
γ=10
γ=∞
0
0.2
0.4
0.6
0.8
1



final density
initial density
0
0.2
0.4
0.6
0.8
1



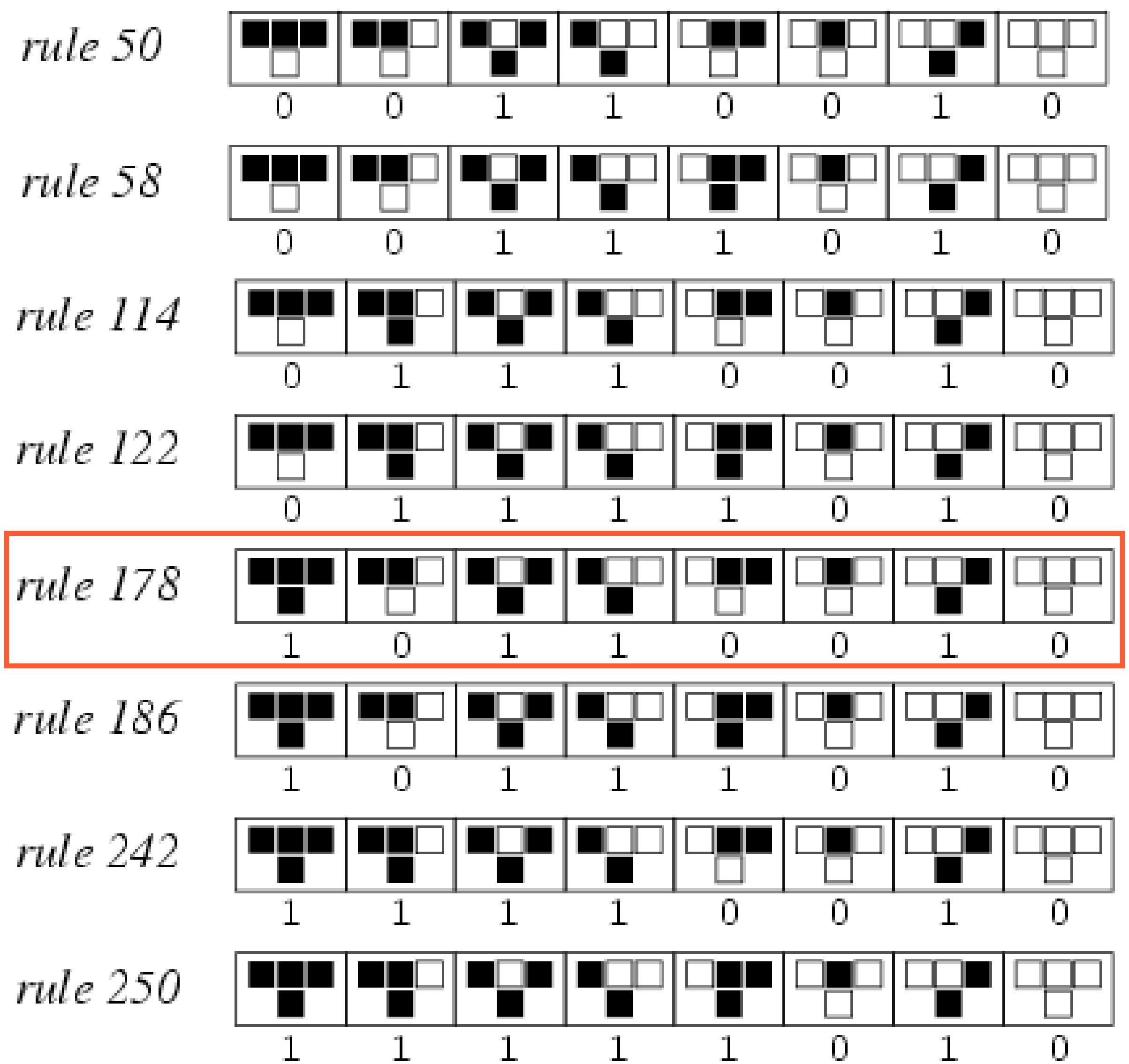
final density
initial density
0
0.2
0.4
0.6
0.8
1



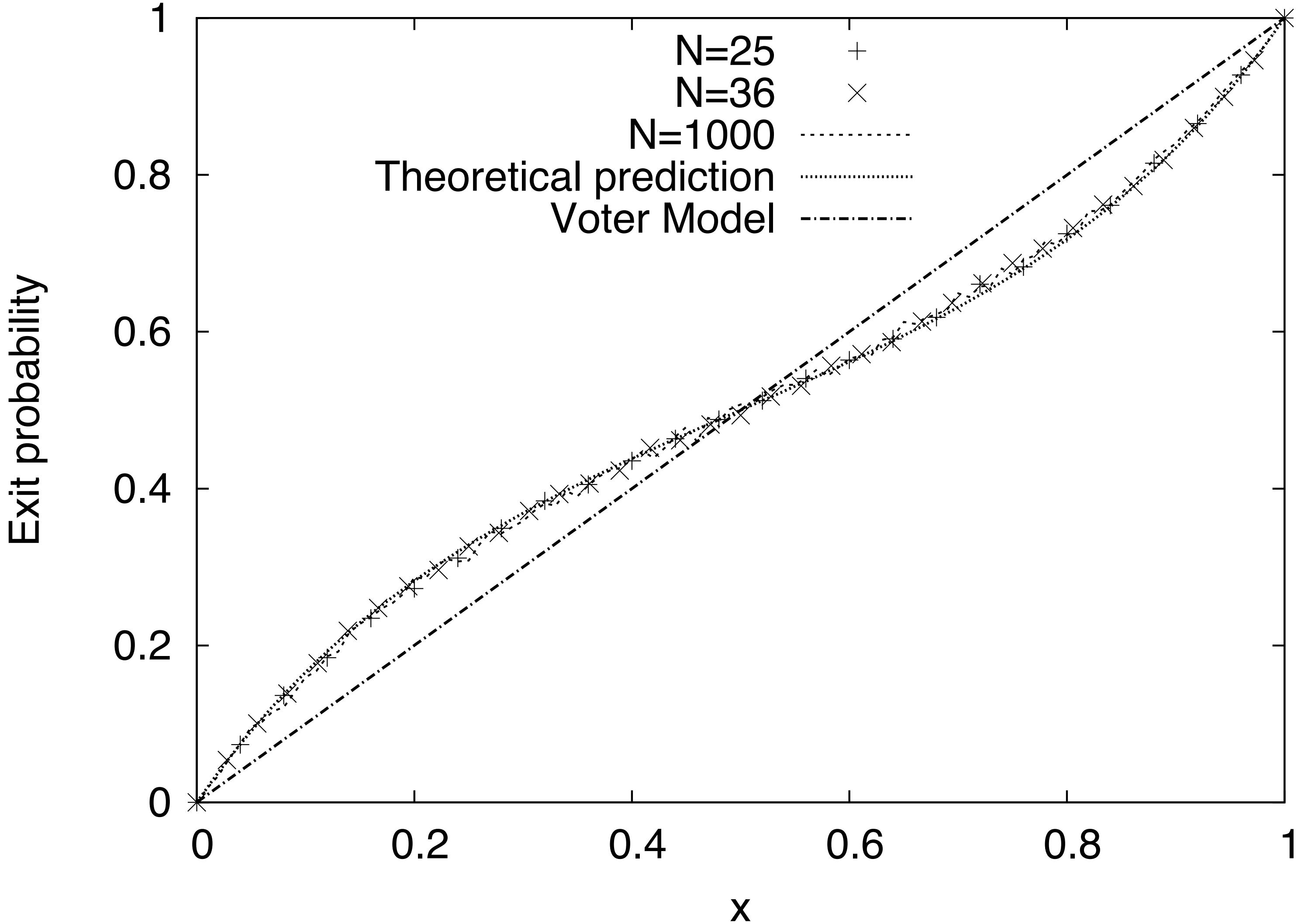
final density
initial density
0
0.2
0.4
0.6
0.8
1

$\gamma = 1$

The *vacillating* voter model where voters change opinion at a fixed rate if either 1 or 2 of their neighbors disagree with them, equivalent to rule 178 of the one-dimensional cellular automaton (see Wolfram).



The probability to reach the final state of +1 consensus has a non-trivial initial state dependence.



Role of Communities: Coupled Random Networks

Distinct communities within networks are defined as subsets of nodes which are more densely linked when compared to the rest of the network.

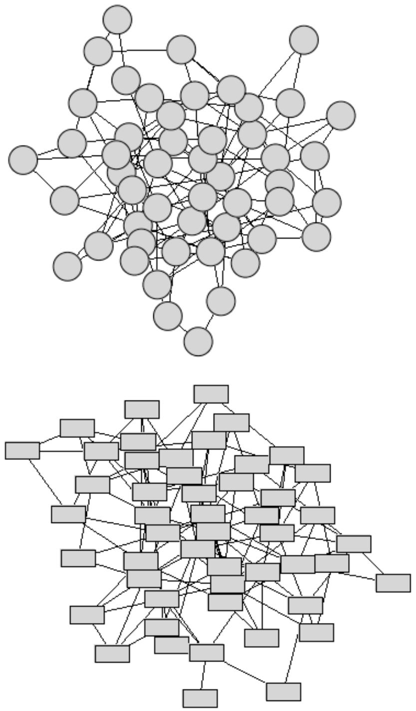
The network is composed of N nodes divided into two types of nodes, 1 and 2. Different types of nodes have a probability p_{cross} to be linked, while nodes of the same type have a probability p_{in} to be linked.

The inter-connectivity between the communities is tunable through the parameter

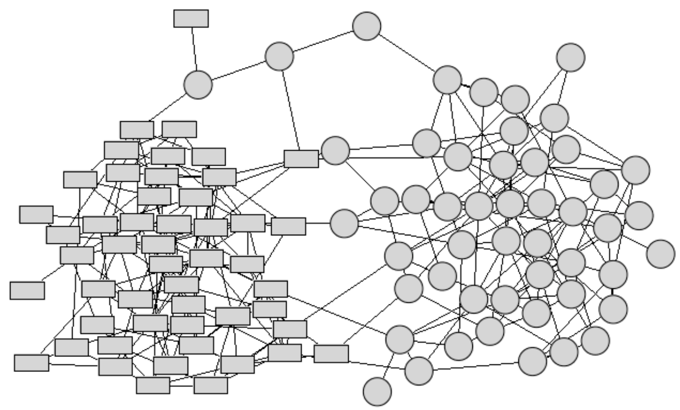
$$\nu = p_{cross}/p_{in}$$

Coexistence of opposite opinions in a network with communities, R. Lambiotte and M. Ausloos, *JSTAT,* P08026 (2007)
Majority Model on a network with communities, R. Lambiotte, M. Ausloos and J. Holyst, *Phys. Rev. E*, **75** (2007) 030101(R)

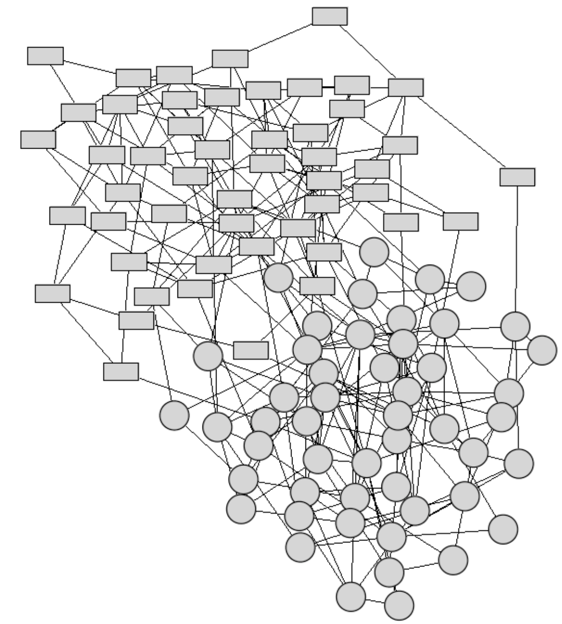
Coupled Random Networks



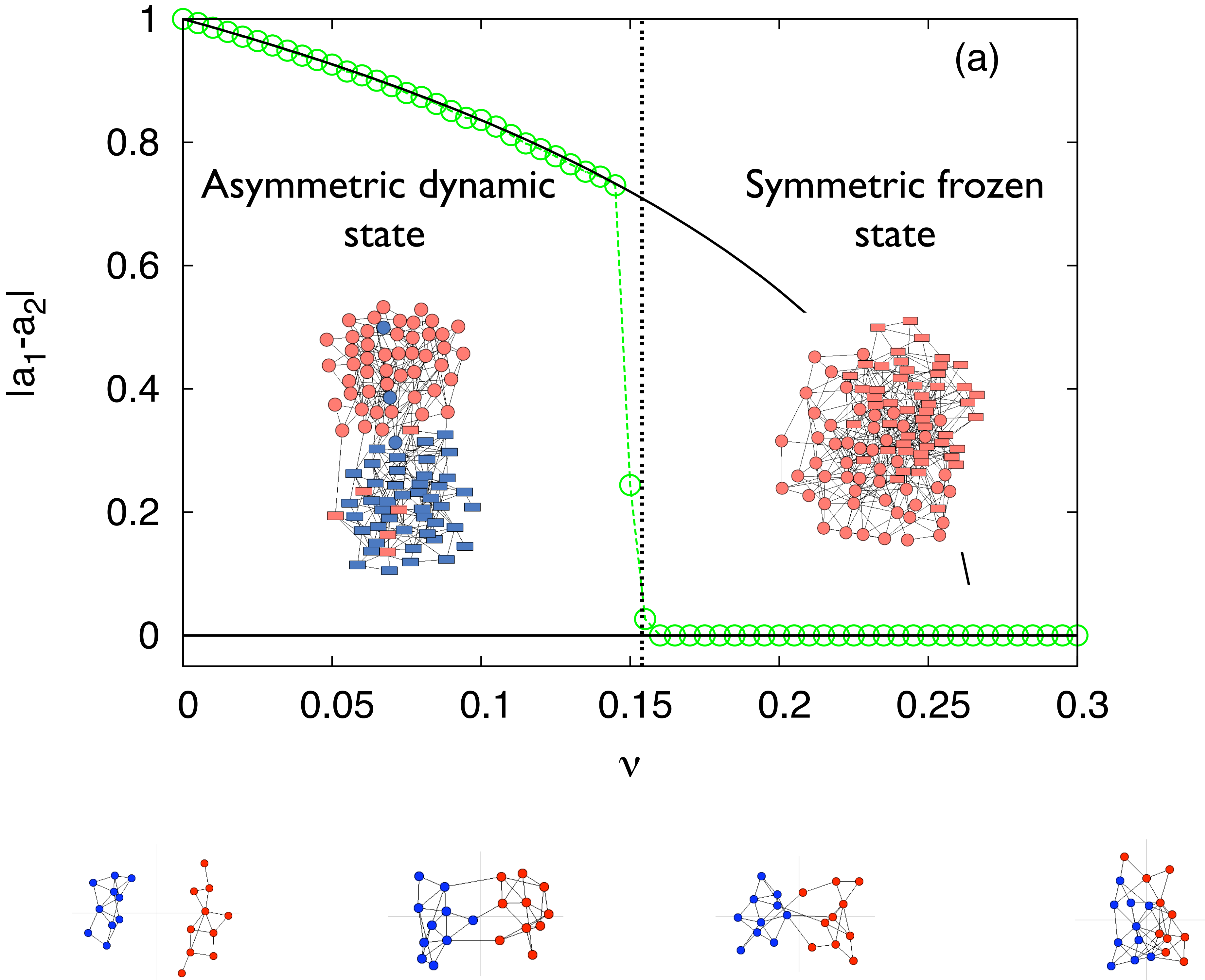
$\nu = 0$



$\nu = 0.05$



$\nu = 0.1$



1
0.8
0.6
0.4
0.2
0
|a1-a2|
0
0.05
0.1
0.15
0.2
0.25
0.3
ν
(a)
Asymmetric dynamic state
Symmetric frozen state

Conclusion

Generalizations of the Voter and the ICM

Imitation/infection depends on the number of neighbours/contacts

Role of the network structure:

Degree Heterogeneity

Presence of communities

Random structure vs. ordered structure (lattice)